COVARIANT FORMULATION OF DYNAMICAL EQUATIONS OF QUANTUM VORTICES IN TYPE II SUPERCONDUCTORS

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Superfluid in Newtonian theory

A.D.Sedrakian, D.M.Sedrakian (1994)

1. $rot\vec{V}_s = \vec{\chi} n$

2. $\frac{\partial n}{\partial t} + div\, n\vec{V}_L = 0$

3. $\rho_s \left[\left(\vec{V}_s - \vec{V}_L \right), \vec{\chi} \right] = \eta \left(\vec{V}_L - \vec{V}_n \right)$

$$\vec{\chi} = \vec{\nu} \cdot \frac{\hbar}{2m}$$

$$V_{Lr} = \frac{\kappa}{1+\kappa^2} \left(\Omega_s - \Omega_n \right) r \qquad V_{L\varphi} = \frac{\Omega_n + \kappa^2 \Omega_s}{1+\kappa^2}$$

$$\kappa = \frac{\rho_s}{\rho_n} \frac{\hbar n}{4m\eta}$$

Superfluid in Genaral Relativity Theory

D.M.Sedrakian, B.Carter, D.Langlois

(1998)

1. $\eta^{\mu\nu\rho\sigma} \nabla_{[\mu}\mu_{\sigma]} = -\omega D^{\mu\nu}$

2. $\nabla_\rho \left(\omega D^{\mu\nu}\right) = 0$

3. $\omega_{\rho\sigma} n^\rho(s) = \eta \perp_{\sigma\rho} n^\rho(n)$

$$\frac{1}{\omega} \frac{k^\rho \omega_{\rho\sigma} m^\sigma}{m^\rho \perp_{\rho\sigma} m^\sigma} = \frac{\kappa}{1+\kappa^2}\left(\Omega_n - \Omega_s\right)$$

$$\frac{k^\rho \perp_{\rho\sigma} m^\sigma}{m^\rho \perp_{\rho\sigma} m^\sigma} = \frac{\Omega_n + \kappa^2 \Omega_s}{1+\kappa^2}$$

a) $\mu_\nu = \rho_s u_\nu(s)$

b) $\omega_{\mu\nu} = 2 \cdot \nabla_{[\mu}\mu_{\nu]}$

c) $\omega = \omega^{\mu\nu}\omega_{\mu\nu}$

d) $D^{\mu\nu} = -u^\mu(s) V^\nu(L) + u^\nu(s) V^\mu(L)$

e) $\kappa = \dfrac{n(s)\alpha(s)}{\eta n(n)\alpha(n)}\omega$

f) $n^\rho(s) = \rho_s \alpha(s)\left[k^\rho + \Omega_s m^\rho\right]$

$n^\rho(n) = \rho_n \alpha(n)\left[k^\rho + \Omega_n m^\rho\right]$

theory

D.M.Sedrakian (2006)

$$1.\quad rot\vec{M} = \kappa\vec{n} \qquad \left(\vec{\kappa} = \Phi_0 \vec{\nu}\right)$$

$$2.\quad \frac{\partial n}{\partial t} + div\vec{V}_L = 0$$

$$3.\quad \frac{en_s}{c}\left[\left(\vec{V}_s - \vec{V}_L\right), \vec{\kappa}\right] = -\eta \vec{V}_L$$

$$a)\ \vec{M} = \frac{cm}{e^2 n_s}\vec{j} + \vec{A}$$

$$b)\ rot\vec{A} = \vec{B}$$

$$c)\ rot\vec{B} = \frac{4\pi}{c}\vec{j}$$

$$d)\ \vec{j}_s = en_s\vec{V}_s$$

$$V_{Lr} = \frac{\kappa^2 / en_s}{1+\kappa^2} j_{s\varphi} \qquad V_{L\varphi} = \frac{\kappa / en_s}{1+\kappa^2} j_{s\varphi}$$

Type II superconductor in General Relativity Theory

D.M.Sedrakian, R.Krikorian (2007)

Consider a static universe with metric of the form

$$ds^2 = g_{ij} dx^i dx^j + g_{44} \left(dx^4 \right)^2 \tag{1}$$

where the g ’s are independent of the time coordinate x^4. In this universe we have a type II superconductor with world lines of the normal part along the x^4 lines; consequently

$$u^i(n) = 0, \quad g_{44} \left[u^4(n) \right]^2 = -1, \quad u^4(n) = \frac{1}{\sqrt{-g_{44}}} \tag{2}$$

Type II superconductor in GRT

As it is well known, when the intensity of the applied magnetic field is less than the critical value for the creation of quantum vortices, the equations which relate the supercurrent to the electromagnetic field are the London equations which, in covariant form, read

$$\nabla_{[\mu} M_{\nu]} \equiv s_{\mu\nu} = 0 \qquad (3)$$

where ▽ is the operator of covariant derivation and

$$M_\nu = \frac{mc}{e^2 n(s)} j_\nu + A_\nu \qquad (4)$$

with e (e <0), m and $n(s)$ denoting respectively the charge, mass and number density of superelectrons. j_ν is the 4-current defined by

$$j_\nu = en(s)\, u_\nu(s) \qquad (5)$$

and the 4-potential connected to the electromagnetic field tensor by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (6)$$

Type II superconductor in GRT

- In the presence of vortices, the right-hand side of Eq.(3) must be set equal to a non zero antisymmetric tensor characterizing the system of vortices. This tensor may be written in the form

$$D^{\rho\sigma} = -u^{\rho}(L)\,v^{\sigma}(L) + u^{\sigma}(L)\,v^{\rho}(L) \qquad (7)$$

where $u^{\rho}(L)$ and $v^{\rho}(L)$ are respectively the unit 4-velocity of the vortex and the unit spacelike vector defining the direction of the vortex.

Accordingly, in the presence of vortices Eq.(3) must be replaced by

$$\nabla_{[\mu}M_{\nu]} \equiv s_{\mu\nu} = \frac{s}{2}\eta_{\mu\nu\rho\sigma}D^{\rho\sigma} \quad \text{or} \quad \eta^{\mu\nu\rho\sigma}\nabla_{[\rho}M_{\sigma]} = -2sD^{\mu\nu} \qquad (8)$$

where

$$\eta_{\mu\nu\rho\sigma} = -\sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}, \qquad s^2 = \frac{1}{2}s^{\mu\nu}s_{\mu\nu} \qquad (9)$$

From Eq.(8) one easily derive the equation of conservation of vortex number

$$\nabla_{\mu}\left(sD^{\mu\nu}\right) = 0 \qquad (10)$$

Type II superconductor in GRT

$$\eta^{\mu\nu\rho\sigma}\nabla_{[\rho}M_{\sigma]} = -2sD^{\mu\nu} \qquad (8) \qquad \nabla_{\mu}\left(sD^{\mu\nu}\right) = 0 \qquad (10)$$

Equations (8) and (10) have the following Galilean limits:

$$rot\vec{M} = n\phi_0\vec{\chi} \qquad \frac{\partial n}{\partial t} + div\left(n\vec{v}\left(L\right)\right) = 0$$

where $\phi_0 = s/n = hc/2e,\ \vec{\chi},\ \text{and}\ \ n$ are respectively the quantized magnetic flux of a vortex, the unit vector in the direction of the vortex, and the density of vortices. $\vec{v}\left(L\right)$ is the velocity of the vortex, which is perpendicular to $\dot{\chi}$

COVARIANT FORMULATION OF THE FORCES ACTING ON VORTICES

1.Friction force

Friction is a consequence of the interaction between the normal matter in the core of a vortex and the normal component of the superconductor. By classical physics analogy we adopt the following covariant expression of the friction force

$$F_\rho = \eta n(n)\, \tilde{u}_\rho(n,L) \quad (11) \qquad \tilde{u}_\rho(n,L) = \left[\delta_\rho^\alpha + u_\rho(L)\, u^\alpha(L)\right] u_\alpha(n) \quad (12)$$

where $\eta,\ n(n)$ and $\tilde{u}_\rho(n,L)$ are respectively the friction coefficient, the number density of the normal component, and the velocity of the normal comonent relative to the vortex.

Taking into account the orthogonality of $u^\sigma(L)$ and $v^\sigma(L)$, and introducing projection tensor

$$\perp_\rho^\sigma = \delta_\rho^\sigma - \eta_\rho^\sigma, \quad \eta_\rho^\sigma = -u_\rho(L)\, u^\sigma(L) + v_\rho(L)\, v^\sigma(L)$$

we finally obtain for the friction force the following expression

$$F_\rho = \eta n(n) \perp_{\rho\sigma} u^\sigma(n) \qquad (13)$$

COVARIANT FORMULATION OF THE FORCES ACTING ON VORTICES

2.Lorenz force

The second force $\bar{F}_\rho$, acting on a quantum vortex is due to the superconducting current. By analogy with the classical case, is taken proportional to the velocity of the superconducting component relative to the vortex, i.e. $\tilde{u}_\rho(s,L)$. Moreover, $\bar{F}_\rho$ being orthogonal to the relative velocity $\tilde{u}_\rho(s,L)$, the velocity of vortex $u_\rho(L)$, and the vortex direction $v_\rho(L)$:

$$\bar{F}_\rho = \frac{en(s)}{2c} s\eta_{\rho\mu\nu\sigma}\tilde{u}^\mu(s,L)\, D^{\nu\sigma} = \frac{1}{c} j^\sigma(s)\, s_{\rho\sigma} \qquad (14)$$

Galilean limit

$$F_\rho = \eta n(n) \perp_{\rho\sigma} u^\sigma(n)$$

$$\vec{F}_{fr} = -\eta \vec{v}(L)$$

$$\bar{F}_\rho = \frac{1}{c} j^\sigma(s)\, s_{\rho\sigma}$$

$$\vec{F}_s = \frac{en(s)\,\phi_0}{c}\left[\vec{v}(s) - \vec{v}(L),\, \vec{\chi}\right]$$

DETERMINATION OF THE VORTEX 4-VELOCITY

The 4-velocity of vortices can be obtained from the condition

$$F_\rho + \bar{F}_\rho = 0 \quad \Rightarrow \quad \frac{1}{c} j^\sigma(s) s_{\sigma\rho} = \eta n(n) \perp_{\rho\sigma} u^\sigma(n) \qquad (15)$$

$$j^\rho(s) = en(s)\gamma(s)\left[k^\rho + \Omega(s) m^\rho\right] \qquad (16)$$

$$u^\rho(n) = \gamma(n) k^\rho \qquad (17)$$

$$\frac{1}{s}\frac{k^\rho s_{\lambda\rho} m^\lambda}{m^\rho \perp_{\lambda\rho} m^\lambda} = \frac{\kappa\Omega(s)}{1+\kappa^2} \qquad (18)$$

$$\frac{k^\rho \perp_{\lambda\rho} m^\lambda}{m^\rho \perp_{\lambda\rho} m^\lambda} = -\frac{\kappa^2\Omega(s)}{1+\kappa^2} \qquad (19)$$

$$\kappa = \frac{en(s)\gamma(s)s}{c\eta n(n)}$$

One easily verifies that the left-hand side of Eqs. (18) and (19) correspond respectively to the radial and azimuthal components of the 4-vector $u^\mu(L)$

DETERMINATION OF THE VORTEX 4-VELOCITY

(i) the presence of vortices does not modify the static gravitational field

(ii) The homogeneous applied magnetic field is directed along the x^3 - coordinate axis

Substitution of the tensor components

$$m^{\rho} \perp_{\rho\sigma} m^{\sigma} = g_{22} + \eta_{22} = g_{22}\left[1 + u^2(L)\, u_2(L)\right]$$

$$m^{\rho} \perp_{\rho\sigma} k^{\sigma} = -\eta_{24} = g_{22} u^2(L)\, u_4(L)$$

$$k^{\rho} s_{\rho\sigma} m^{\sigma} = s\sqrt{-g}\, u^1(L)$$

in Eqs. (18) and (19) yields

$$u^1(L) = -\frac{g_{22}\Omega(s)/c}{\sqrt{-g}} \frac{\kappa}{1+\kappa^2}\left[1 + u^2(L)\, u_2(L)\right]$$

$$u^2(L) = -\frac{\Omega(s)}{c} \frac{\kappa^2}{1+\kappa^2} \frac{1 + u^2(L)\, u_2(L)}{u_4(L)} \qquad (20)$$

normalization condition

$$u^4(L)\, u_4(L) + u^2(L)\, u_2(L) + u^1(L)\, u_1(L) = -1 \qquad (21)$$

RELAXATION EQUATION

$$\frac{\partial M_2}{\partial x^1} = -s\frac{\sqrt{-g}}{\sqrt{g_{11}}}u^4(L) \qquad (22)$$

The equation of conservation of vortex number

$$\frac{\partial}{\partial x^4}\left(s\frac{\sqrt{-g}}{\sqrt{g_{11}}}u^4(L)\right) = -\frac{\partial}{\partial x^1}\left(s\frac{\sqrt{-g}}{\sqrt{g_{11}}}u^1(L)\right) \qquad (23)$$

From (22) and (23) we obtain

$$\frac{\partial M_2}{\partial x^4} = s\frac{\sqrt{-g}}{\sqrt{g_{11}}}u^1(L) \qquad (24)$$

RELAXATION EQUATION

Eq. (24) may be exhibited in the form

$$\frac{\partial j_2}{\partial T}+\frac{j_2}{\tau}=-\frac{e^2 n(s)}{mc^2}\frac{\partial A_2}{\partial T} \qquad (25)$$

where the relaxation time τ is defined by

$$\frac{1}{\tau}=\frac{es}{2mc}\frac{\kappa}{1+\kappa^2}\frac{1+u^2(L)u_2(L)}{\sqrt{-g_{44}}\sqrt{g_{11}}u^4(s)} \qquad (26)$$

$$s=-\frac{\sqrt{g_{11}}}{\sqrt{-g}}\frac{1}{u^4(L)}\frac{\partial M_2}{\partial x^1} \qquad (27)$$

As we see, the quantity s depends on j_2, consequently, the relaxation equation (25) is a nonlinear equation with respect to j_2.

RELAXATION EQUATION

$$j_2 << \frac{1}{4\pi\lambda^2} A_2 \qquad (28)$$

Condition (28) states that the relaxation current is much smaller than the Meissner current. When the nonequilibrium vortex tends to equilibrium, the relaxation current j_2 tends to zero; accordingly, the last stage of the relaxation process may be regarded as linear. Linearity of the process is conserved, when the change in the applied magnetic field ΔH is small compared to the magnetic field H.

In the case of equilibrium the quantity s reads

$$s_0 = -\frac{\sqrt{g_{11}}}{\sqrt{-g_{44}u^4(L)}} \frac{1}{\sqrt{\gamma}} \frac{\partial A_2}{\partial x^1} \qquad (29)$$

$$B^3 = -\frac{1}{\sqrt{\gamma}} \frac{\partial A_2}{\partial x^1} \qquad (30)$$

$$s_0 = \frac{\sqrt{g_{11}}B}{\sqrt{-g_{44}u^4(L)}} \qquad (31)$$

RELAXATION TIME

The final expression for the relaxation time

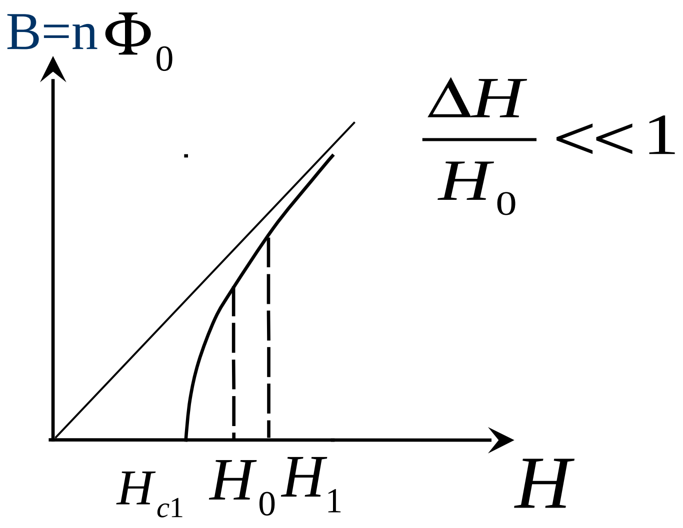
$$\frac{1}{\tau_0} = \frac{eB}{mc}\frac{\kappa}{1+\kappa^2}\frac{1+u^2(L)u_2(L)}{\left\{\left[1+u^i(L)u_i(L)\right]\left[1+u^i(s)u_i(s)\right]\right\}^{1/2}} \quad i=1,2 \qquad (32)$$

In the case of small velocities $u^i(L)u_i(L) \ll 1, \quad u^i(s)u_i(s) \ll 1$

the nonrelativistic expression of τ_0 reads

$$\frac{1}{\tau_0} = \frac{eB}{mc}\frac{\kappa}{1+\kappa^2} \qquad (33)$$

Let us now find a solution of the relaxation equation when the value of the magnetic field undergoes a discrete jump from the value H_0 to H_1, $\left(H_1 - H_0\right)/H_0 \ll 1$



$$\frac{\partial j_2}{\partial T} + \frac{j_2}{\tau_0} = j \tag{34}$$

$$\text{where} \qquad j = -\frac{1}{4\pi\lambda^2} \left.\frac{\partial A_2}{\partial H}\right|_{H_0} \frac{dH}{dT}$$

$$j_2 = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{j(\omega)\, e^{i\omega T}}{\omega - i/\tau_0} d\omega$$

$$j(\omega) = J = -\frac{1}{4\pi\lambda^2} \left.\frac{\partial A_2}{\partial H}\right|_{H_0} \left(H_1 - H_0\right)$$

$$j_2 = J e^{-T/\tau_0} \tag{35}$$