Superfluid Properties of Neutron Star Crust

Jérôme MARGUERON

Institut de Physique Nucléaire et Université Paris-Sud, Orsay, France

The Modern Physics of Compact Stars

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Why the crust is important ?

Cf. previous talks...

Crust = interface between the core and the observed surface.

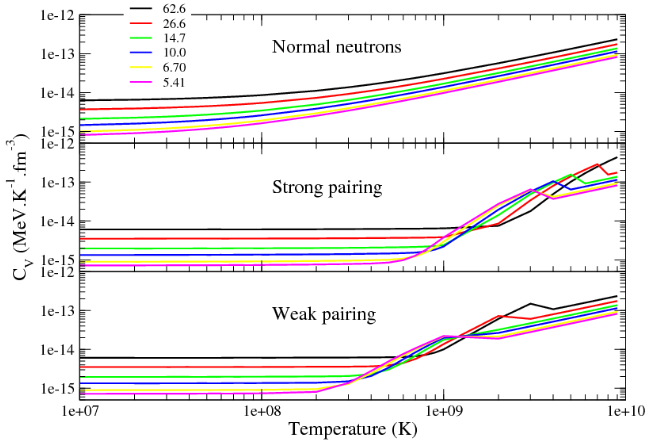
Some observations:

- rotation and spin jumps: Glitches (vortex pinning) ,
- surface temperature: cooling, thermal relaxation of LMXRT (specific heat) ,
- flares: crust shear modes, ...

Superfluidity of the unbound neutrons might play an important role.

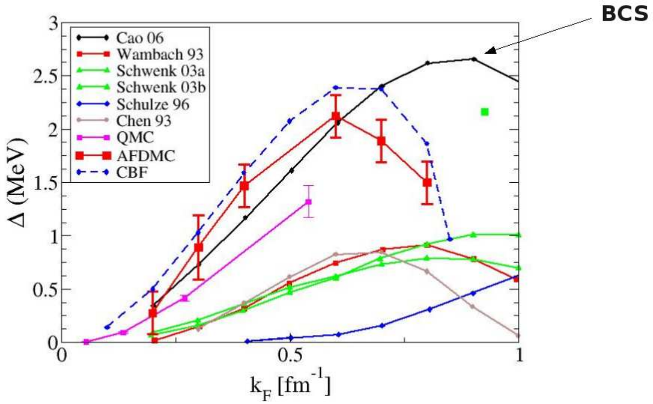
Specific heat in the inner crust

inner crust = nuclear clusters (lattice) + unbound neutrons + electrons



Do we know the density dependence of the pairing gap?

In neutron matter:

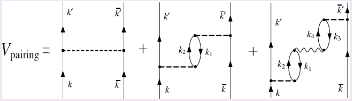


In symmetric matter??

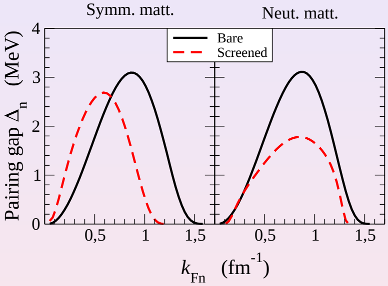
and its isospin dependence?

Microscopic treatment based on the realistic N-N interaction.
`Cao, Lombardo, Schuck, PRC 74, 064301 (2006)`

Bare + medium polarization:



- reference calculation including only the bare NN interaction (bare),
- additional contribution from medium polarization effects (screened).



Medium polarization effects:

- In symmetric matter: shift the peak to lower densities.
- In neutron matter: reduction of the peak (/2).

How does the gap influence the specific heat ?

Levenfish & Yakovlev, Astron. Rep. 38 (1994) 247:

$$c_V^{\mathrm{sf}}(T,\Delta) \approx R(T/T_c) \times c_V^{\mathrm{normal}}(T). \tag{1}$$

- $R \longrightarrow$ superfluid effects ,

$$R \propto \exp -(T_c/T)^2 \tag{2}$$

- $c_V^{\mathrm{normal}} \longrightarrow$ density of states effects.
  (boundary conditions, discretization of the continuum, ...).

$$c_V^{\mathrm{normal}} \propto T \tag{3}$$

What are the effects due to the nuclear clusters (lattice)?

The crust of neutron stars

Lindemann melting criterion: mean fluctuation of ions should be small compared with average ion spacing (r_z).
Then matter is solid if

$$T < T_m \approx Z^2 e^2 / 100 r_z$$

.

For ^{56}Fe, solid if $\rho > 10^7$ g/cm^{-3}.

Consequences:
Crust: Coulomb lattice made of nuclear clusters.
Inner crust: lattice + unbound neutrons ($\rho > \rho_{drip}$).

$\rightarrow$ Proper description: band theory.
Wigner-Seitz approximation (in most practical calculations).

Effects of the lattice

$$\Delta = 0$$

Band theory

Hamiltonian:

$$h_0^{(q)} \equiv -\boldsymbol{\nabla} \cdot \frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r})}\boldsymbol{\nabla} + U_q(\boldsymbol{r}) - \mathrm{i}\boldsymbol{W_q}(\boldsymbol{r}) \cdot \boldsymbol{\nabla} \times \boldsymbol{\sigma}\,, \tag{4}$$

Band theory:
Periodic potential: define an irreducible cell.
Floquet-Bloch theorem,

$$\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = u_{\alpha\boldsymbol{k}}(\boldsymbol{r})e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\,, \tag{5}$$

$u_{\alpha\boldsymbol{k}}(\boldsymbol{r})$ have the full periodicity of the lattice.
α is discrete, $\boldsymbol{k}$ is continuous.
Satisfy the boundary conditions:

$$\varphi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r}+\boldsymbol{T}) = e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{T}}\varphi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r})\,, \tag{6}$$

Schroedinger eq.:

$$h_0^{(q)}\varphi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r}) = \varepsilon_{\alpha\boldsymbol{k}}^{(q)}\,\varphi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r}) \tag{7}$$

Equation for $u_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r})$:

$$(h_0^{(q)} + h_{\boldsymbol{k}}^{(q)})u_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r}) = \varepsilon_{\alpha\boldsymbol{k}}^{(q)}\,u_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r}) \tag{8}$$

with

$$h_{\boldsymbol{k}}^{(q)} \equiv \frac{\hbar^2 k^2}{2m_q^{\oplus}(\boldsymbol{r})} + \boldsymbol{v_q}\cdot\hbar\boldsymbol{k}\,, \tag{9}$$

$$\boldsymbol{v_q} \equiv \frac{1}{\mathrm{i}\hbar}[\boldsymbol{r}, h_0^{(q)}]\,. \tag{10}$$

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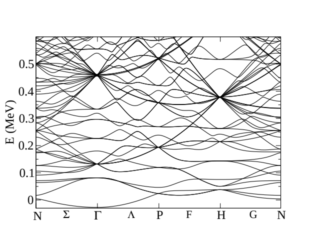
Comparison between band theory and WS approximation

Band theory:
unit cell depends on the geometry of the lattice
Floquet-Bloch theorem,

$$\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = u_{\alpha\boldsymbol{k}}(\boldsymbol{r})e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \, ,$$

Equation for $u^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r})$:

$$(h^{(q)}_0 + h^{(q)}_{\boldsymbol{k}})u^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}}\, u^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r})$$



WS approximations:

- spherical unit cell
- Dirichlet / Neumann boundary conditions
- $h^{(q)}_{\boldsymbol{k}} = 0$

Comparison band theory/WS:
Self-consistent HF+WS $\rightarrow$ potential $\rightarrow$ band theory (1 iteration).

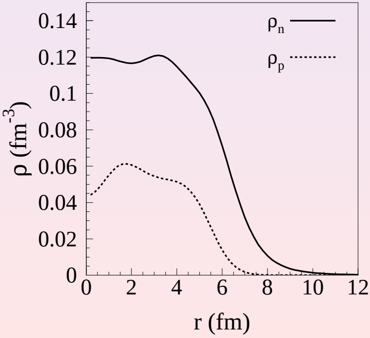
`N. Chamel et al., PRC 75, 055806 (2007)`

Comparison between band theory and WS approximation

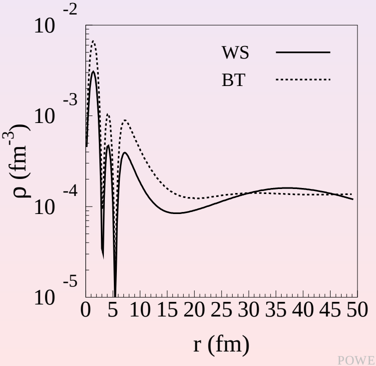
From Negele-Vautherin: ^{200}Zr (40 protons, 90 bound neutrons and 70 unbound neutrons).

Comparison of the densities:

Bound neutron density



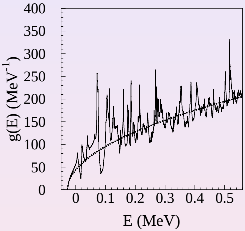
Unbound neutron density



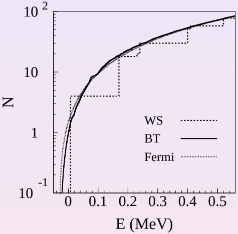
Comparison between band theory and WS approximation

Single particle spectrum of the unbound neutrons

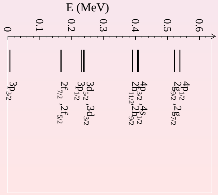
Density of states (unbound neutrons)



Integrated density of states (unbound neutrons)



WS approximation:



$\longrightarrow$ Shell effects $\sim$ 100 keV

Preliminary conclusions

WS approximation is justified if

- Static properties: $T > 100$ keV,
- Dynamical processes: typical energy > 100 keV.
  neutrino scattering, ...

N. Chamel, S. Naimi, E. Khan, J.M., PRC 75, 055806 (2007)

Application: specific heat (normal state)

$$c_V(T) = \left.\frac{\partial U}{\partial T}\right|_V = T\left.\frac{\partial S}{\partial T}\right|_V , \tag{11}$$

$$S = -\sum_\alpha \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi^3)} \left[f_{\alpha\boldsymbol{k}} \ln f_{\alpha\boldsymbol{k}} + (1 - f_{\alpha\boldsymbol{k}}) \ln(1 - f_{\alpha\boldsymbol{k}}) \right] \tag{12}$$

We consider the 3 first layers of Negele & Vautherin:

ρ [g.cm^{-3}]	Z	N	R_{cell} [fm]	ρ_n^G [fm^{-3}]	$m_n^\star/m_n$	ξ_F
6.69×10^{11}	40	160	49.24	$1.3\ 10^{-4}$	4.0	0.44
1.00×10^{12}	40	210	46.33	$2.6\ 10^{-4}$	3.6	0.49
1.47×10^{12}	40	280	44.30	$4.9\ 10^{-4}$	3.2	0.52

What do we expect?

Lattice shall be important if $\lambda_F = 2\pi/k_F \sim$ lattice spacing.
λ_F=40, 32, 26 fm.
$\rightarrow$ unbound neutrons shall be strongly scattered through the lattice

Effective mass for 1 electron in a solid:

$$\left.\frac{1}{m_e^\star}\right|_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon^{(e)}_{\alpha\boldsymbol{k}}}{\partial k_i \partial k_j} \tag{13}$$

Average effective massfor the unbounds neutrons:

$$m_n^\star = \rho_n^G/\mathcal{K}\,, \qquad \mathcal{K} = \frac{1}{3}\sum_{\alpha,i}\int_{\mathrm{F}} \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\frac{1}{\hbar^2}\frac{\partial^2 \varepsilon_{\alpha\boldsymbol{k}}}{\partial k_i \partial k_i}\,, \tag{14}$$

Deformation of the Fermi surface:

$$\mathcal{S}_{\mathrm{F}} = \xi_{\mathrm{F}} 4\pi k_{\mathrm{F}}^2 \tag{15}$$

Entropy for the unbound neutrons in the crust

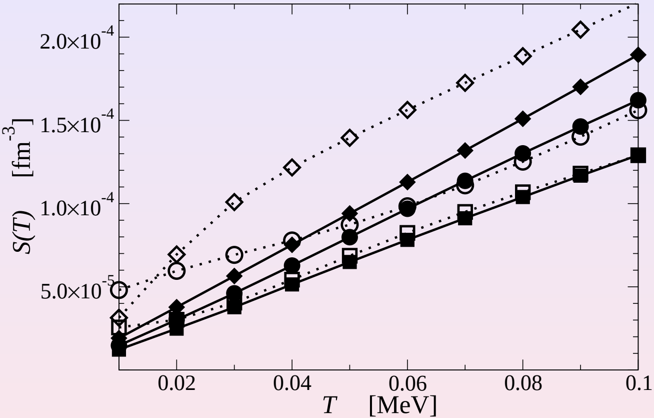
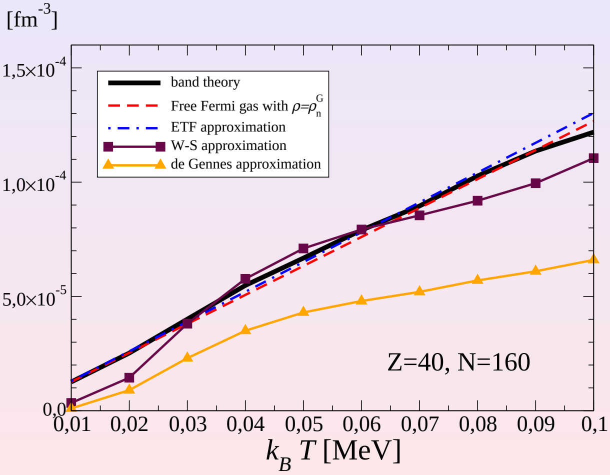


Figure: Entropy per unit volume calculated with the band theory (solid thick line) and with the Wigner-Seitz approximation (dashed line) for the three layers with $N = 160$ (squares), $N = 210$ (circles) and $N = 280$ (diamonds).

Results for N=160



Various approximations:

Fermi gas:

$$c_V^{\mathrm{FG}}(T) = \left(\frac{\pi}{3}\right)^{2/3} \frac{m_n^{\oplus} T}{\hbar^2} \left(\rho_n^G\right)^{1/3} \quad , \tag{16}$$

Semi-classical Extended-Thomas-Fermi

$$c_V^{\mathrm{TF}}(T) = \left(\frac{\pi}{3}\right)^{2/3} \frac{T}{\hbar^2} \int \frac{\mathrm{d}^3\boldsymbol{r}}{V_{\mathrm{cell}}} m_n^{\oplus}(\boldsymbol{r}) \rho_n(\boldsymbol{r})^{1/3} \, . \tag{17}$$

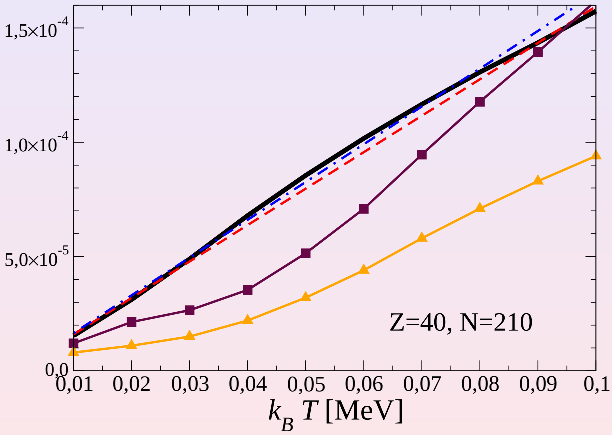
Wigner-Seitz:

$$c_V^{\mathrm{WS}} = T \left.\frac{\partial S^{\mathrm{WS}}}{\partial T}\right|_V \, . \tag{18}$$

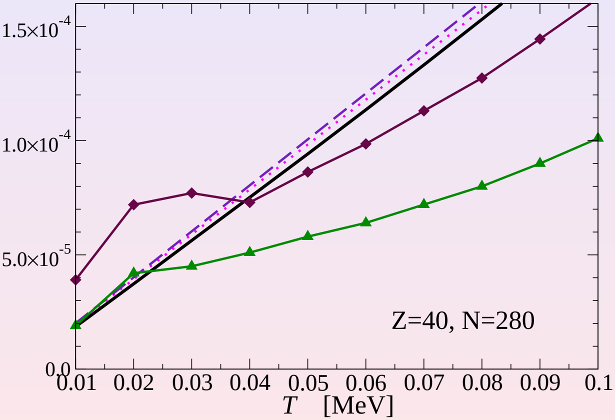
De Gennes (neglecting $\varepsilon(T)$ and $\mu(T)$):

$$c_V^{\mathrm{DG}}(T) = \frac{1}{V_{\mathrm{cell}}} \sum_{\alpha} g_\alpha f_\alpha (1 - f_\alpha) \left(\frac{\varepsilon_\alpha - \mu_n}{T}\right)^2 \, . \tag{19}$$

Results for N=210



Results for N=280



Are neutrons "Free"?

Group velocity of the unbound neutrons: $\boldsymbol{v}_{\alpha \boldsymbol{k}} = \hbar^{-1} \nabla_{\boldsymbol{k}} \varepsilon_{\alpha \boldsymbol{k}}$.
Effective mass:

$$\mathcal{K} = \frac{1}{3(2\pi)^3 \hbar} \sum_{\alpha} \oint_{\mathcal{S}_{\rm F}} |\boldsymbol{v}_{\alpha \boldsymbol{k}}| \mathrm{d}\mathcal{S} \,. \tag{20}$$

Consequence: $\boldsymbol{v}_{\alpha \boldsymbol{k}} \rightarrow \boldsymbol{v}_{\alpha \boldsymbol{k}}/2$ due to the lattice.

Specific heat:

$$c_V(T) \simeq \frac{\pi^2}{3} g(\varepsilon_{\rm F}) T \,. \tag{21}$$

Density of states:

$$g(\varepsilon_{\rm F}) = \frac{1}{(2\pi)^3 \hbar} \sum_{\alpha} \oint_{\mathcal{S}_{\rm F}} \frac{d\mathcal{S}}{|\boldsymbol{v}_{\alpha \boldsymbol{k}}|} \,. \tag{22}$$

$\longrightarrow$ Cancellation effect.

summary

Unbound neutrons in the shallow layers of the inner crust are strongly scattered by the lattice, but due to cancellation effect, the specific is very similar to that of a free Fermi gas.

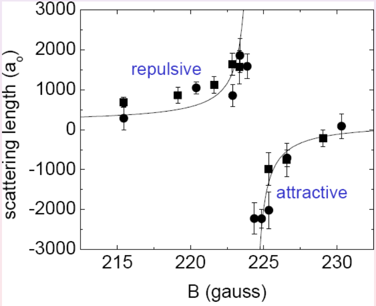
`N. Chamel, J. M., E. Khan, submitted`

$\Delta \neq 0$

BCS-BEC crossover

BCS-BEC crossover in cold atomic gas

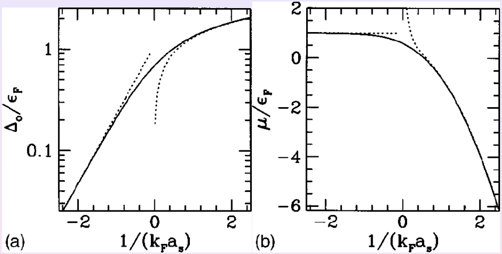
Magnetic Feshbach resonance
Interaction characterized by s-wave scattering length:
$a > 0$ repulsion, $a < 0$ attraction, large $|a|$ strong interaction.



C. A. Regal and D. S. Jin, PRL **90**, 230404 (2003)

- BEC state of diatomic molecules (a >0)
  condensation in the ground state
  narrow in r, large in E.
- BCS-BEC crossover region
  Generalized "Cooper pairs"
  Unitary limit: $1/k_F a \to 0$
- BCS state of cooper pairs (a <0)
  pairing in momentum space near the Fermi energy
  narrow in E, large in r.

General properties of the BCS-BEC phase tansition



J.R. Engelbrecht, et al., PRB 55, 15153 (1997)
M. Matsuo, PRC 73, 044309 (2006)

$(k_{Fn}a_{nn})^{-1}$	$P(d_n)$	ξ_{rms}/d_n	Δ_n/ϵ_{Fn}	ν_n/ϵ_{Fn}	
−1	0.81	1.10	0.21	0.97	BCS boundary
0	0.99	0.36	0.69	0.60	unitarity limit
1	1.00	0.19	1.33	−0.77	BEC boundary

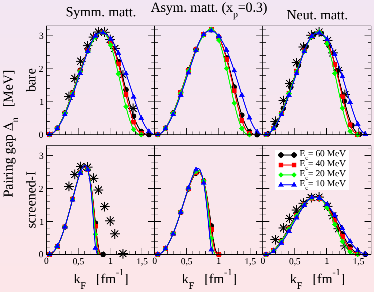
Adjustment of a simple pairing interaction

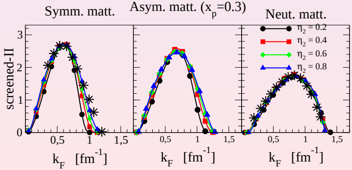
Reproduces the scattering length and the Pairing gap in uniform matter obtained from microscopic treatment based on the realistic N-N interaction.

self-consistent calculations of nuclear matter and nuclei.

Result of the adjustment:

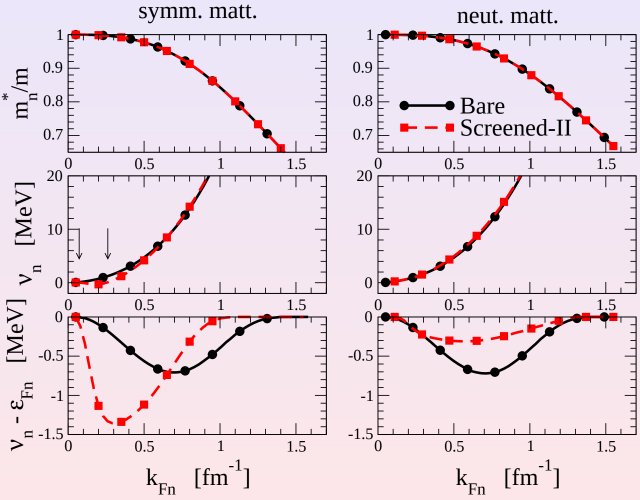
$$g = g_1 \qquad\qquad g = g_1 + g_2$$





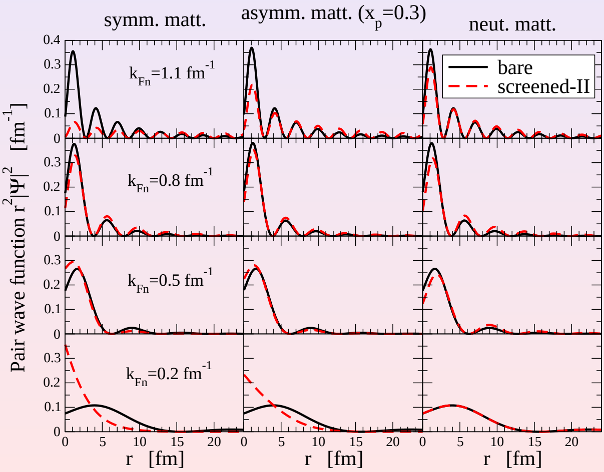
Neutron Fermi momentum k_{Fn}: $\rho_n \equiv k_{Fn}^3/3\pi^2$.

Effective mass (Sly4) and chemical potential



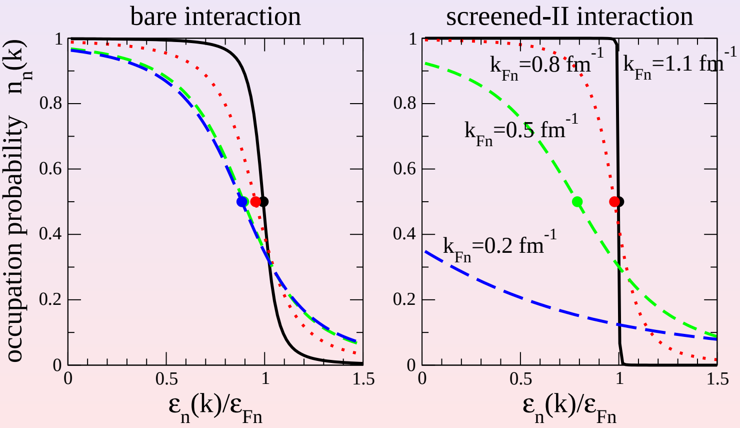
Evolution of the Cooper pair wave function

$$\Psi_{\text{pair}}(k) = C\, u_k v_k \tag{23}$$



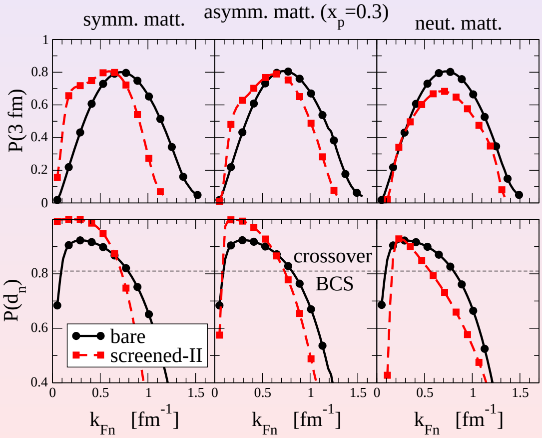
Occupation probabilities $n_n(k)$ and chemical potential ν_n

$$n_n(k) = \frac{1}{2}\left[1 - \frac{\epsilon_n(k) - \nu_n}{E_n(k)}\right] \tag{24}$$



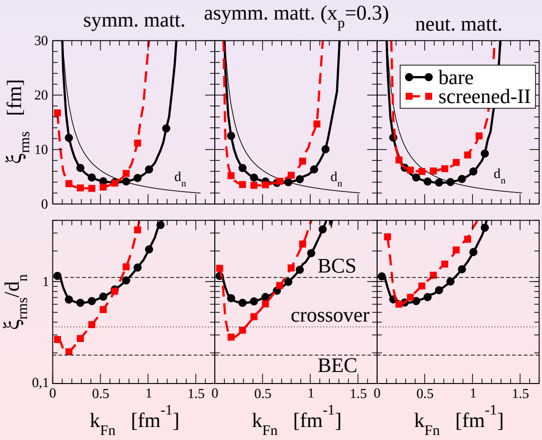
Properties of the probability $P(r)$

$$P(r) = \int_0^r dr' r'^2 |\Psi_{\text{pair}}(r')|^2 \tag{25}$$

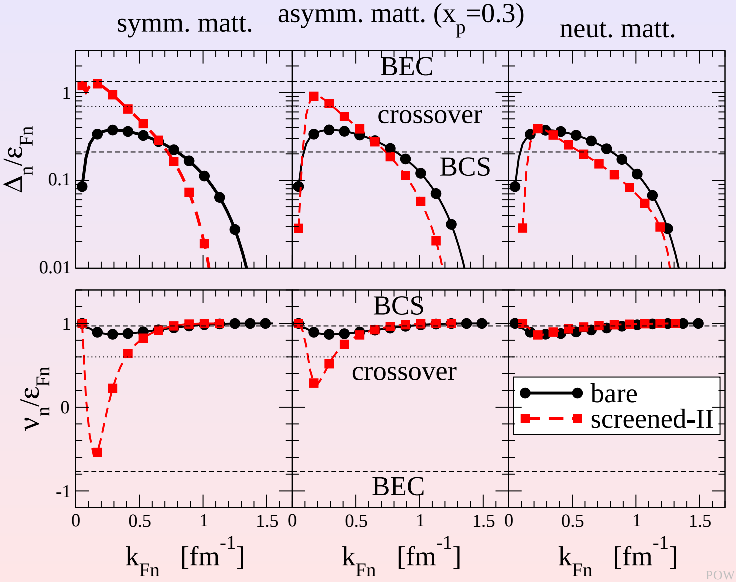


Spatial correlations

$$\xi_{\rm rms} = \int_0^\infty dr r^4 |\Psi_{\rm pair}(r)|^2 \text{ and } d_n = \rho_n^{1/3} \tag{26}$$



BCS-BEC phase-diagram and pairing interactions



Summary

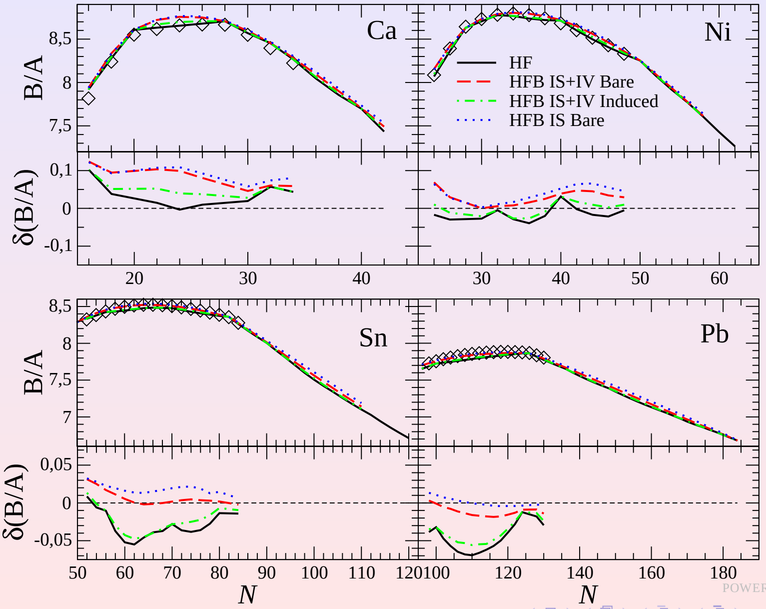
- New effective pairing interaction which reproduces microscopic pairing gaps in symmetric and neutron matter.
- Medium polarization effects:
  $\longrightarrow$ reduction of the bare gap in neutron matter,
  $\longrightarrow$ strong attraction (quasi-BEC state) in low density symmetric matter.

Consequences: Strong correlations $\longrightarrow$ modification of the EoS
What about constraints in nuclei ?

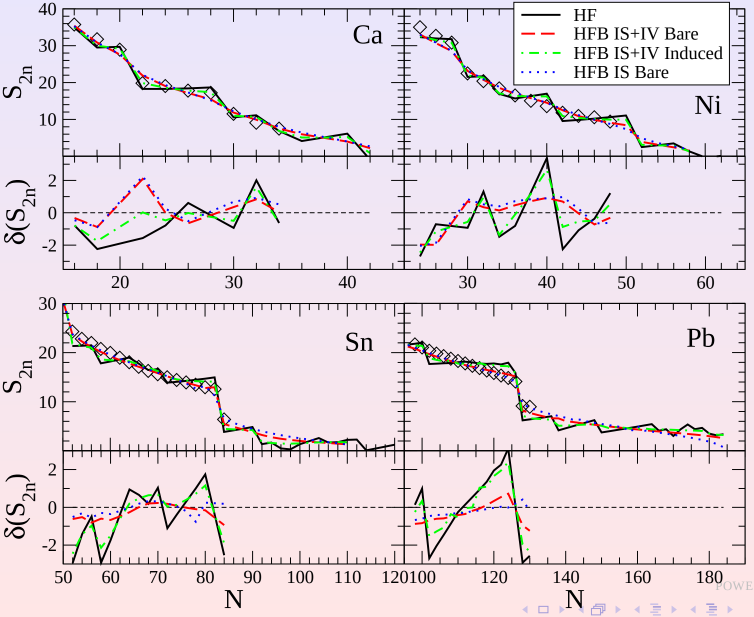
`J. M., H. Sagawa, K. Hagino, Phys. Rev. C 76, 064316 (2007)`

Neutron pairs in semi-magic nuclei

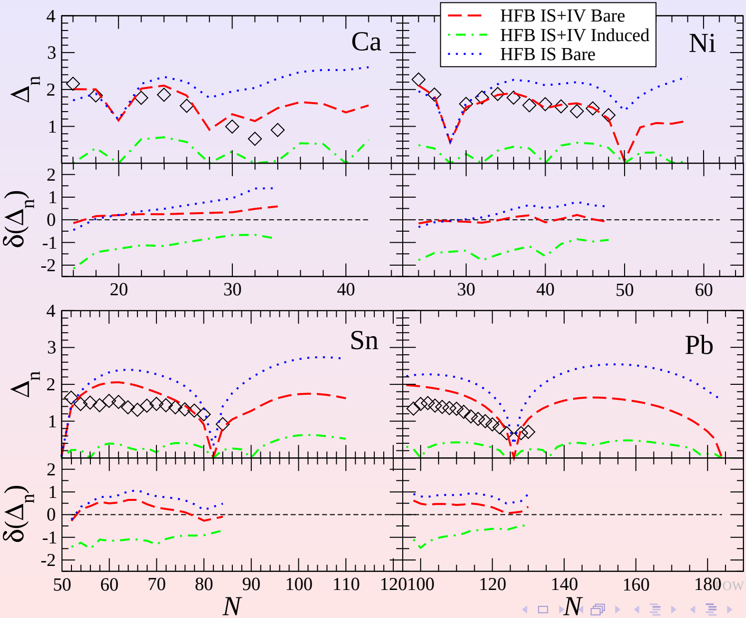
Binding energies



Two neutrons separation energies



Pairing gaps



General Conclusions

Unbound neutrons in the crust has very interesting properties:

- scattered by the lattice
- specific heat close to that of a Fermi gas
- BCS-BEC crossover triggered by the density

We set up a treatment of the pairing which could be used in nuclear matter and in nuclei.
$\rightarrow$ Comparison with nuclei might provide constraints to the models.
In the futur, expect more microscopic calculations in symmetric matter.

Outlooks

Developpement of a Band theory including pairing.

Continue the link pairing in matter and in nuclei (inclusion of Coulomb repulsion between protons, of the particle-vibration coupling, ...)