Fedora GNU/Linux; LaTeX 2ϵ; xfig

Superconductor coupled to a superfluid: flux tubes and the type-I/type-II transition

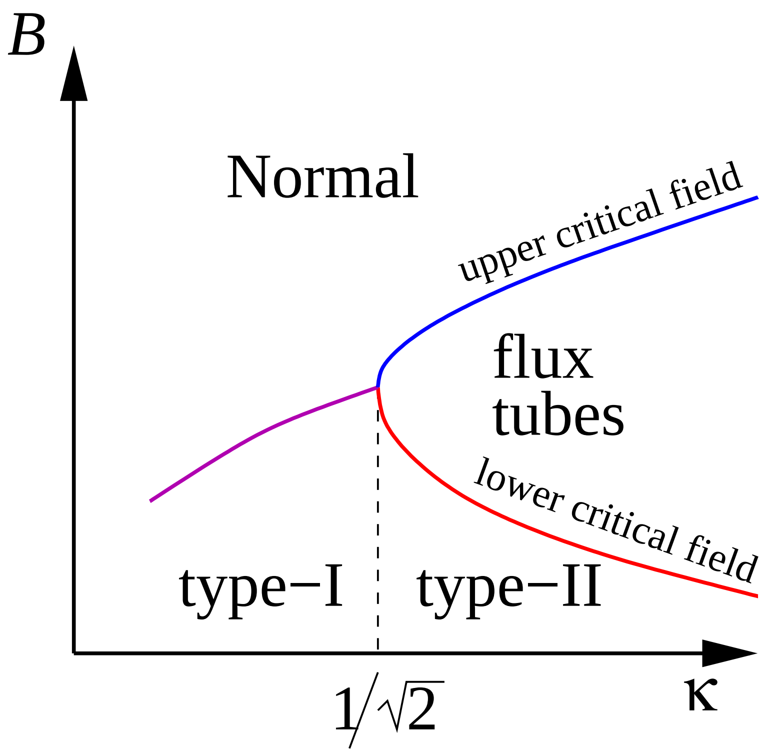
Mark Alford

Washington University
Saint Louis, USA

M. Alford, G. Good, arXiv:0712.1810 (Phys. Rev. B78:024510, 2008).

Phys. Rev. Focus 22, 3 (July 2008)

Superconductor at low temperature



- Meissner effect: excludes mag field, London penetration depth λ
- Coherence length ξ; define $\kappa = \lambda/\xi$.
- $\kappa > \frac{1}{\sqrt{2}}$: type-II, flux tubes repel; flux tube lattice
- $\kappa < \frac{1}{\sqrt{2}}$, type-I, flux tubes attract; macroscopic normal region

We calculated flux tube energetics, i.e. behavior at lower critical field.

What if the superconducting condensate is coupled to a superfluid?

E.g. protons and neutrons in nuclear matter.

Ginzburg-Landau theory for superconductor coupled to superfluid

ϕ_p and ϕ_n are proton and neutron pair condensates.

$$\mathcal{F} = \frac{\hbar^2}{4m_p}(|\nabla\phi_n|^2 + |D\phi_p|^2) + \frac{|\nabla \times \mathbf{A}|^2}{8\pi} + V(|\phi_p|^2, |\phi_n|^2) + U_{ent}(\phi_p, \phi_n) \qquad \left[D = (\nabla - \frac{2ie}{\hbar c}\mathbf{A})\right]$$

$$V(|\phi_p|^2, |\phi_n|^2) = \tfrac{a_{pp}}{2}\left(|\phi_p|^2 - \langle\phi_p\rangle^2\right)^2 + \tfrac{a_{nn}}{2}\left(|\phi_n|^2 - \langle\phi_n\rangle^2\right)^2 + a_{pn}\left(|\phi_p|^2 - \langle\phi_p\rangle^2\right)\left(|\phi_n|^2 - \langle\phi_n\rangle^2\right) \ .$$

$$U_{ent} = -\frac{\hbar^2}{4m_p}\frac{\sigma}{2\langle\phi_p\rangle\langle\phi_n\rangle}\Big[\phi_p^*\phi_n^*\left(D\phi_p\cdot\nabla\phi_n\right) + \phi_p^*\phi_n\left(D\phi_p\cdot\nabla\phi_n^*\right) + \phi_p\phi_n\left((D\phi_p)^*\cdot\nabla\phi_n^*\right) + \phi_p\phi_n^*\left((D\phi_p)^*\cdot\nabla\phi_n\right)\Big]$$

Alpar, Langer, Sauls, Astrophys. J. 282, 533 (1984); Comer and Joynt, gr-qc/0212083.

σ is gradient coupling, a_{pp}, a_{nn}, and a_{pn} are density couplings

Flux tube solution with n flux quanta

$$\begin{aligned} \phi_p &= \langle\phi_p\rangle\ f(r)e^{in\theta} \\ \phi_n &= \langle\phi_n\rangle\ g(r) \\ \mathbf{A} &= \frac{n\hbar c}{2er} a(r)\hat{\theta} \end{aligned} \qquad \begin{aligned} \lambda &\equiv \sqrt{\frac{2m_p c^2}{4\pi(2e)^2\langle\phi_p\rangle^2}} \\ \xi &\equiv \sqrt{\frac{\hbar^2}{4m_p a_{pp}\langle\phi_p\rangle^2}} \\ \kappa &= \lambda/\xi, \end{aligned}$$

Rescale to $\tilde{r} = r/\xi$, set $a_{nn} = a_{pp}$, $\beta = a_{pn}/a_{pp}$

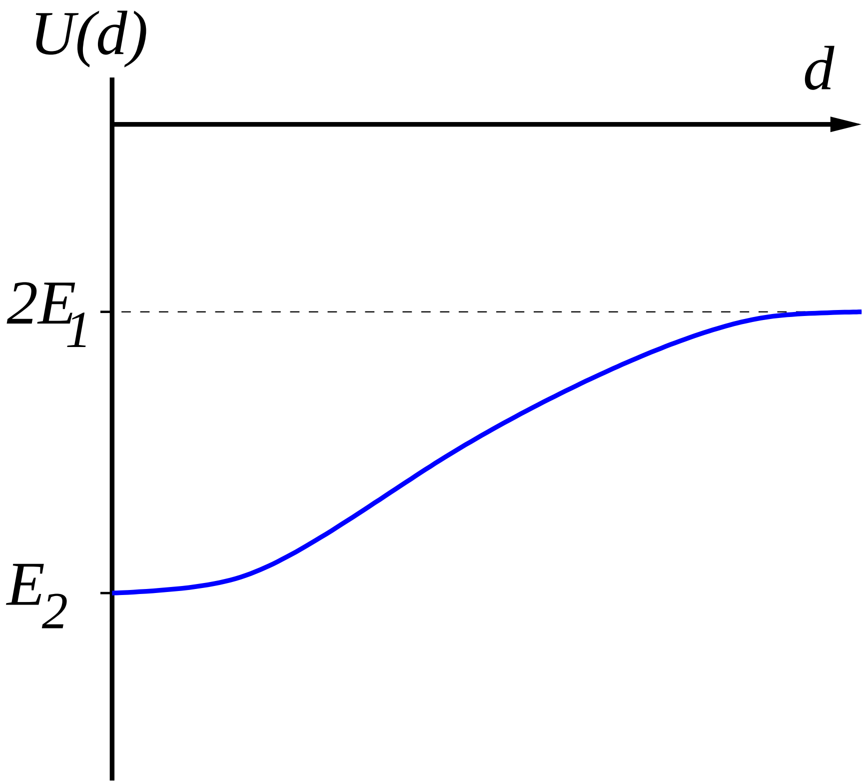
$$\begin{aligned} E_n &= 2\pi a_{pp}\langle\phi_p\rangle^4\xi^2 \int_0^\infty \tilde{r}d\tilde{r}\Bigg\{(f')^2 + \frac{n^2 f^2(1-a)^2}{\tilde{r}} + \frac{\langle\phi_n\rangle^2}{\langle\phi_p\rangle^2}(g')^2 \\ &\quad + n^2\kappa^2\frac{(a')^2}{\tilde{r}^2} + \frac{1}{2}\left(f^2-1\right)^2 + \frac{1}{2}\frac{\langle\phi_n\rangle^4}{\langle\phi_p\rangle^4}\left(g^2-1\right)^2 \\ &\quad + \beta\frac{\langle\phi_n\rangle^2}{\langle\phi_p\rangle^2}\left(f^2-1\right)\left(g^2-1\right) - 2\sigma\frac{\langle\phi_n\rangle}{\langle\phi_p\rangle} f\cdot g\cdot f'\cdot g'\Bigg\} \end{aligned}$$

See how E_n depends on κ, β, σ.

Flux tube energetics and type-I/II transition

Favored flux tube "size" n is the one with minimum energy/flux E_n/n.

We are effectively calculating $U(\infty)$ and $U(0)$ in the flux tube interaction potential $U(d)$.



For two $n = 1$ flux tubes,

$E_2 = U(0) \ \Rightarrow E_2/2 = \frac{1}{2}U(0)$

$2E_1 = U(\infty) \ \Rightarrow E_1/1 = \frac{1}{2}U(\infty)$

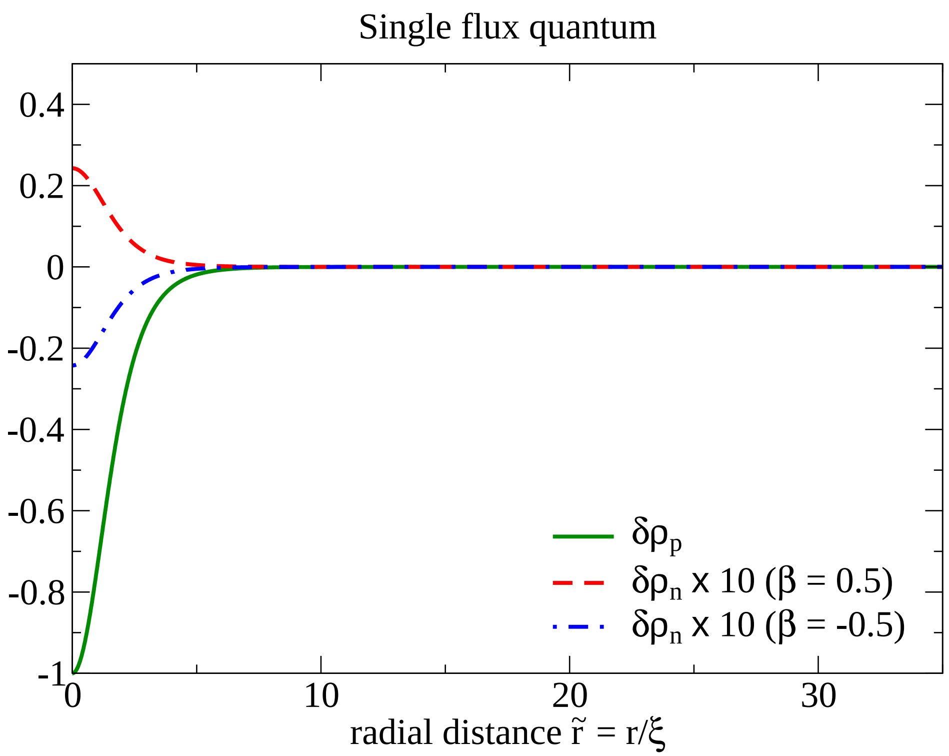
If the potential $U(d)$ is monotonic, then knowing $U(\infty)$ and $U(0)$ we can tell if they attract or repel.

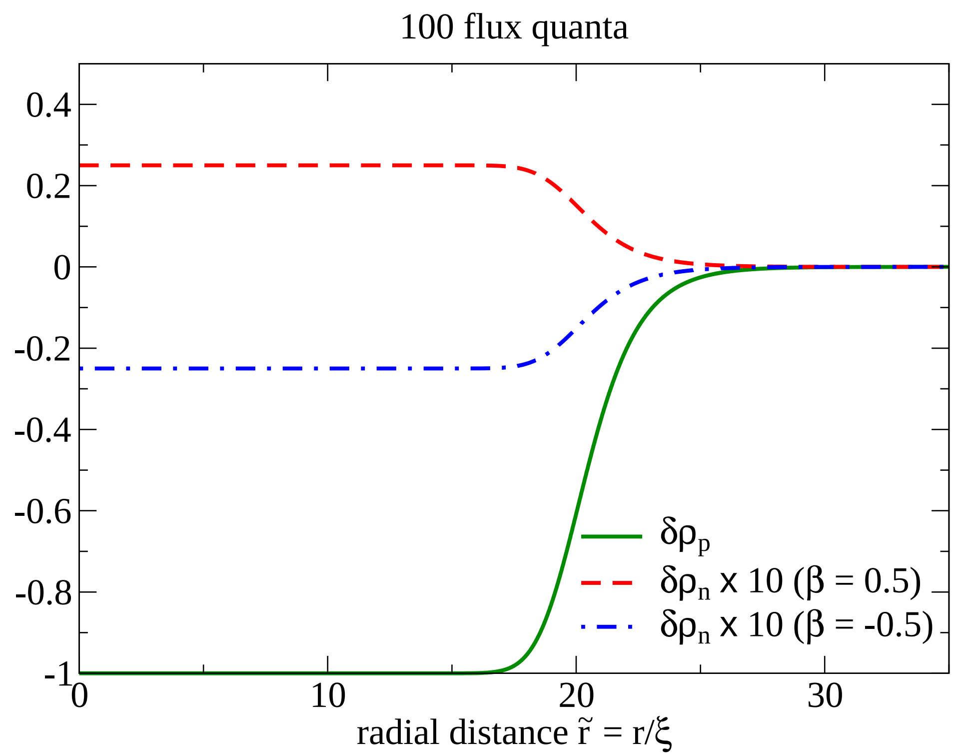
Type-I: flux tubes attract, so E_n/n falls with n (e.g. $\frac{1}{2}E_2 < E_1$)

Type-II: flux tubes repel, so E_n/n rises with n (e.g. $\frac{1}{2}E_2 > E_1$)

Flux tube profiles with nonzero density coupling $\beta = 0.5$

$\kappa = 3.0$, $\sigma = 0$, $\langle\phi_n\rangle^2 / \langle\phi_p\rangle^2 = 20$ as in neutron star.





Deviation $\delta\rho$ of the condensates from their vacuum values.

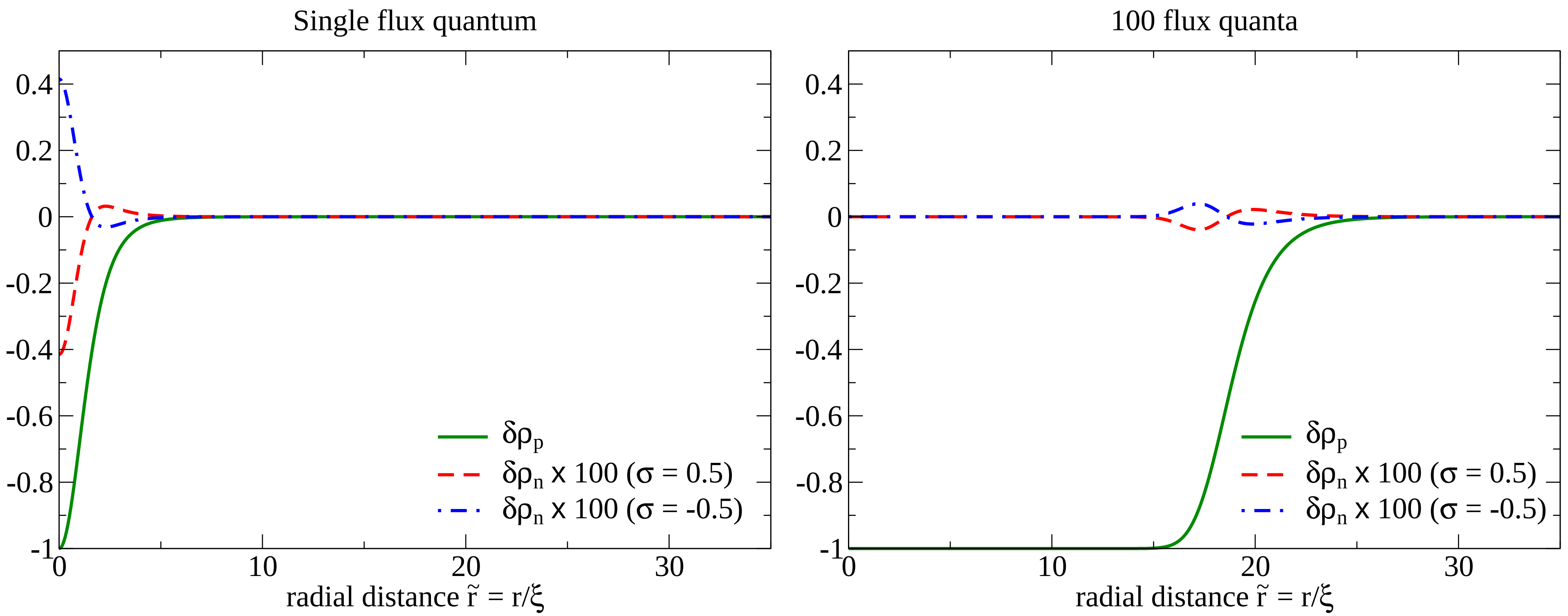
$\beta > 0$ proton condensate repels neutron condensate

$\beta < 0$ proton condensate attracts neutron condensate

Expect core-area contribution to energy, $\sim -\beta^2 n$

Flux tube profiles with nonzero gradient coupling $\sigma = 0.5$

$\kappa = 3.0$, $\beta = 0$, $\langle\phi_n\rangle^2/\langle\phi_p\rangle^2 = 20$ as in neutron star.

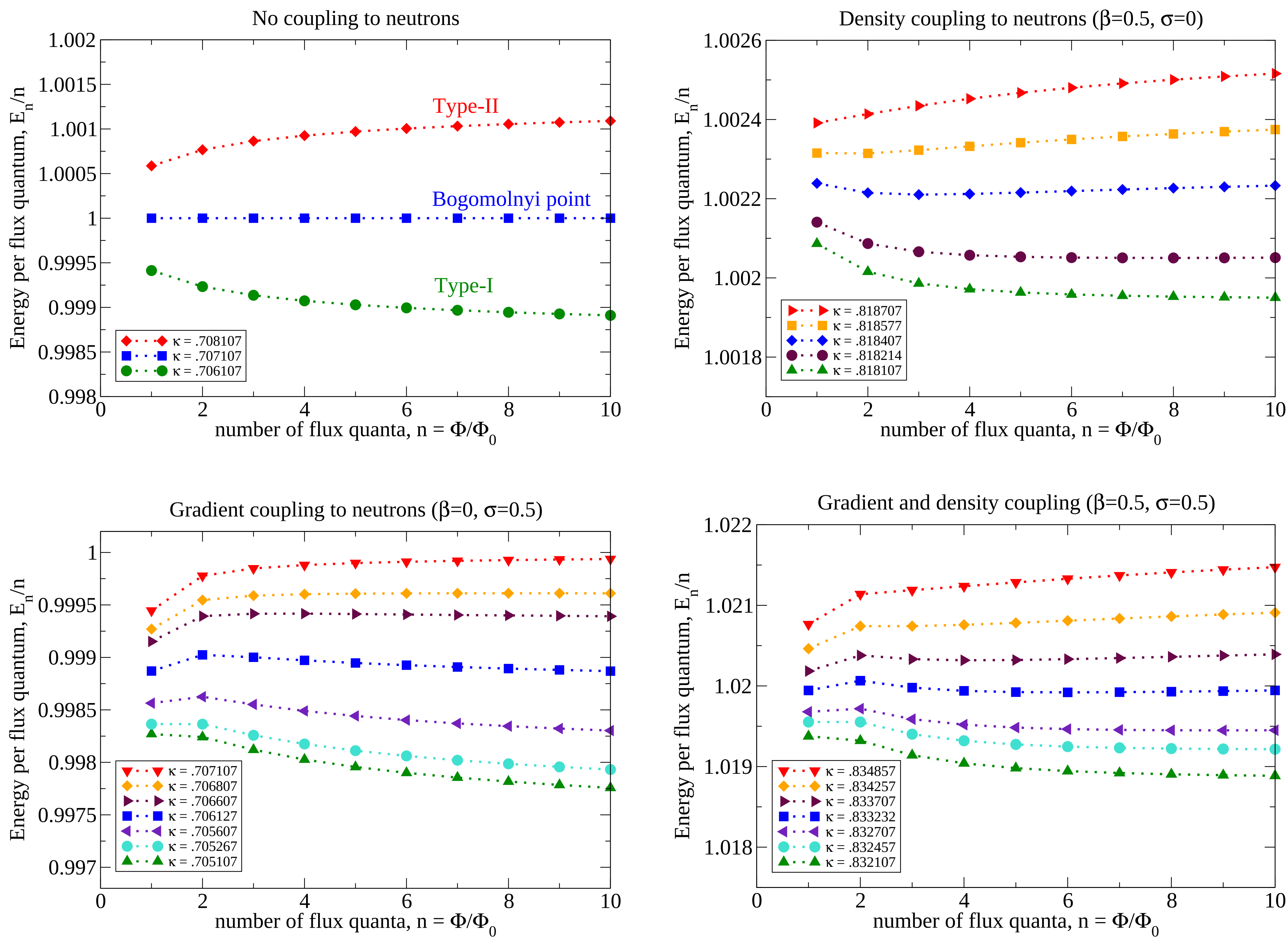


Deviation $\delta\rho$ of the condensates from their vacuum values.

$\sigma > 0$ positive proton gradient favors positive neutron gradient

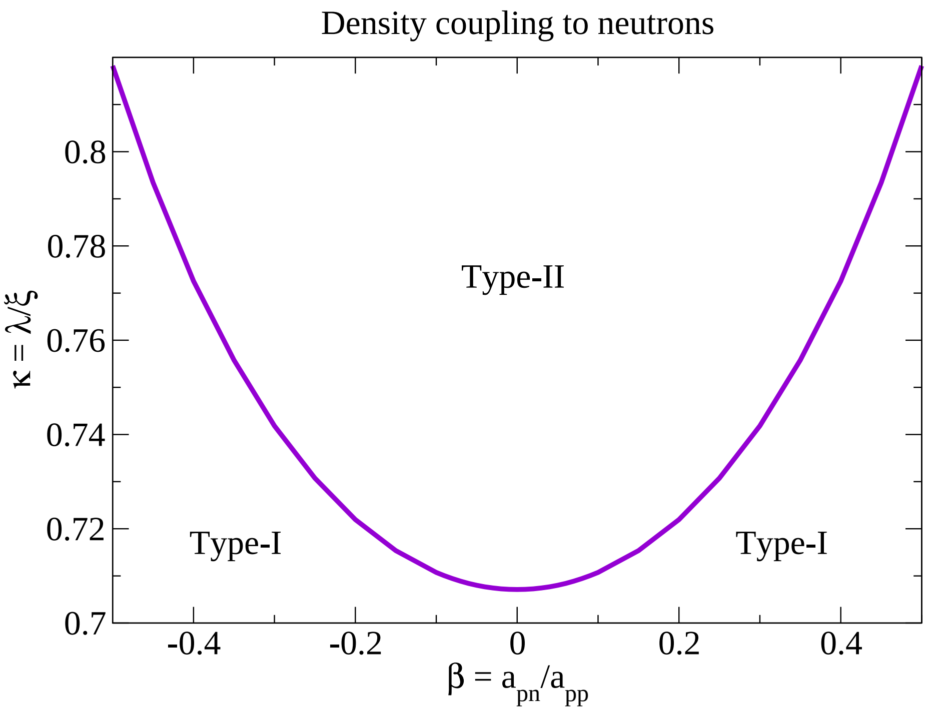
$\sigma < 0$ positive proton gradient favors negative neutron gradient

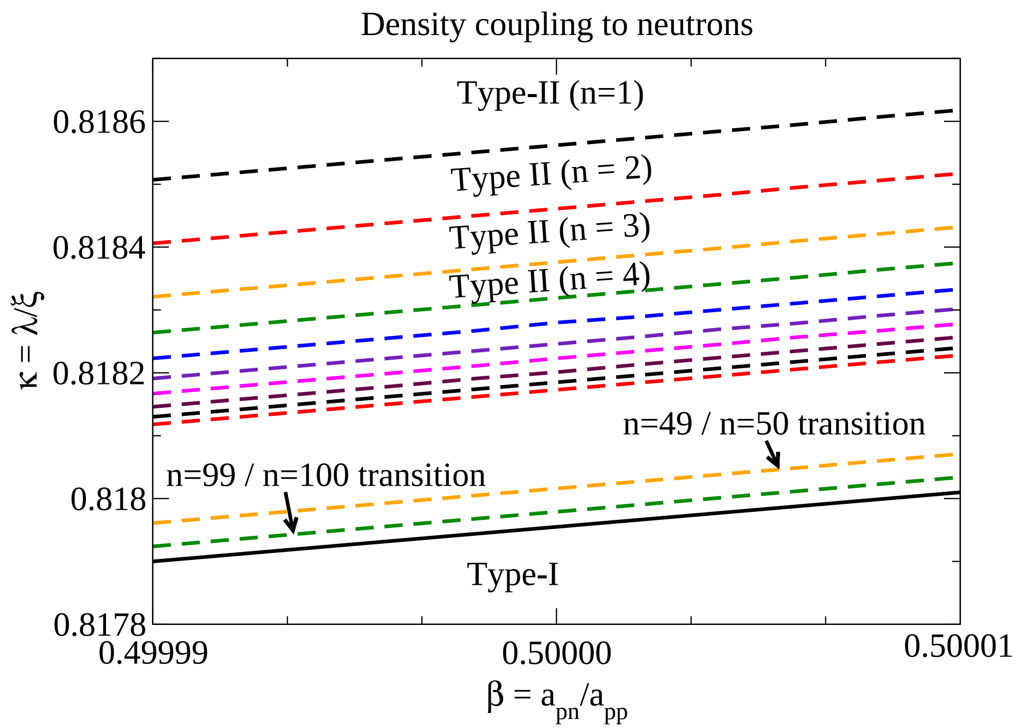
Expect core-perimeter contribution to energy, $\sim -\sigma^2\sqrt{n}$



No coupling to neutrons
Type-II
Bogomolnyi point
Type-I
κ = .708107
κ = .707107
κ = .706107
Density coupling to neutrons (β=0.5, σ=0)
κ = .818707
κ = .818577
κ = .818407
κ = .818214
κ = .818107
Gradient coupling to neutrons (β=0, σ=0.5)
κ = .707107
κ = .706807
κ = .706607
κ = .706127
κ = .705607
κ = .705267
κ = .705107
Gradient and density coupling (β=0.5, σ=0.5)
κ = .834857
κ = .834257
κ = .833707
κ = .833232
κ = .832707
κ = .832457
κ = .832107
Energy per flux quantum, E_n/n
number of flux quanta, n = Φ/Φ_0

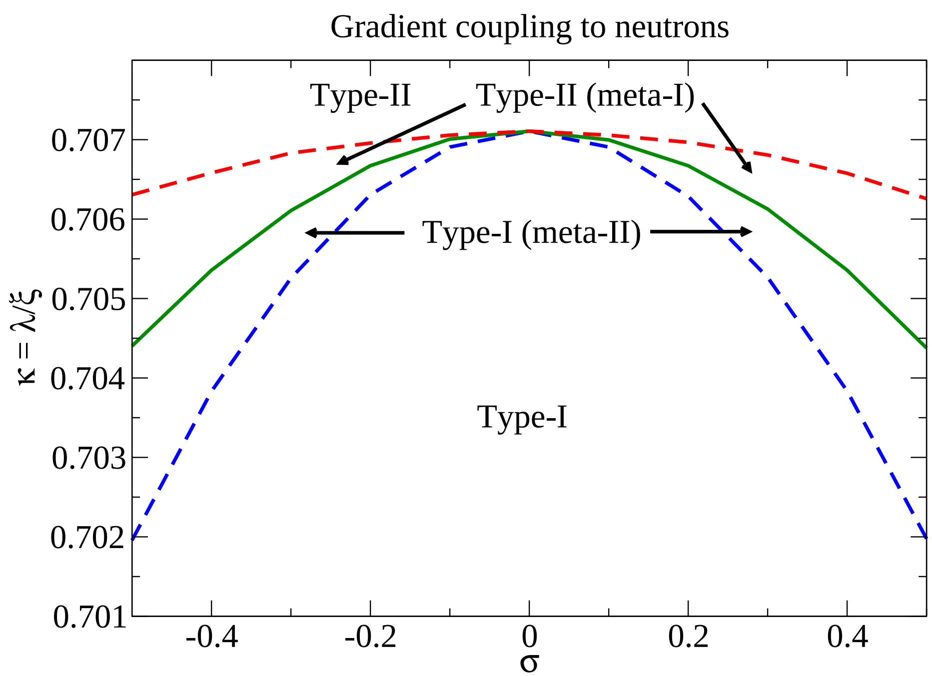
Effect of density coupling on type-I/II boundary





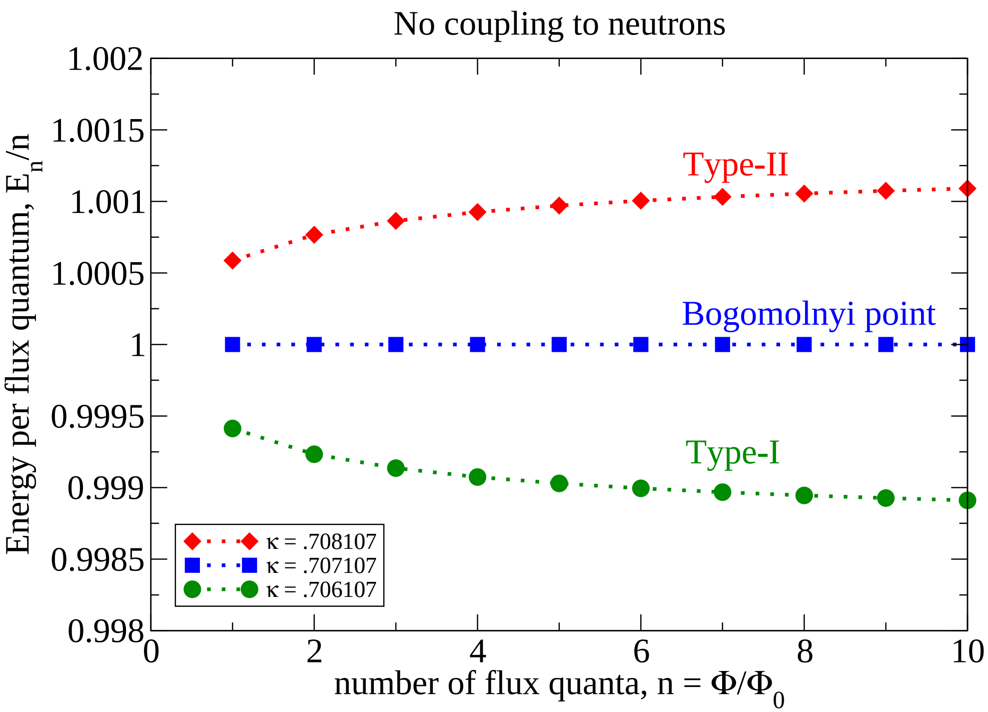
- critical κ shifts upwards, $\kappa_{\text{critical}} \sim 1/\sqrt{2} + \beta^2$
- on the type-II side there is a sequence of "type-II(n)" bands in which the number of flux quanta in the favored flux tube steps through all integer values, reaching infinity when the superconductor becomes type-I.

Effect of gradient coupling on type-I/II boundary



- critical κ shifts downwards
- Spinodal (metastable) regions appear: the type-I/type-II boundary becomes a typical 1st order phase transition

Understanding the uncoupled case



Expand in $\frac{1}{n}$ and $\delta\kappa \equiv \kappa - \frac{1}{\sqrt{2}}$.

$$E_n^{(sc)}(\delta\kappa) = nE_{\text{Bog}} + \delta\kappa M\left(n - c_{\frac{1}{2}}\sqrt{n} + c_1 + \cdots\right)$$

core-area term $\propto n$,

core-perimeter term $\propto \sqrt{n}$.

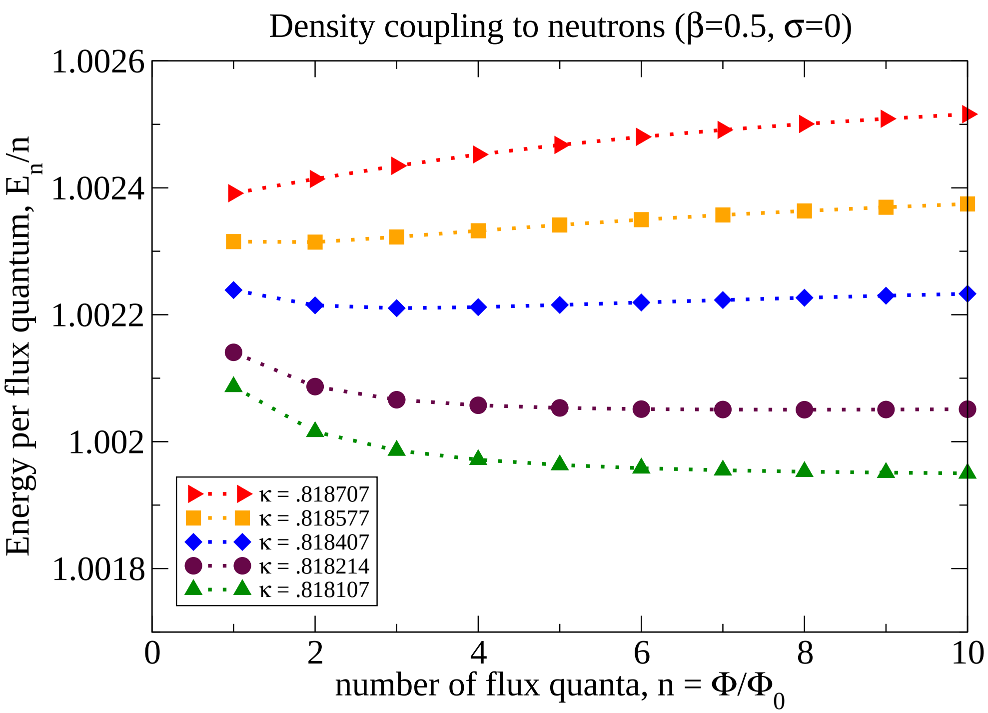
$$\frac{E_n^{(sc)}}{n} = E_{\text{Bog}} + M\delta\kappa - c_{\frac{1}{2}}M\delta\kappa\frac{1}{\sqrt{n}} + c_1 M\delta\kappa\frac{1}{n} + \cdots$$

M and $c_{\frac{1}{2}}$ must be positive:

$\delta\kappa < 0$ type-I $n = \infty$ is favored and stable

$\delta\kappa > 0$ type-II $n = 1$ is favored and stable

Understanding the density-coupled case



$$E_n(\kappa,\beta) = E^{(sc)}(\kappa) + M_\beta\Big(-n + b_{\frac{1}{2}}\sqrt{n} + b_1 + \cdots\Big)$$

core-area term $\propto n$. 2^{nd} order pert $\Rightarrow M_\beta \propto \beta^2$ is positive

core-perimeter term $\propto \sqrt{n}$

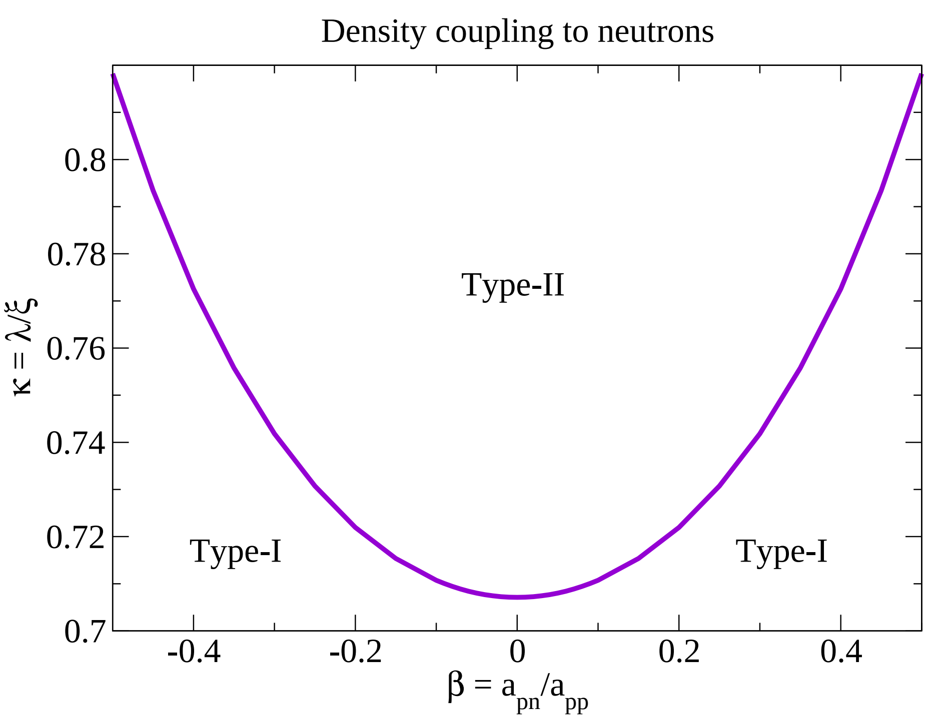
$b_{\frac{1}{2}}$ is positive (gradient costs)

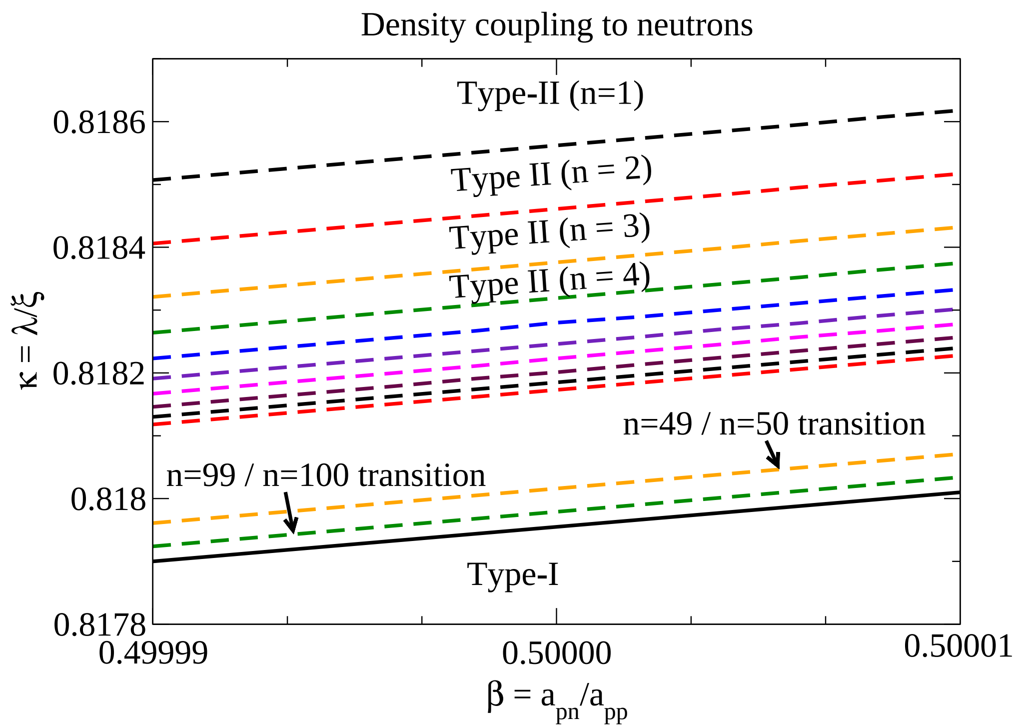
$$\frac{E_n}{n} = E_{\text{Bog}} + (M\delta\kappa - M_\beta) + \frac{M_\beta b_{\frac{1}{2}} - \delta\kappa M c_{\frac{1}{2}}}{\sqrt{n}} + \frac{M_\beta b_1 + \delta\kappa M c_1}{n} + \cdots$$

Type-I/II transition at $\delta\kappa_{\text{crit}}(\beta) = \dfrac{M_\beta\, b_{\frac{1}{2}}}{M c_{\frac{1}{2}}} \propto \beta^2$ (shifts *upwards*)

If $1/n$ term positive then for $\delta\kappa > \delta\kappa_{\text{crit}}$ there is a min in E_n/n.

Density-coupled phase diagram





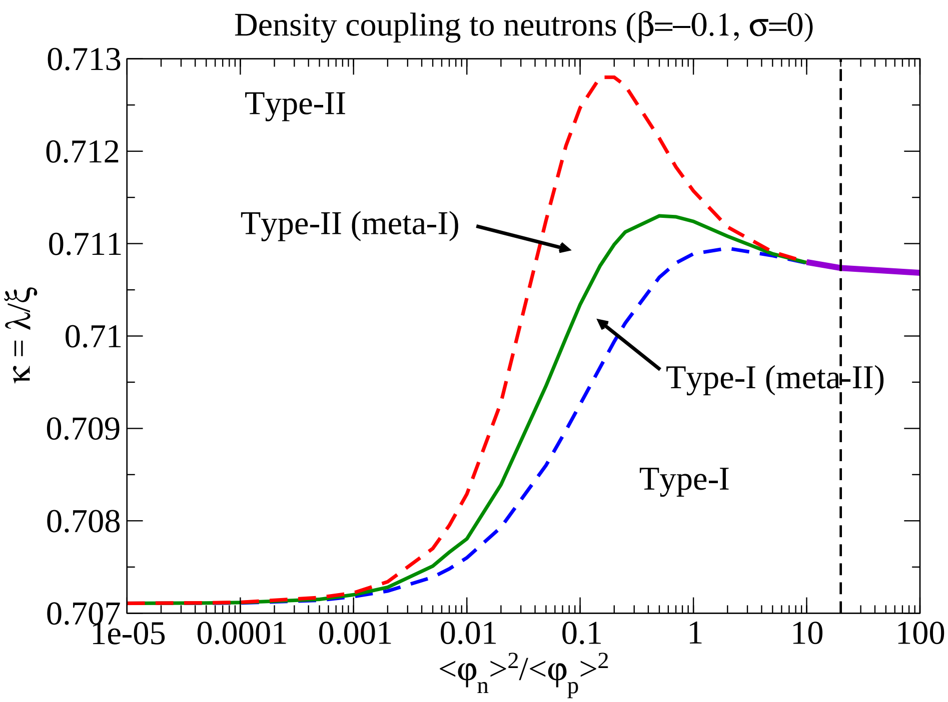
$$\frac{E_n}{n} = \text{const} - \frac{\delta\kappa - \delta\kappa_{\text{crit}}}{\sqrt{n}} + \frac{\text{const}}{n} + \cdots$$

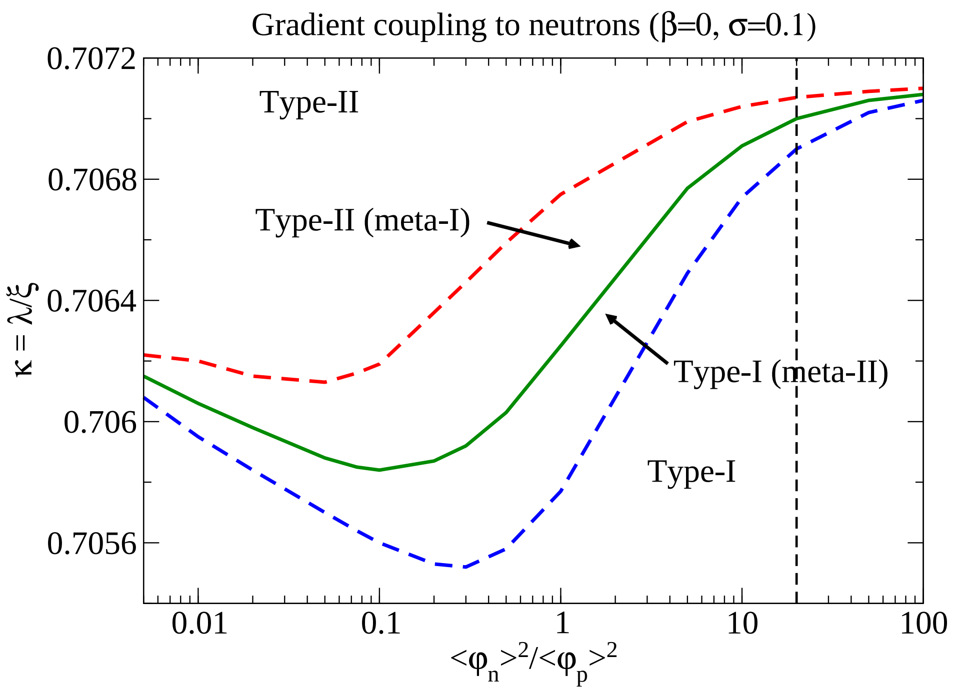
So as $\delta\kappa$ drops towards $\delta\kappa_{\text{crit}}$, there is a minimum in E_n/n at

$$\frac{1}{\sqrt{n_{\text{min}}}} \propto \delta\kappa - \delta\kappa_{\text{crit}}, \quad \Rightarrow n_{\text{min}} \propto (\delta\kappa - \delta\kappa_{\text{crit}})^{-2} \text{ for } \delta\kappa > \delta\kappa_{\text{crit}}$$

Future direction 1: dependence on $\langle\phi_n\rangle/\langle\phi_p\rangle$

So far we assumed $\langle\phi_n\rangle^2/\langle\phi_p\rangle^2 \sim 20$, as in neutron stars. Vary it:

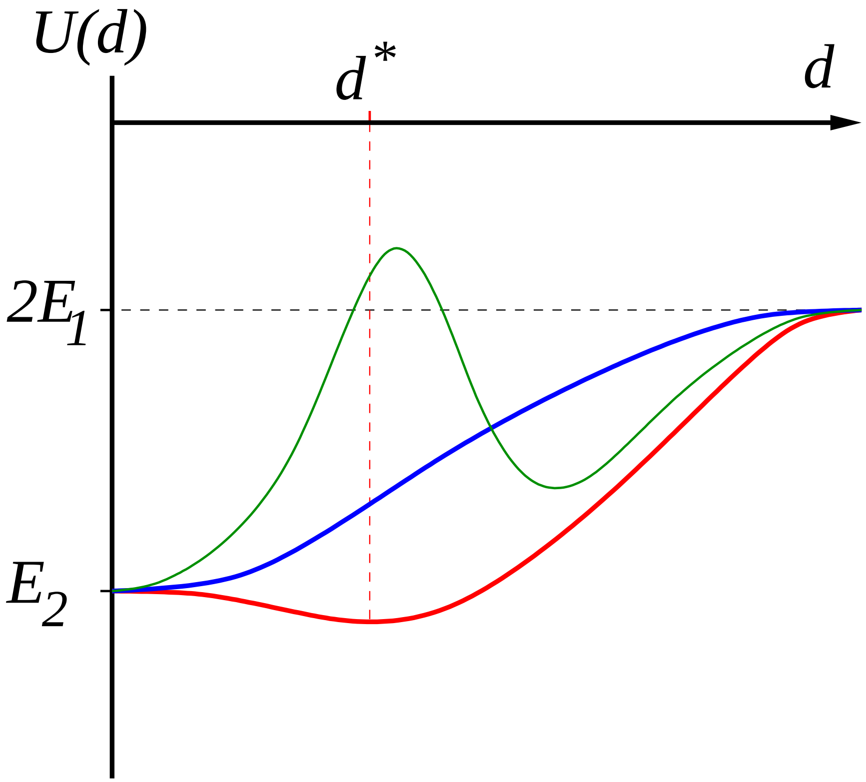




As $\langle\phi_n\rangle/\langle\phi_p\rangle \to 0$, $\kappa_{\rm crit} \to \frac{1}{\sqrt{2}}$. But,

- not at all monotonic
- With density coupling, no type-II(n) if $\langle\phi_n\rangle/\langle\phi_p\rangle \lesssim 1$
- for $\beta > 0$ (repulsion) the $\langle\phi_n\rangle \to 0$ limit is singular!

Future direction 2: type-II(n) or type-II*?



Cylindrical geometry: we only calculate E_1, E_2, etc, i.e. $U(0)$ and $U(\infty)$.
So far: assumed $U(d)$ monotonic; get type II(n) phases.
But $U(d)$ could be much weirder.

Guess: $U(d)$ could have a global minimum at $d = d^*$. Instead of type-II(n) phases we have a type-II^* phase, where inter-flux-tube distance is fixed by the microscopic physics not the magnetic field. Expect mixed phase of flux lattice domains and normal domains.

Need numerical calculations of flux tubes on a lattice (Gleiser, Thorarinson, Walker, in progress). Early results indicate there *is* a global minimum!

Future directions: summary

- Understand dependence on $\langle\phi_n\rangle/\langle\phi_p\rangle$.
- Go beyond the cylindrical geometry.
- Vary a_{nn}/a_{pp}.
- Other generalizations of the single-cmpt superconductor.
  - $SO(5)$ model (Mackenzie, Vachon, Wichowski, `hep-th/0301188`)
  - two charged fields (Babaev and Speight, `cond-mat/0411681`)
  - Higher-order terms in GL for single cmpt superconductor?
- Could any regions of a neutron star be near the type-I/type-II transition? Gaps vary rapidly with density.
- Could experimentalists custom-build a superfluid superconductor in the lab, and measure its properties?