



# Neutron Star Structure with Modern Nucleonic Three-Body Forces

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PRC 77, 034316 (2008) • PRC 78, 028801 (2008)

## Motivation for TBF:

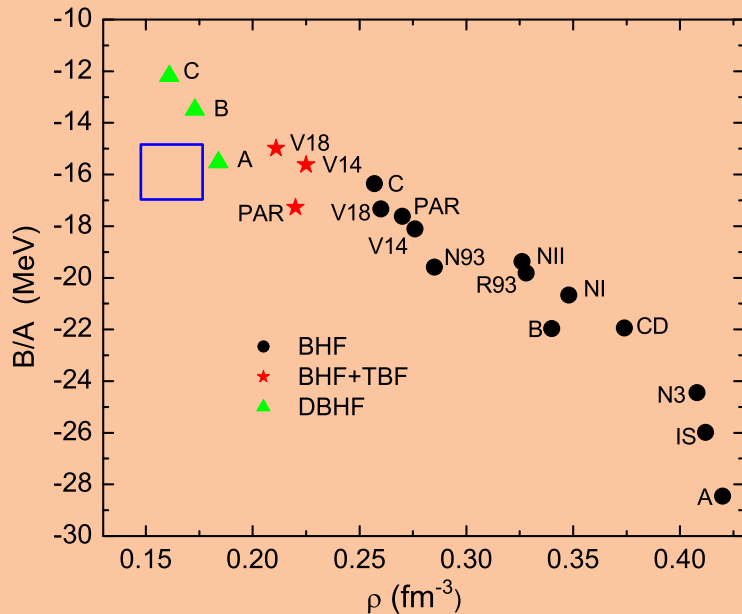
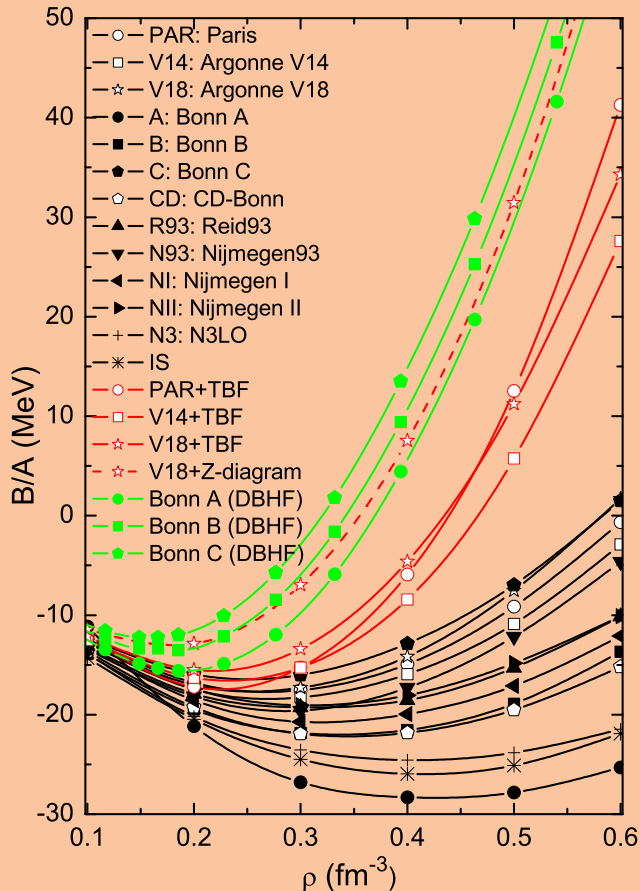
- Structure of (light) nuclei, nucleon-deuteron scattering
- Saturation of nuclear matter
- Nuclear EOS at high density

## Goal:

- Construct nuclear TBF consistent with a given meson-exchange NN potential (Bonn B, Nijmegen 93)
- Use in microscopic BHF calculation of high-density nuclear matter
- Neutron star structure

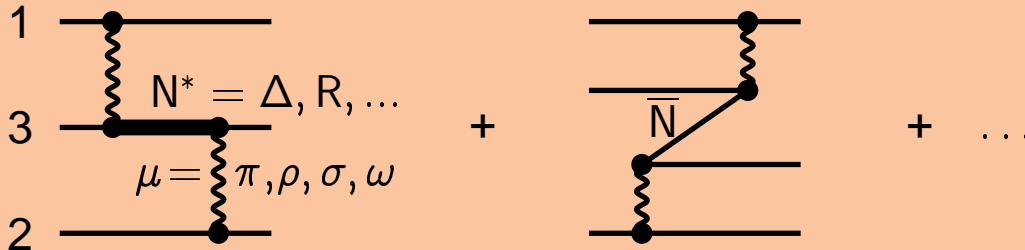
● BHF Binding energy and saturation point of symmetric matter:

PRC 74, 047304 (2006)



↪ TBF substantially improve saturation

# Three-Nucleon Forces:



- Only small effect required [ $\delta(B/A) \approx 1$  MeV at  $\rho_0$ ]
- Model dependent
- Use and compare microscopic and phenomenological TBF...
  - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989):  
Exchange of  $\pi, \rho, \sigma, \omega$  via  $\Delta(1232), R(1440), N\bar{N}$   
Parameters compatible with two-nucleon potential (Paris,  $V_{18}, \dots$ )
  - Urbana IX phenomenological TBF:  
Only  $2\pi$ -TBF + phenomenological repulsion  
Fit saturation point

# Microscopic Meson Exchange TBF:

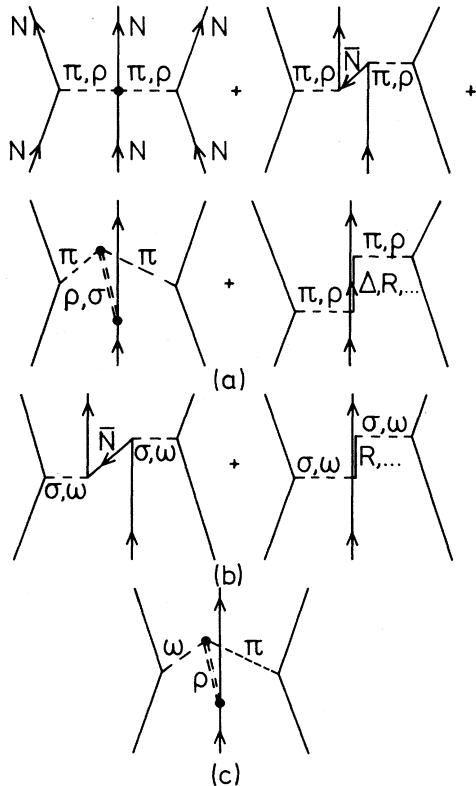


FIG. 3. Leading order contributions to the three-body force deduced from the meson-exchange current operators indicated in Fig. 2. See text for the explanation of the various groups (a)–(c).

P. Grangé, A. Lejeune, M. Martzloff, J.-F. Mathiot,  
 PRC 40, 1040 (1989)

$\pi, \rho$  - part based on Tuscon-Melbourne TBF:

S.A. Coon et al., NPA 317, 242 (1979);

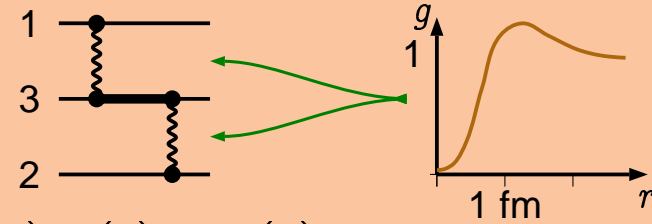
NPA 438, 631 (1985); PRC 48, 2559 (1993); ...

- Effects of  $\Delta(1232)$ ,  $R(1440)$ ,  $N\bar{N}$
- Parameters compatible with two-nucleon (Paris) potential

# Some Details:

- Average over spectator nucleon using BHF defect function:

$$\begin{aligned} \bar{V}_{12}(\mathbf{r}) &= \rho \int d^3 r_3 \sum_{\sigma_3, \tau_3} g(r_{13}) g(r_{23}) V_{123} \\ &= (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) V_C(r) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) V_S(r) + V_I(r) \\ &\quad + S_{12}(\hat{\mathbf{r}}) \left[ (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) V_T(r) + V_Q(r) \right] \end{aligned}$$



: five components

- Example:  $\rho\rho$  contribution:

$$V_O^{\rho\rho}(r) = -\frac{16 m_\rho^2 f_{\rho NN}^2 f_{\rho N\Delta}^2}{81 m_\Delta - m_N} \sum_3 \left[ 2Y_x^\rho Y_y^\rho + P_r T_x^\rho T_y^\rho \right] \quad (O = C)$$

$$\left[ \frac{P}{2} T_x^\rho T_y^\rho - P_x Y_x^\rho T_y^\rho - P_y Y_y^\rho T_x^\rho \right] \quad (O = T)$$

$\rho NN, \rho N\Delta$  form factors and kinematical factors

# Meson Exchange Parameters:

Table 1: Meson-exchange parameters of the Bonn B and Argonne  $V_{18}$  potentials. The letter in brackets behind the form factor cutoff denotes the type of form factor: (M)onopole, (D)ipole, (R)oper. We use the baryon masses  $m_N = 938.4$  MeV,  $m_\Delta = 1232$  MeV,  $m_R = 1440$  MeV.

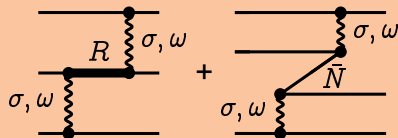
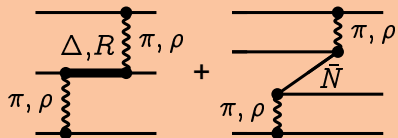
|          |             | $m$ (MeV) | $g^2/4\pi$ | $\Lambda$ (MeV) |   |
|----------|-------------|-----------|------------|-----------------|---|
| Bonn B   | $\pi$       | 138       | 14.4       | 1700 (M)        | $a = 1.38, b = -2.80, c = 1.25$ [TM(99)]  |
|          | $\rho$      | 769       | 0.90       | 1850 (D)        | $\kappa = 6.1, g_{\pi N\Delta}/g_{\pi NN} = g_{\rho N\Delta}/g_{\rho NN} = 1.8$ |
|          | $\sigma NN$ | 550       | 8.94       | 1900 (M)        |   |
|          | $\omega NN$ | 783       | 24.5       | 1850 (D)        |   |
|          | $\sigma NR$ | 550       | 0.8        | 2000 (R)        | $\alpha = 1$  |
|          | $\omega NR$ | 783       | 1.0        | 1850 (R)        | $\alpha = 1$  |
| $V_{18}$ | $\pi$       | 138       | 14.43      | 1580 (M)        | $a = 1.12, b = -2.49, c = 0.98$ [TM(81)]  |
|          | $\rho$      | 776       | 0.55       | 1400 (M)        | $\kappa = 6.6, g_{\pi N\Delta}/g_{\pi NN} = g_{\rho N\Delta}/g_{\rho NN} = 1.8$ |
|          | $\sigma NN$ | 540       | 11.9       | 1100 (M)        |   |
|          | $\omega NN$ | 780       | 33.0       | 1300 (M)        |   |
|          | $\sigma NR$ | 540       | 2.58       | 1450 (R)        | $\alpha = -2.35$  |
|          | $\omega NR$ | 780       | 4.23       | 1550 (R)        | $\alpha = -2.33$  |

Nij93 has many more parameters: 2 scalar mesons, wide  $\sigma$  and  $\rho$  mesons, “pomeron”

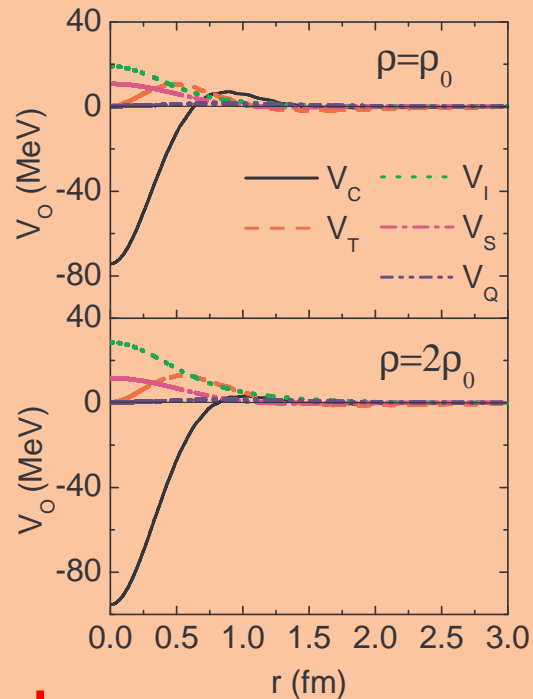
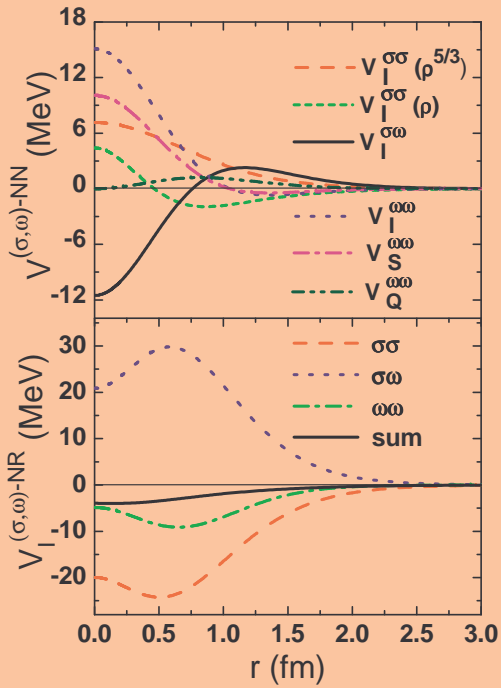
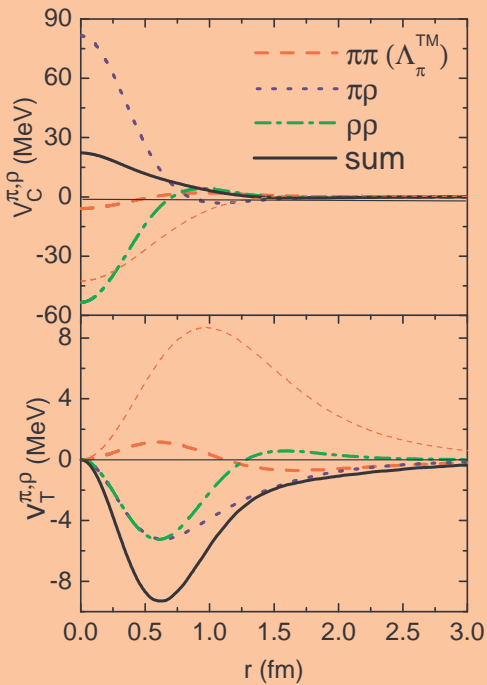
- Results with Bonn B potential ...



# Individual Meson Exchange Contributions:



Total



 Strong compensation !



# Phenomenological TBF (Urbana Model):

- Two pion exchange + phenomenological repulsion:

$$V_{ijk} = \sum_{\text{cyc.}} \left[ \begin{aligned} & \mathbf{A} \{X_{ij}, X_{jk}\} \{ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k \} \\ & + \frac{\mathbf{A}}{4} [X_{ij}, X_{jk}] [ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k ] + \mathbf{U} T_{ij}^2 T_{jk}^2 \end{aligned} \right]$$

$$X_{ij} = Y(m_\pi r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T(m_\pi r_{ij}) S_{ij}$$

$$Y(x) = \frac{e^{-x}}{x} \left( 1 - e^{-cr^2} \right), \quad T(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{x} \left( 1 - e^{-cr^2} \right)^2$$

- Corresponds to micro TBF with only  $\pi\pi$  contribution and

$$\mathbf{A} = -\frac{2}{81} \frac{(m_\pi f_{\pi NN} f_{\pi N\Delta})^2}{m_\Delta - m_N}$$

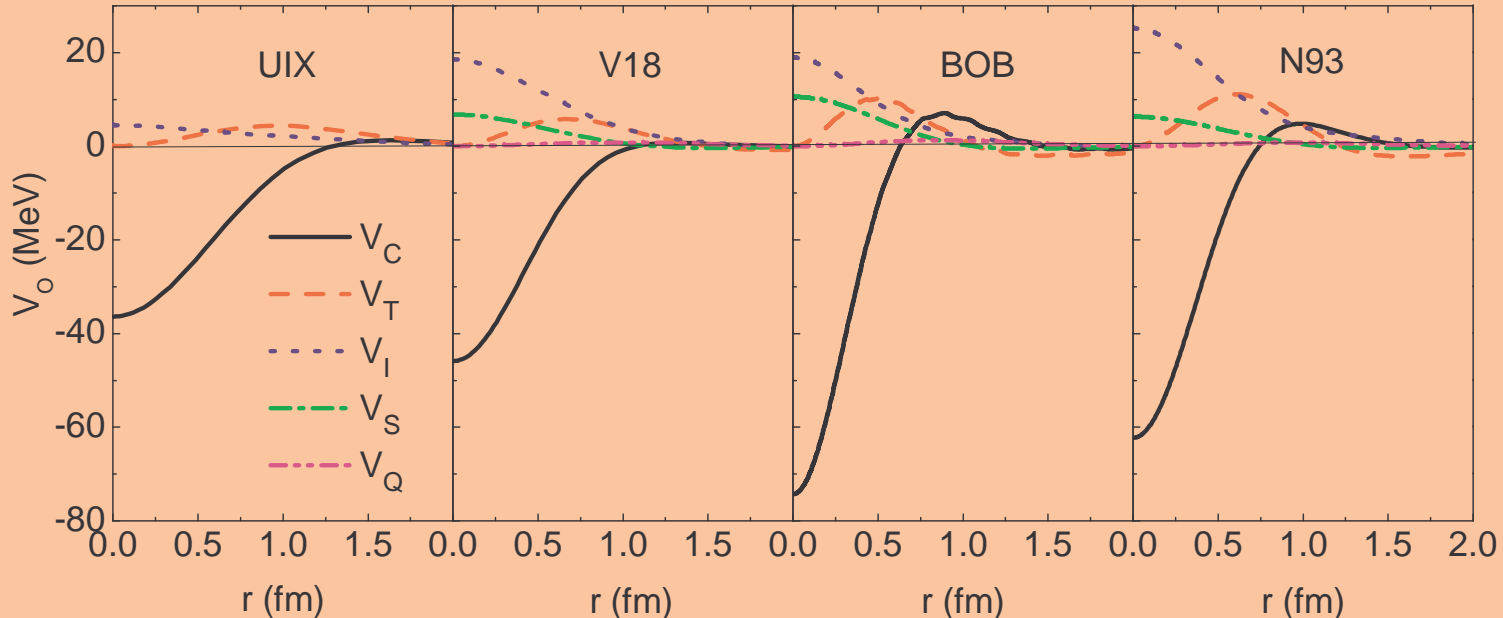
- Optimal parameters for BHF+TBF (with  $V_{18}$  2BF):

$$\mathbf{A} \approx -0.0500 \text{ MeV}, \quad \mathbf{U} \approx 0.00042 \text{ MeV}$$

# Phenomenological vs. Microscopic TBF:

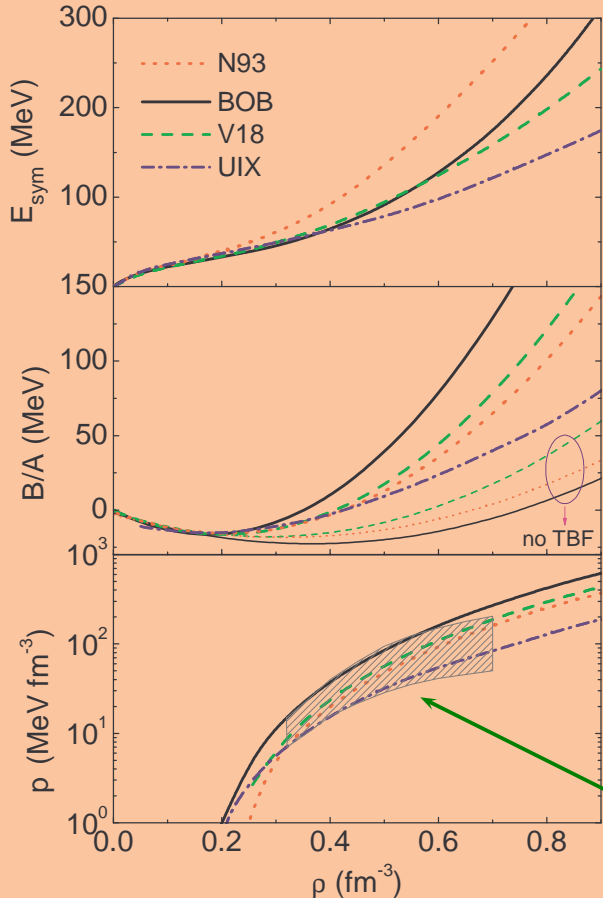
- Compare micro TBF with  $V_{18}$ , Bonn B, or Nijmegen 93 potential and UIX TBF (with  $V_{18}$ ):

$$\bar{V}_{ij}(r) = (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)V_C(r) + (\sigma_1 \cdot \sigma_2)V_S(r) + V_I(r) \\ + S_{ij}(\hat{r}) \left[ (\tau_1 \cdot \tau_2)V_T(r) + V_Q(r) \right] \quad \text{at } \rho = \rho_0 :$$



# Results of BHF Approach:

- Symmetry energy, EOS, Saturation properties:

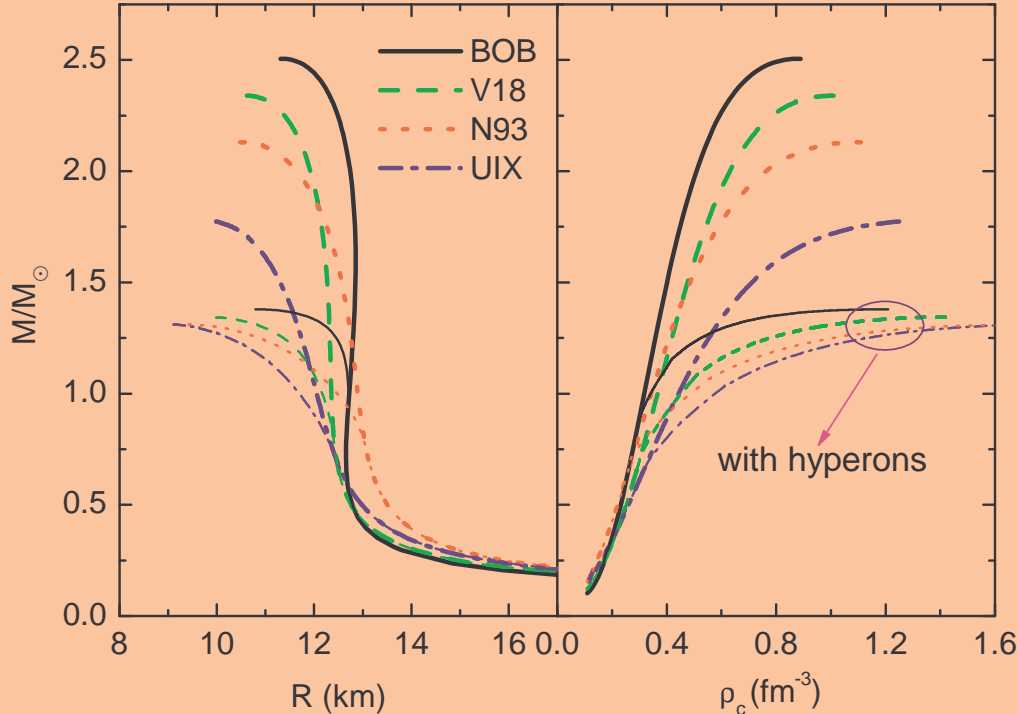


|            | $[\rho, B/A]_0$<br>[ $\text{fm}^{-3}$ , MeV] | $K$<br>MeV | $E_{\text{sym}}$<br>MeV | $E'_{\text{sym}}$<br>MeV |
|------------|--|------------|-------------------------|--------------------------|
| <b>N93</b> | [0.18, -15.4]                                | 216        | 34.0                    | 35.5                     |
| <b>BOB</b> | [0.17, -15.9]                                | 244        | 29.4                    | 24.8                     |
| <b>V18</b> | [0.20, -14.7]                                | 226        | 30.6                    | 33.8                     |
| <b>UIX</b> | [0.18, -15.3]                                | 192        | 33.5                    | 24.5                     |

Nuclear flow analysis of Science 298, 1592 (2002)

# Neutron star structure:

- Solve TOV equations:



NSC89 NY potential  
No YY  
No hyperon TBF

↪ Large variation with nucleonic TBF  
Self-regulating softening due to hyperon appearance

## Summary:

- Consistent microscopic TBF + BHF provide reasonable saturation
- Uncertain high-density behaviour:  $M_{\max} \approx 1.8, \dots, 2.5 M_{\odot}$
- BHF EOS including hyperons predicts  $M_{\max} \approx 1.3, \dots, 1.5 M_{\odot}$
- Inclusion of quark matter phase raises  $M_{\max}$  to less than  $1.7 M_{\odot}$
- Masses above  $2 M_{\odot}$  not explainable in our theoretical frame !

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## Future:

- Technical improvements: 3rd nucleon average, static approx., ...
- BHF with TBF + 3 hole line corrections
- Micro TBF in light nuclei ?