BEC/BCS Crossover in strongly interacting systems

Daniel Zabłocki

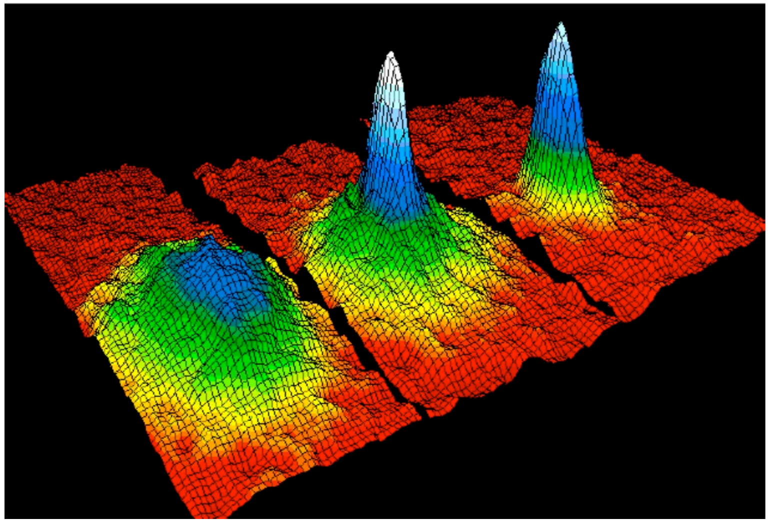
Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Poland
Institut für Physik, Universität Rostock, Germany
Bogoliubov Laboratory for Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia

The Modern Physics of Compact Stars

Երևան (Yerevan), 22 September 2008

in collaboration with David Blaschke and Roberto Anglani

0. background



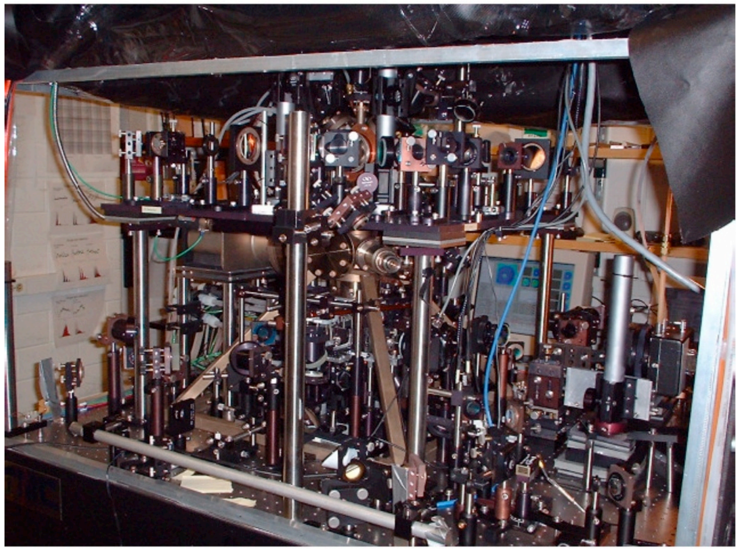
Bose-Einstein Condensation (BEC)
theoretically predicted already in 1924
first observed in 1995 in atomic traps
Nobel price in 2001

Davis et al., Phys. Rev. Lett. **75** (1995) 3969

Tuning of the coupling in (nonrelativistic) low-temperature systems of fermionic atoms with Feshbach resonances in an external magnetic field

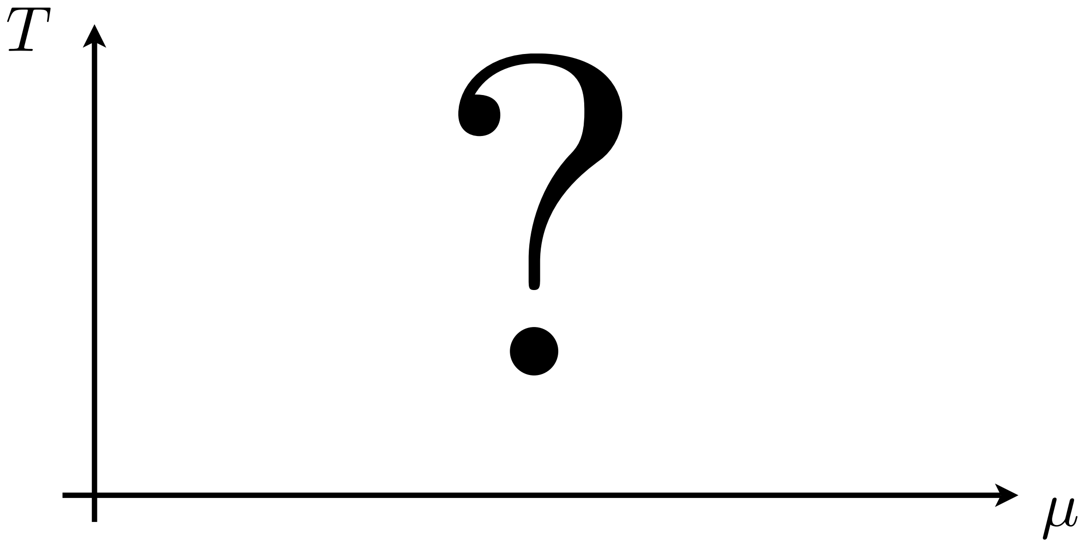
investigate the BEC-BCS crossover in the laboratory

V. Gurarie et al., Ann. Phys. **322** (2007) 2

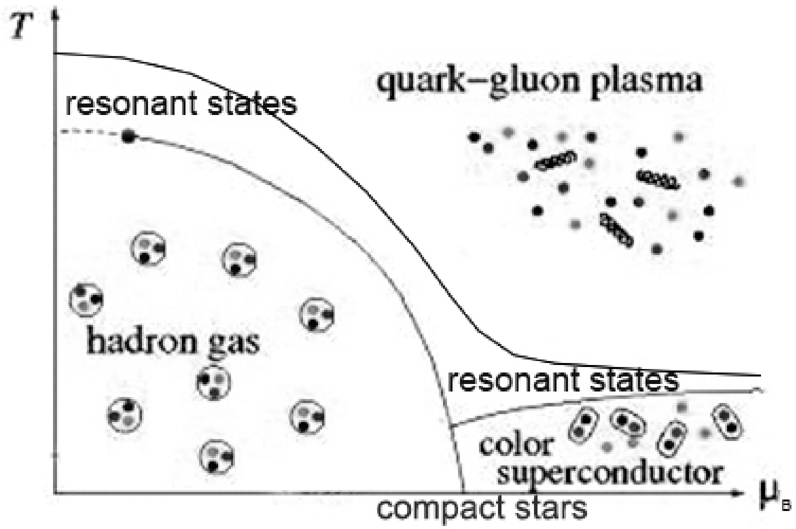


one challenging problem of quantum chromodynamics is

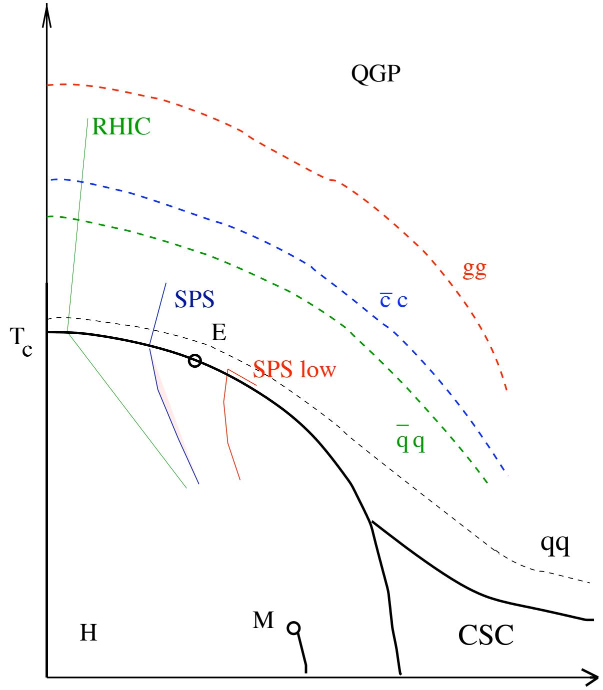
the study of phase diagram



1.1. what we know about phase diagram



Zhuang, P.F. et al. 0710.3634 [hep-ph]



Shuryak and Zahed hep-ph/
0403127

how to give a reliable description in the region around the critical values of chemical potential?

perturbation theory cannot be applied in this region

we have to accept a good compromise.
an effective model:
the Nambu--Jona-Lasinio

The NJL model of QCD mimics the quark-quark interaction mediated by gluons with an effective point-like four fermion interaction

<u>cons</u>

absence of gluon in the Lagrangian;
quarks are not confined (-> PNJL);
etc.

<u>pro</u>

a simple approach to the description of chiral symmetry breaking and phase transitions;
analytical calculations possible

2.2 the starting point: the NJL Lagrangian

For the description of hot, dense Fermi-systems,
with strong short-range interactions we consider a Lagrangian
with internal degrees of freedom (2-flavor, 3-color),
with a current-current-type four-Fermion interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{qq} + \mathcal{L}_{q\bar{q}}$$

$$\begin{aligned}
\mathcal{L}_0 &= \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q \\
\mathcal{L}_{q\bar{q}} &= G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right] \\
\mathcal{L}_{qq} &= G_D \sum_{A=2,5,7} \left[\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T\right]\left[q^T iC\gamma_5\tau_2\lambda_A q\right]
\end{aligned}$$

$$q = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} \otimes \begin{pmatrix} u\\ d\end{pmatrix} \otimes \begin{pmatrix} r\\ g\\ b\end{pmatrix}$$

$$\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3) \quad C = i\gamma_2\gamma_0$$

$m_{0,u} = m_{0,d} = m_0$
$\mu_u = \mu_d = \mu$

G_S Scalar and pseudoscalar coupling strength

G_D Scalar diquark coupling strength

2.3 the partition function

the partition function $\mathcal{Z} = \int [dq]\,[d\bar{q}]\exp\left[\int_0^\beta d\tau \int d^3x\;\mathcal{L}\right]$ $\quad\Omega = -T\ln Z$

Hubbard–Stratonovich auxiliary fields

$$\mathcal{Z} \;=\; \int [dq]\,[d\bar{q}]\,[d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\exp\left[\int_0^\beta d\tau \int d^3x\,\mathcal{L}\right]$$

$$\mathcal{L}_{\text{eff}} = -\frac{\sigma^2+\pi^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D} + \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q - \bar{q}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})q + i\frac{\Delta_A^*}{2}q^T iC\gamma_5\tau_2\lambda_A q - i\frac{\Delta_A}{2}\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T$$

Nambu–Gorkov formalism

$\Psi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^c \end{pmatrix}$ $\quad\bar{\Psi} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{q} & \bar{q}^c \end{pmatrix}$ $\quad q^c(x) \equiv C\bar{q}^T(x)$

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\exp\left[\int_0^\beta d\tau \int d^3x\left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right]\int [d\Psi]\,[d\bar{\Psi}]\exp\left[\int_0^\beta d\tau \int d^3x\,\bar{\Psi}\,S^{-1}\Psi\right]$$

$$\boxed{S^{-1} \equiv \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} & \Delta_A\gamma_5\tau_2\lambda_A \\ -\Delta_A^*\gamma_5\tau_2\lambda_A & i\partial\!\!\!/ - \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}^t\cdot\boldsymbol{\pi} \end{pmatrix}}$$

$$\boxed{\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\left\{\exp\left[\int_0^\beta d\tau \int d^3x\left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right]\exp\left[\mathrm{Tr}\left(\ln S^{-1}\right)\right]\right\}}$$

3. the mean field approximation

how to calculate this?

$$\boxed{\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2 + \boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^* \Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}}$$

the mean field approximation (MFA)

decompose bosonic collective fields into a

homogeneous MF part + ~~fluctuation part~~

$\downarrow$

order parameter:
characterization of phase structure

~~correlations~~

$$\Delta \rightarrow \Delta_{MF} + \delta \quad \sigma \rightarrow \sigma_{MF} + \sigma$$

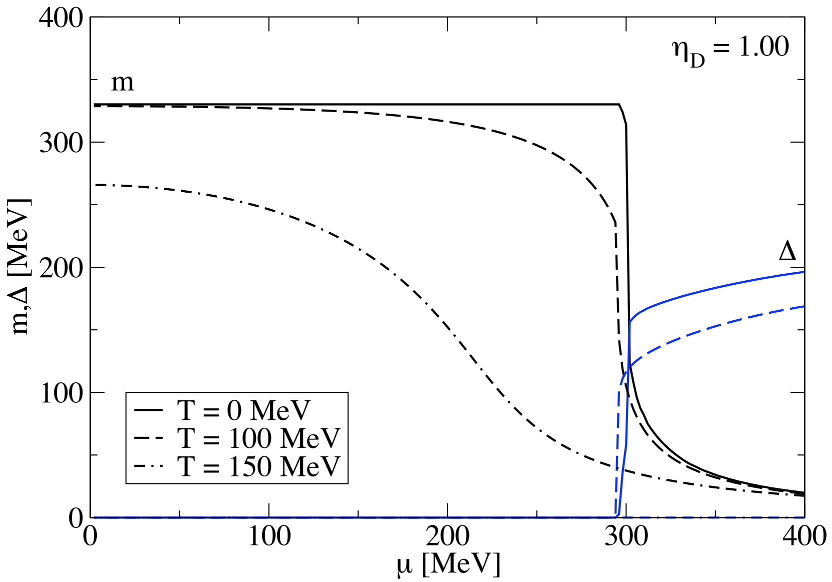
$$\boxed{\frac{\partial \Omega}{\partial m} = \frac{\partial \Omega}{\partial \Delta} = 0} \quad \mathcal{Z}_{MF} = \exp\left[\beta V \left(-\frac{\sigma_{MF}^2 + \boldsymbol{\pi}_{MF}^2}{4G_S} - \frac{\Delta_{MF}^* \Delta_{MF}}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S_{MF}^{-1} \right) \right]$$

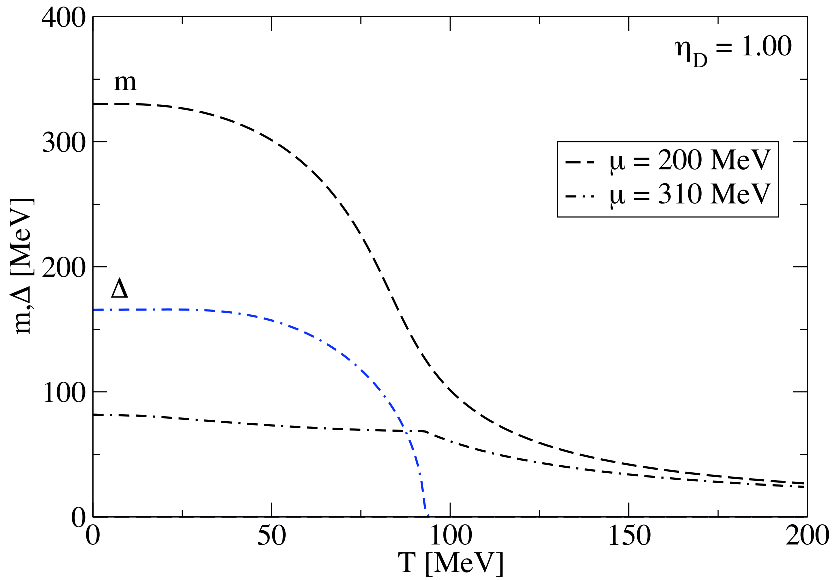
$$m - m_0 = 8G_S\, m \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_{\mathbf{p}}} \left\{ \left[1 - 2n_F(E_{\mathbf{p}}^-)\right] \frac{\xi_{\mathbf{p}}^-}{E_{\mathbf{p}}^-} + \left[1 - 2n_F(E_{\mathbf{p}}^+)\right] \frac{\xi_{\mathbf{p}}^+}{E_{\mathbf{p}}^+} + n_F(-\xi_{\mathbf{p}}^+) - n_F(\xi_{\mathbf{p}}^-) \right\}$$

$$\Delta = 8G_D \int \frac{d^3p}{(2\pi)^3} \left[\frac{1 - 2n_F(E_{\mathbf{p}}^-)}{E_{\mathbf{p}}^-} + \frac{1 - 2n_F(E_{\mathbf{p}}^+)}{E_{\mathbf{p}}^+} \right]$$

$E_{\mathbf{p}}^{\pm} = \sqrt{(\xi_{\mathbf{p}}^{\pm})^2 + \Delta^2}$ with $\xi_{\mathbf{p}}^{\pm} = E_{\mathbf{p}} \pm \mu$, $E_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$

3.1 results of MFA





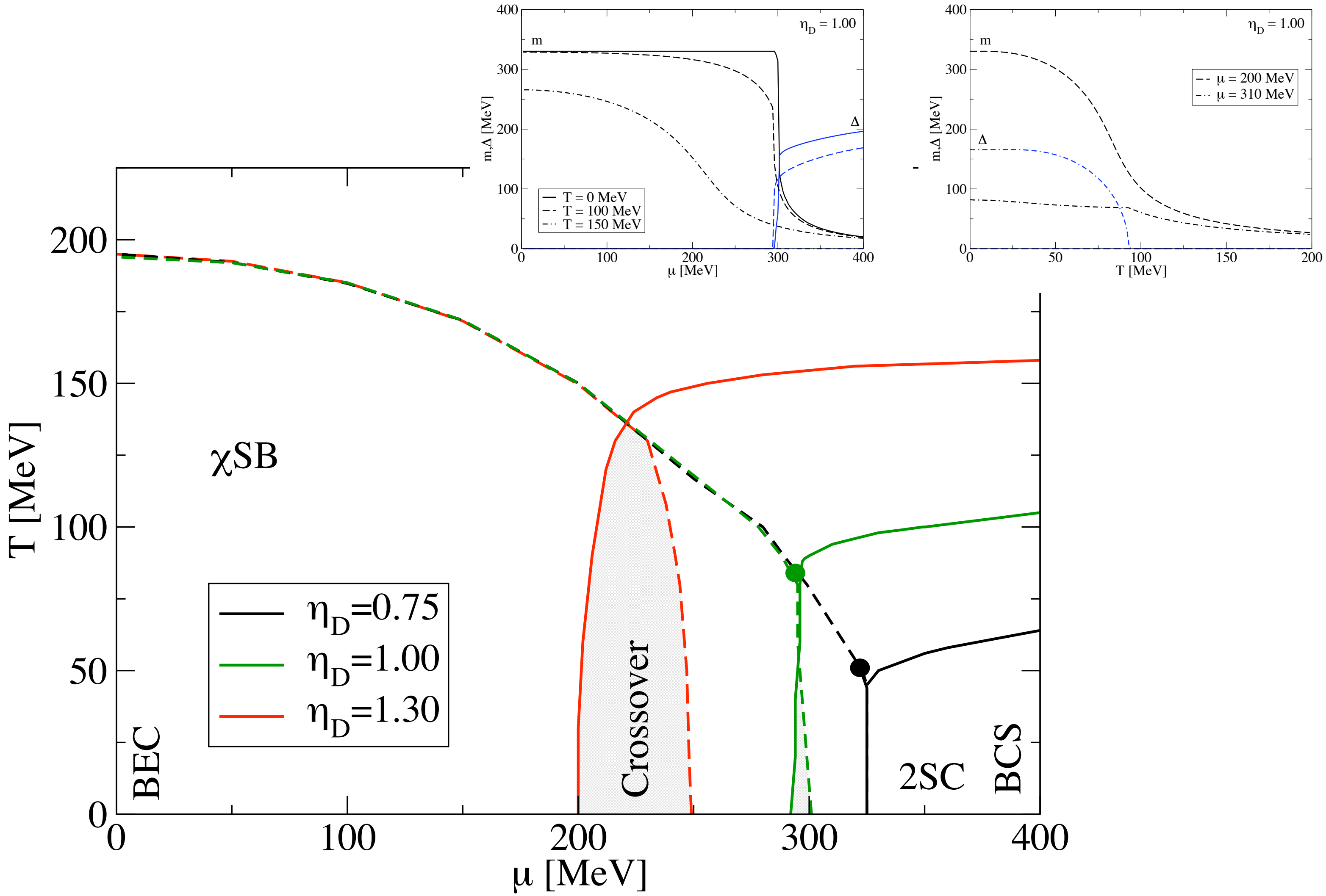
$$\Lambda = 629.540 \text{ MeV}$$

$$m_0 = 5.27697 \text{ MeV}$$

$$G_S \Lambda^2 = 2.17576$$

H. Grigorian, Phys. Part. Nucl. Lett. **4**, 223 (2007) [arXiv:hep-ph/0602238].

3.2 the phase diagram MF



achieved a crossover region by artificially increasing the diquark coupling strength

4 what about fluctuations?

$$\boxed{\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}}$$

$$S^{-1} = S^{-1}_{MF} + \Sigma$$

$$\Sigma \equiv \begin{pmatrix} -\sigma - i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi} & \delta_A \gamma_5 \tau_2 \lambda_A \\ -\delta_A^* \gamma_5 \tau_2 \lambda_A & -\sigma - i\gamma_5 \boldsymbol{\tau}^t \cdot \boldsymbol{\pi} \end{pmatrix}$$

ln - expansion around MF values

$$\begin{aligned} \mathrm{Tr}[\ln(S^{-1})] &= \mathrm{Tr}[\ln(S^{-1}_{MF} + \Sigma)] \\ &= \mathrm{Tr}\{\ln[S^{-1}_{MF}(1 + S_{MF}\Sigma)]\} \\ &= \mathrm{Tr}\ln S^{-1}_{MF} + \mathrm{Tr}\ln[1 + S_{MF}\Sigma] \\ &= \mathrm{Tr}\ln S^{-1}_{MF} + \mathrm{Tr}[S_{MF}\Sigma - \underline{\frac{1}{2} S_{MF}\Sigma S_{MF}\Sigma + \ldots}\,] \end{aligned}$$

$$\mathrm{Tr}\left(S_{MF}\Sigma S_{MF}\Sigma\right) = (\boldsymbol{\pi}, \sigma, \delta_2^*, \delta_2, \delta_5^*, \delta_7^*) \begin{pmatrix} \Pi_{\pi\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Pi_{\sigma\sigma} & \Pi_{\sigma\delta_2} & \Pi_{\sigma\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2^*\sigma} & \Pi_{\delta_2^*\delta_2} & \Pi_{\delta_2^*\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2\sigma} & \Pi_{\delta_2\delta_2} & \Pi_{\delta_2\delta_2^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{\delta_5^*\delta_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{\delta_7^*\delta_7} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \\ \delta_2 \\ \delta_2^* \\ \delta_5 \\ \delta_7 \end{pmatrix}$$

$$\Pi_{\pi\pi}(q_0,\mathbf{q}) \;=\; 2\int\frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_{-}^{+}(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0-s_k\xi_{\mathbf{p+q}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}-\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0+s_k\xi_{\mathbf{p+q}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$

$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{p+q}}^{s_k})}{q_0-t_kE_{\mathbf{p+q}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{p+q}}^{s_k}-|\Delta|^2\right)\right\}$$

$$\Pi_{\sigma\sigma}(q_0,\mathbf{q}) = 2\int\frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_{-}^{-}(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0-s_k\xi_{\mathbf{p+q}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}+\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0+s_k\xi_{\mathbf{p+q}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$

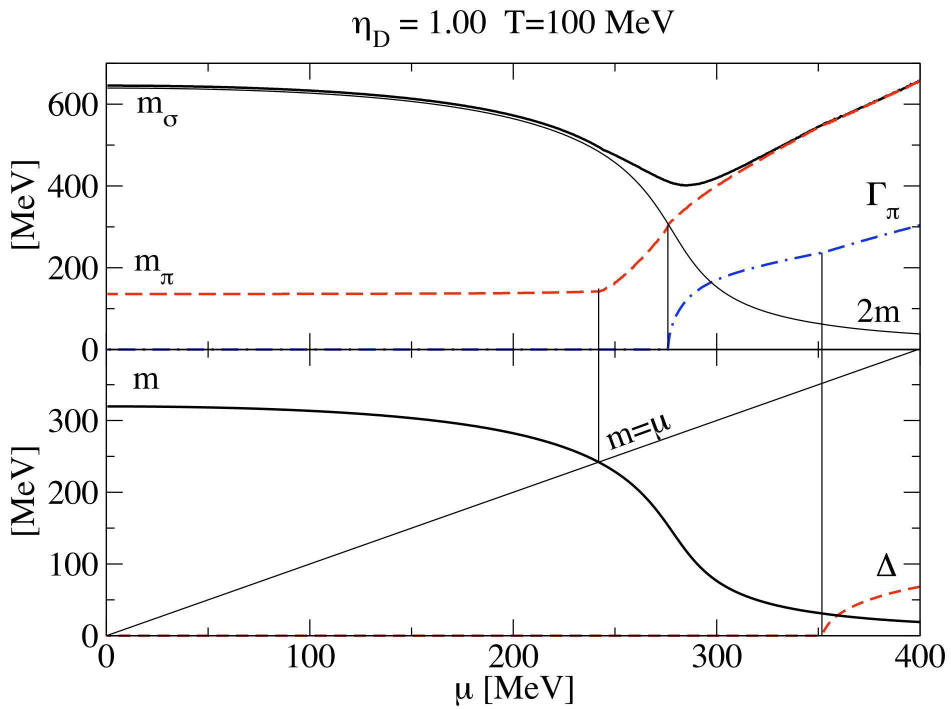
$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{p+q}}^{s_k})}{q_0-t_kE_{\mathbf{p+q}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\times\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{p+q}}^{s_k}-|\Delta|^2\right)\right\}$$

Similar equations can be derived for the other matrix elements

Sun et al. Phys. Rev. D **75** 096004 (2007)
in the 2-color limit
Ebert et al. Phys. Rev. C **72** 015201 (2005)
in the T=0 limit

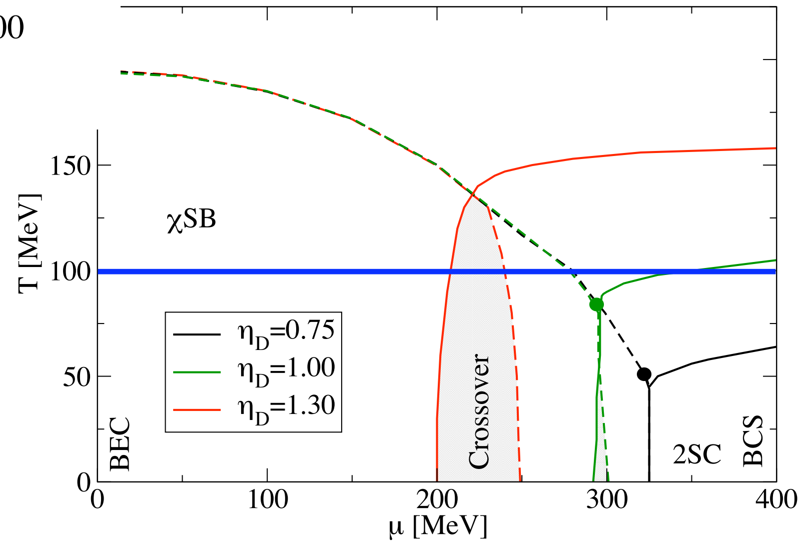
$$\mathcal{T}_{\mp}^{\pm}(s_p,s_k) = 1\pm s_ps_k\frac{\mathbf{p}\cdot\mathbf{k}\mp m^2}{E_{\mathbf{p}}E_{\mathbf{k}}}$$

4.2 the pion mass

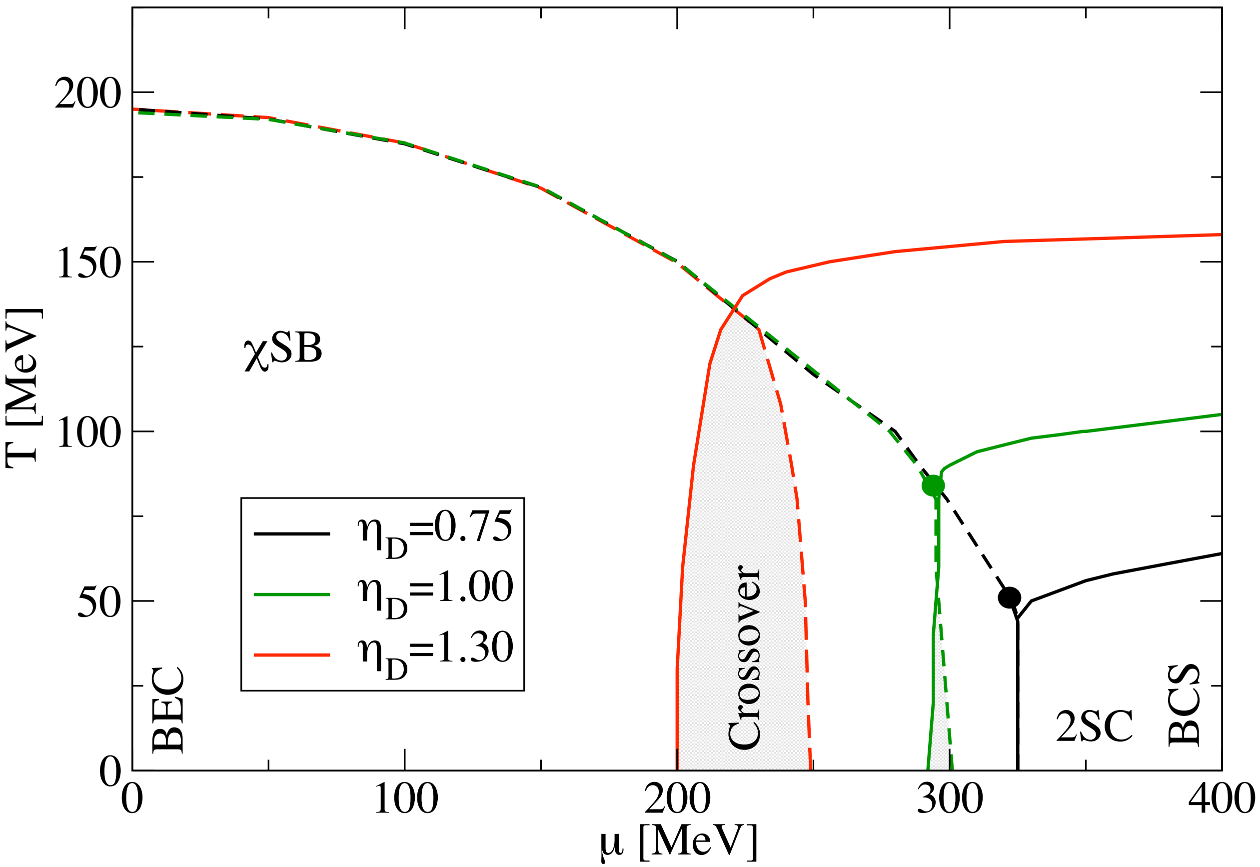


$$\Gamma_\pi = \left(\frac{\partial \mathrm{Re}\Pi_{\pi\pi}}{\partial m_\pi^2}\right)^{-1} \frac{\mathrm{Im}\Pi_{\pi\pi}}{m_\pi}$$

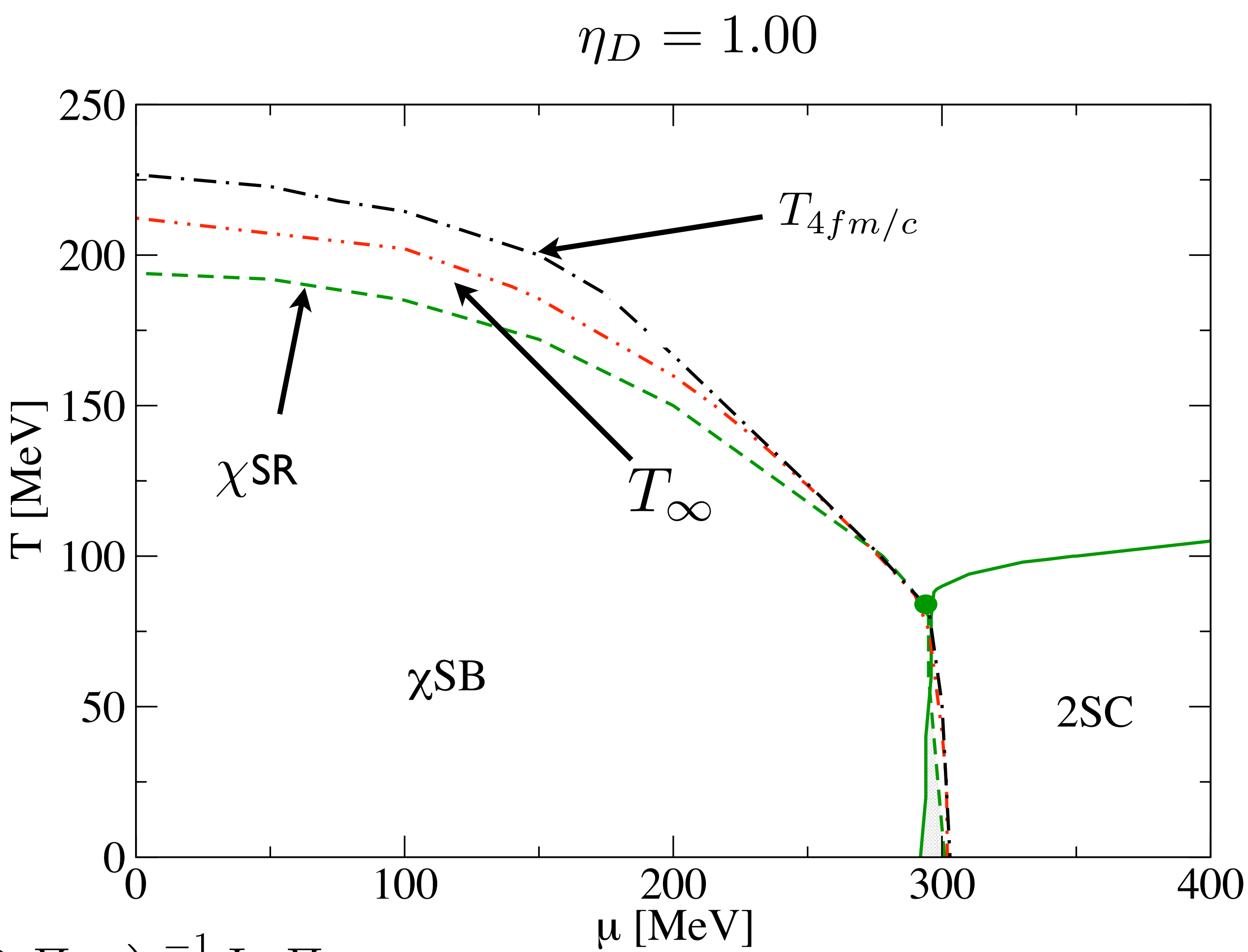
$$1 - 2G_S\Pi(q_0, \underline{\boldsymbol{q} = \boldsymbol{0}}) = 0$$



the phase diagram MF



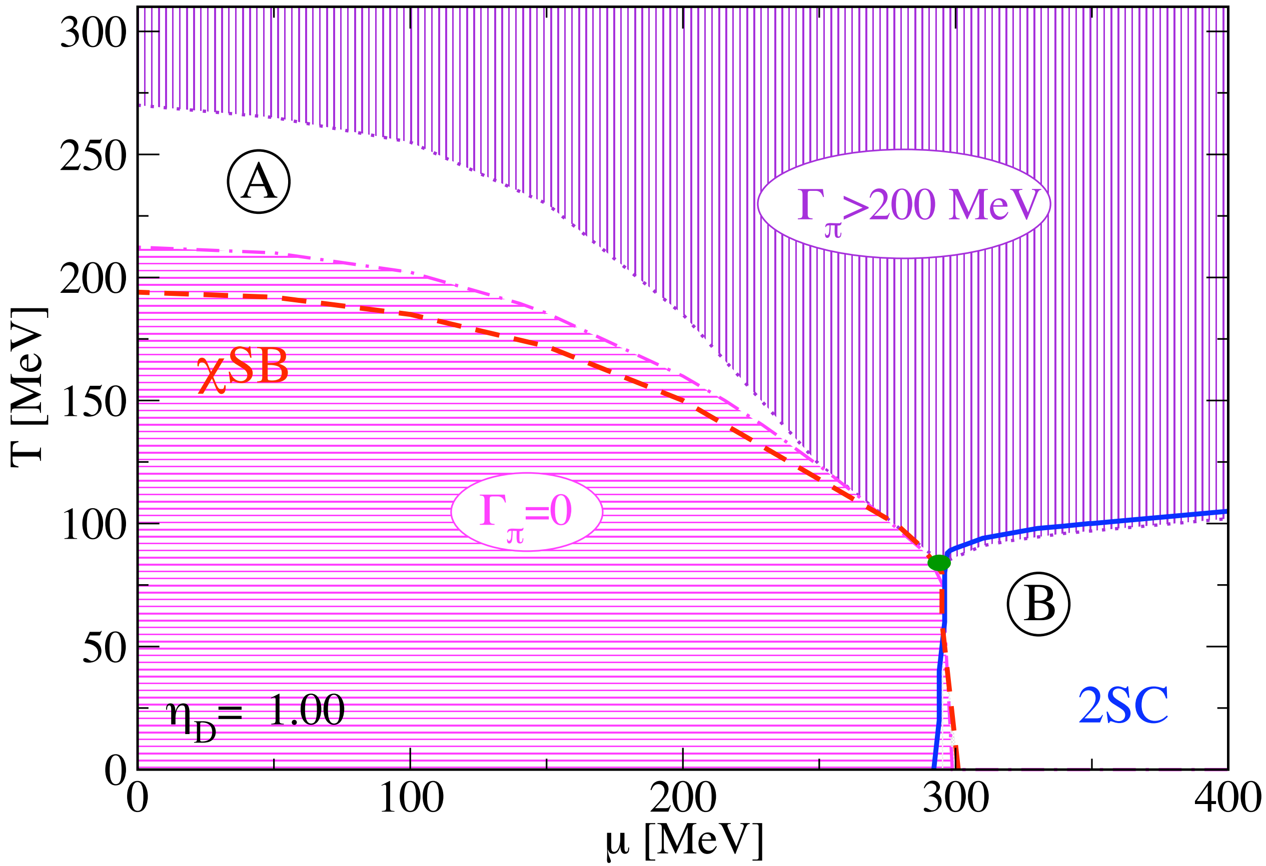
the phase diagram revisited



$$\Gamma_\pi = \left(\frac{\partial \mathrm{Re}\Pi_{\pi\pi}}{\partial m_\pi^2}\right)^{-1} \frac{\mathrm{Im}\Pi_{\pi\pi}}{m_\pi}$$

$$T(\tau = \Gamma_\pi^{-1})$$

RHIC phenomenology



QGP probed at RHIC is far to be a perfect liquid; an explanation: strong correlations in the plasma

Shuryak and Zahed (PRL, 2003)

Region dominated by strong correlated states with a lifetime > 1 fm/c

summary and outlook

fluctuations are included in Gaussian approximation beyond MF;
systematical treatment in the non-perturbative regime possible

some properties of mesons are studied
diquark calculations more complicated, under investigation
new insight for phase diagram; important for HIC and CSs

investigate $\sigma - \delta$-mixing

constraints of color and electrical neutrality and β-equilibrium to be implemented (HIC and CSs)

the same formalism can be applied to Nuclear MF theory
under investigation together with G. Röpke and D. Blaschke

Slightly out of topic

Happy Birthday David!!!

Time to sing!

acknowledgments

Thanks for your attention