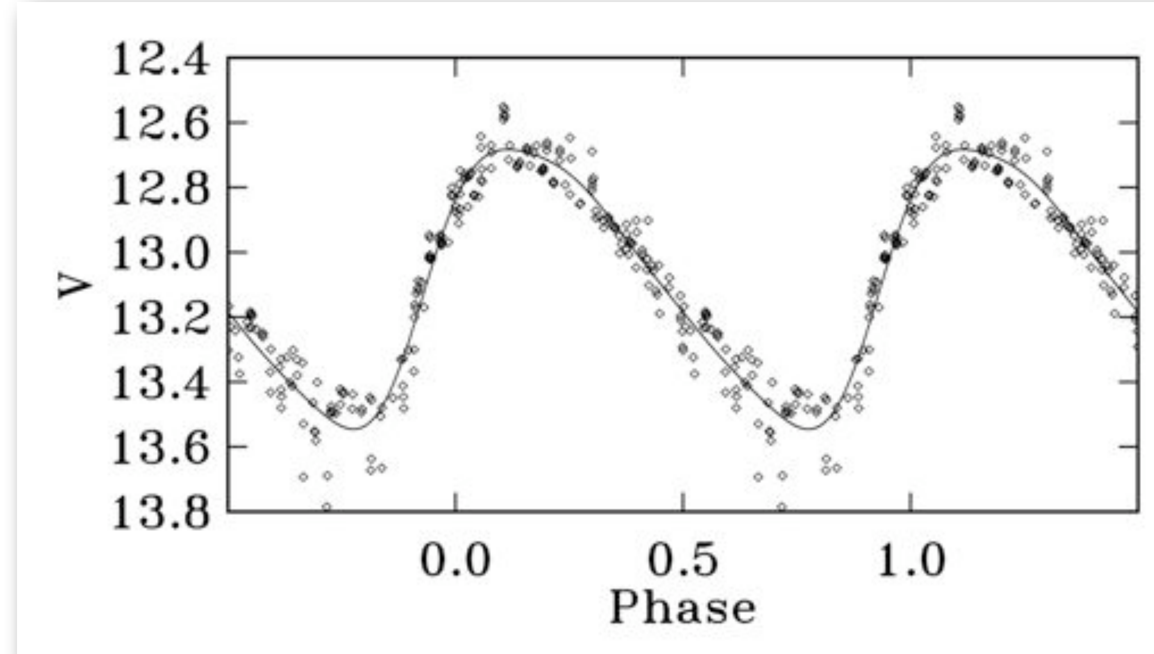
# Oscillations of Compact Stars

Erich Gaertig
Theoretical Astrophysics Department
University of Tübingen

Yerevan, 2008





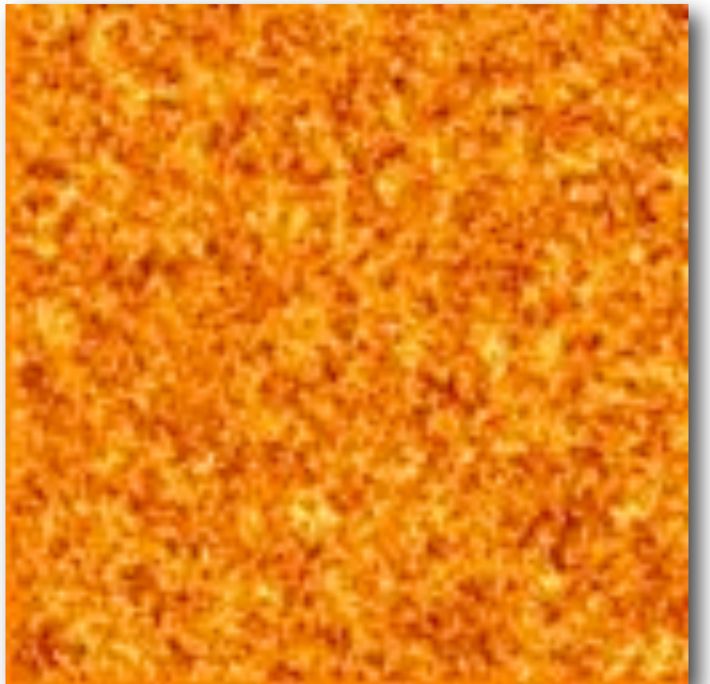

Variable stars have been known for a long time (Cepheids, RR Lyrae, ...)

Many attempts to explain their variability (eclipting binaries, giant cold spots etc.), but in the case of cepheids it is pulsation (Eddington 1930‘s)

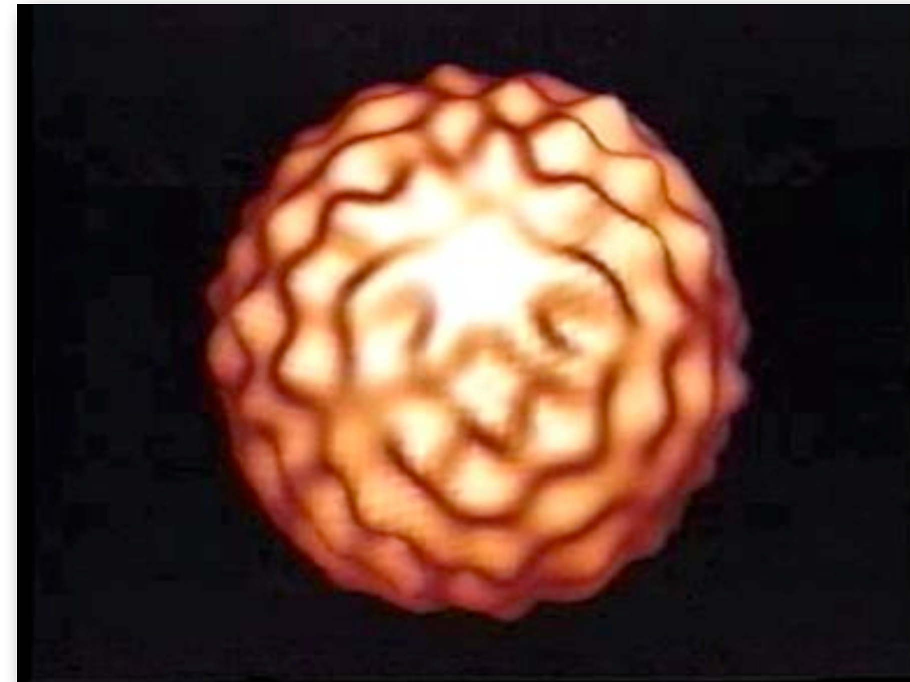
In nowadays terms: Cepheids oscillate mainly in their fundamental and first overtone with frequencies

$$\sigma \sim \left(\frac{GM}{R^3}\right)^{1/2}$$

# Our Sun oscillates as well



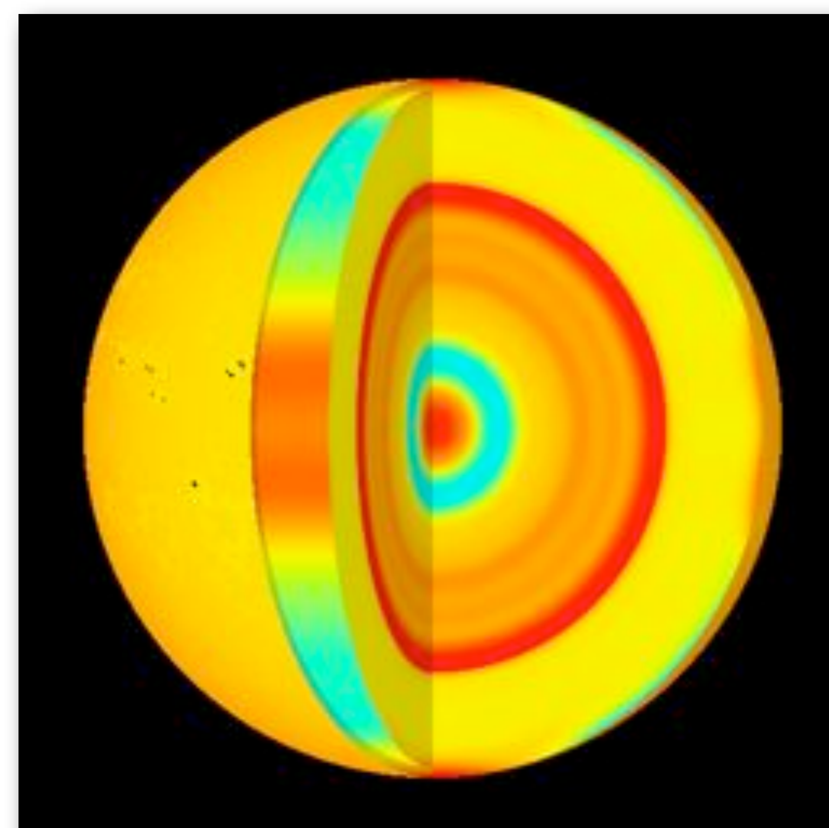
SOHO Doppler Image of the Sun's surface



© D. Hathaway

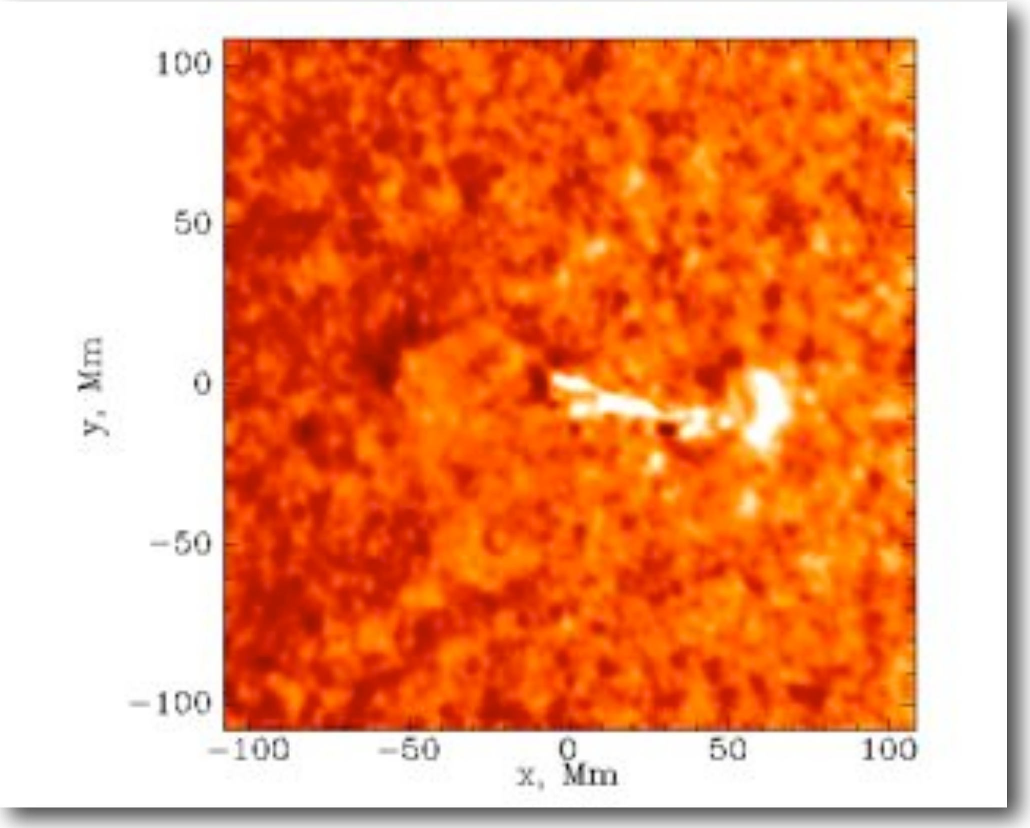
exaggerated high order mode

...and provides valuable
information about its interior
->
Helioseismology



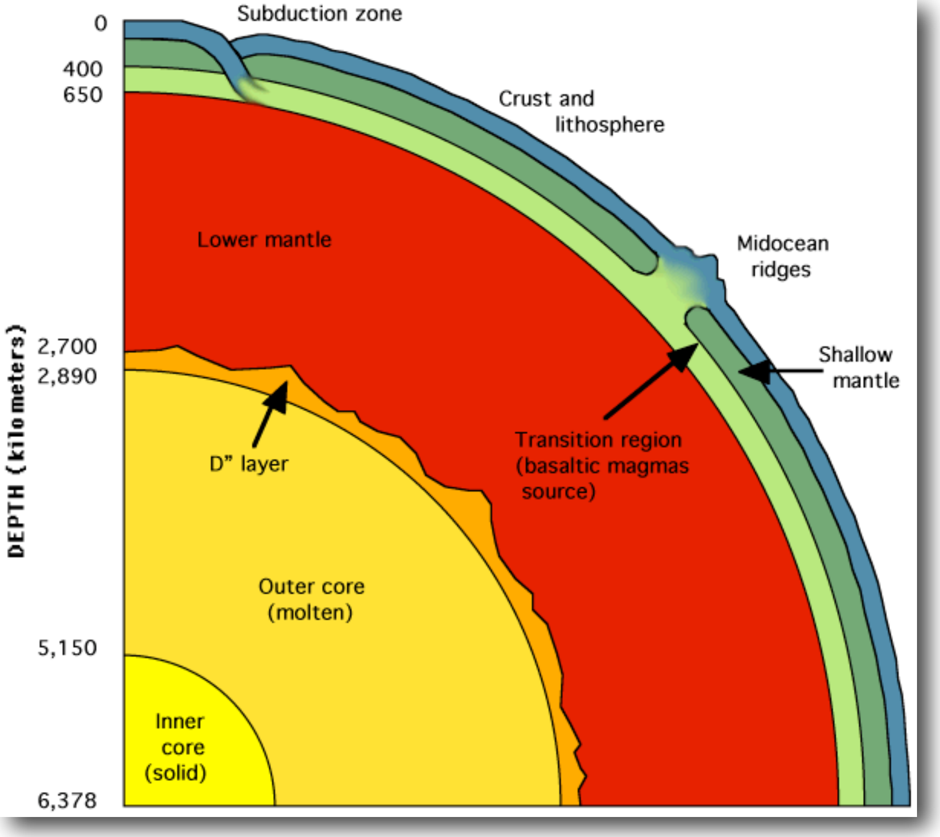
© A. Kosovichev

# One also observes solar quakes









## Mode classification:

For nonrotating Newtonian stars, we have

- *(p)ressure-modes:* Acoustic oscillations, pretty much like sound waves in the air with larger amplitudes towards the surface. Higher radial order means higher frequency

- *(g)ravity-modes:* These modes are driven by gravity/buoyancy in convection-like manner with large amplitudes towards the center. Higher radial order means lower frequency (only for non-isentropic stars)

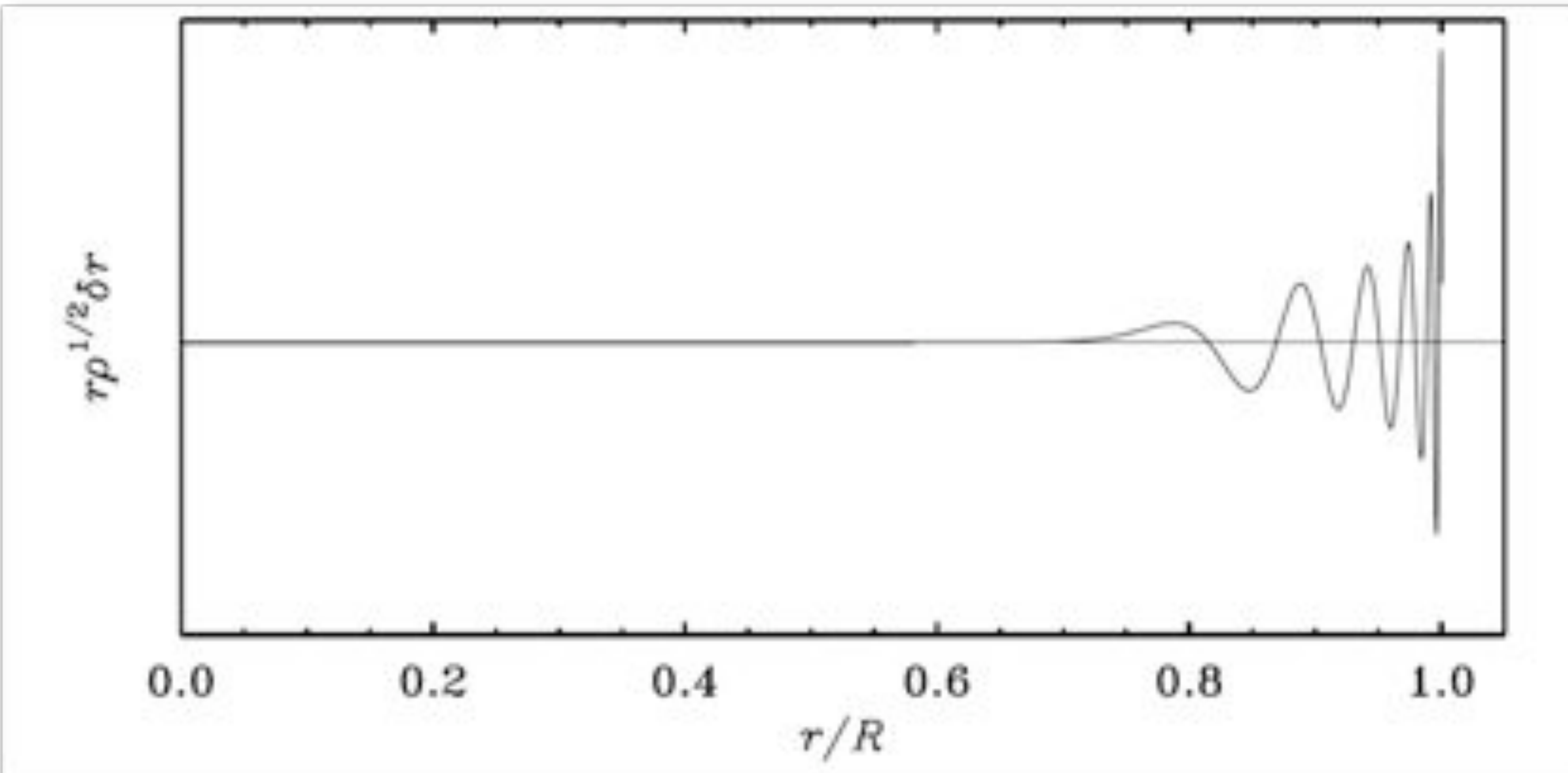
- *(f)undamental mode:* Intermediate mode with no radial nodes; lies between p- and g-modes

Rotation introduces

- *(r)otational-modes:* The Coriolis force is the restoring agent in this case; frequencies are proportional to angular velocity

# Mode classification:

For nonrotating Newtonian stars, we have



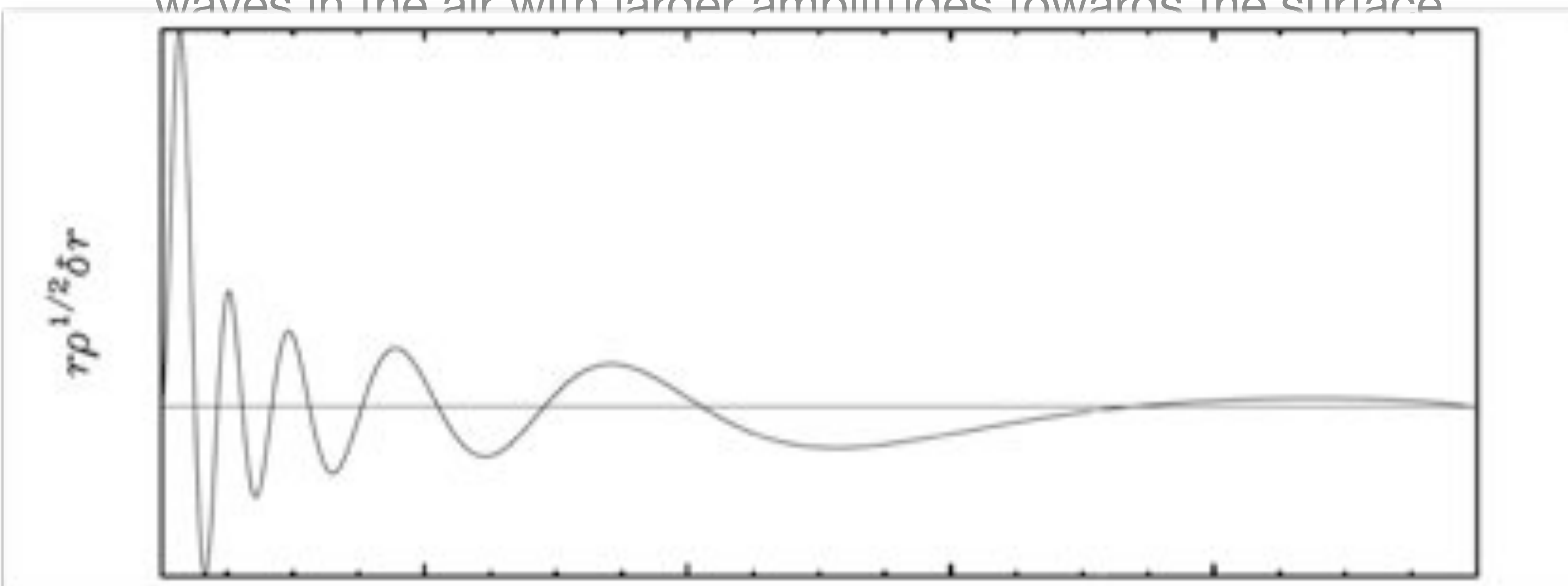

Rotation introduces

- *(r)otational-modes:* The Coriolis force is the restoring agent in this case; frequencies are proportional to angular velocity

## Mode classification:

For nonrotating Newtonian stars, we have

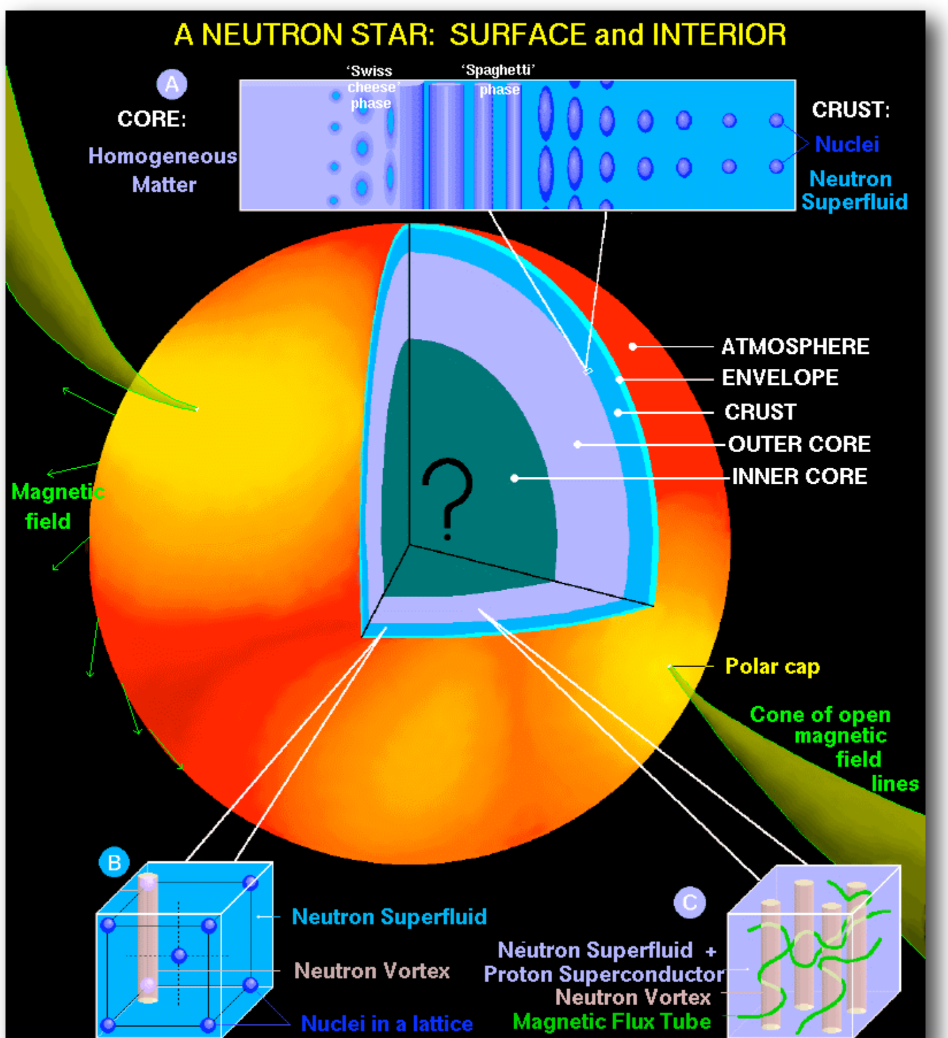
- *(p)ressure-modes:* Acoustic oscillations, pretty much like sound waves in the air with larger amplitudes towards the surface




Rotation introduces

- *(r)otational-modes:* The Coriolis force is the restoring agent in this case; frequencies are proportional to angular velocity

# Why one should deal with Neutron Stars:




© D.Page

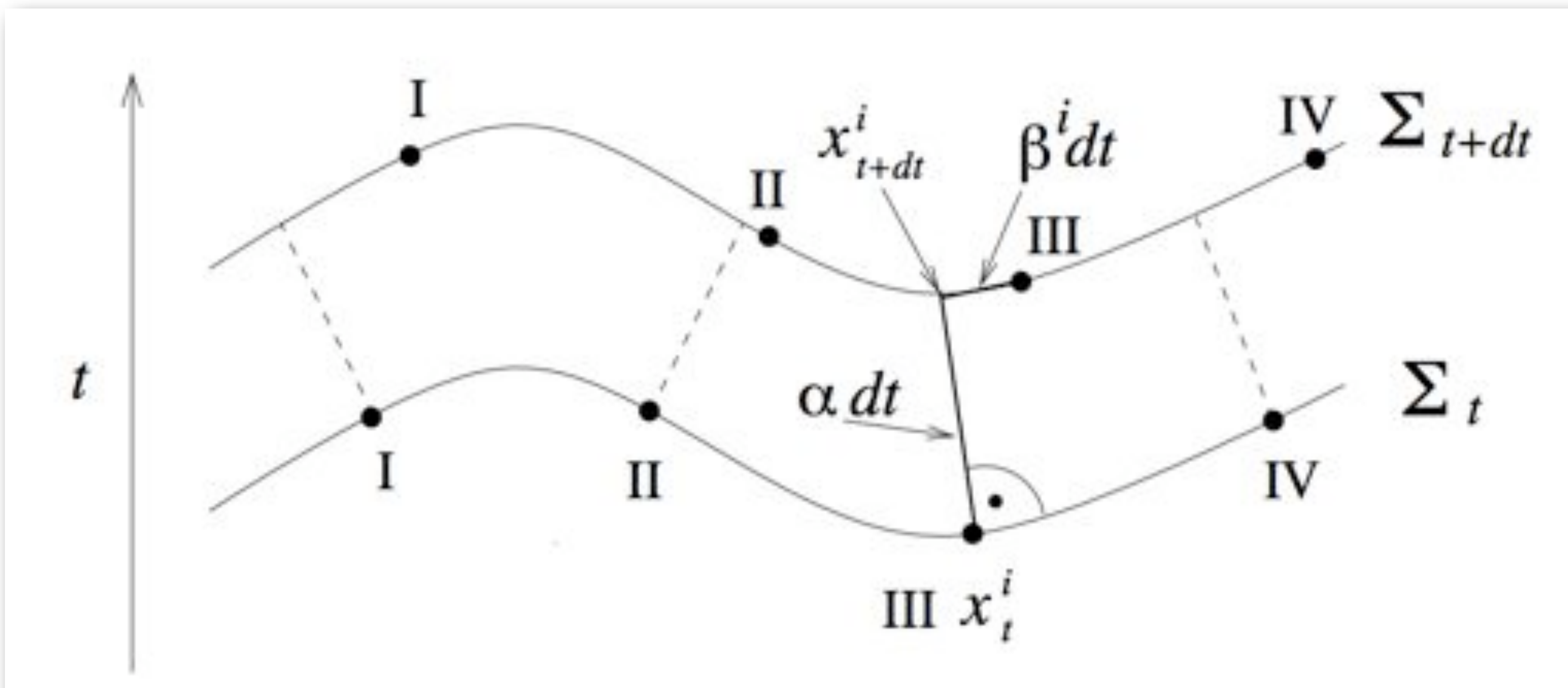
- Most exotic objects in the universe
  - rapid (differential) rotation
  - General Relativity
  - superfluidity
  - strong magnetic fields
  - exotic nuclear physics

# General Relativistic Neutron Star Oscillations:

Solve

$$
\begin{aligned}
G_{\mu\nu} &= 8\pi T_{\mu\nu} \\
\nabla_\mu T^{\mu\nu} &= 0
\end{aligned}
$$

directly (CACTUS, Whisky, Pizza...)






or linearize...

## Relativistic Stellar Perturbation Theory:

$$\delta \left( \begin{array}{ccc} G_{\mu\nu} & = & 8\pi\, T_{\mu\nu} \\ \nabla_\mu T^{\mu\nu} & = & 0 \end{array} \right)$$

with energy-momentum tensor

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p g_{\mu\nu}$$

and line-element

$$ds^2 = -e^{\nu(r,\theta)} dt^2 + e^{\mu(r,\theta)} (dr^2 + r^2 d\theta^2) + e^{\psi(r,\theta)} r^2 \sin^2\theta (d\phi - \omega(r,\theta) dt)^2$$

Decompose the angular part of the perturbations in spherical harmonics

$$\delta p = p(r,t) Y^l_m(\theta,\phi) \quad , \quad \delta u_\nu = u_\nu(r,t) Y^l_m(\theta,\phi)$$

and neglect products of perturbations

## Still much room for approximations

Most commonly used are:

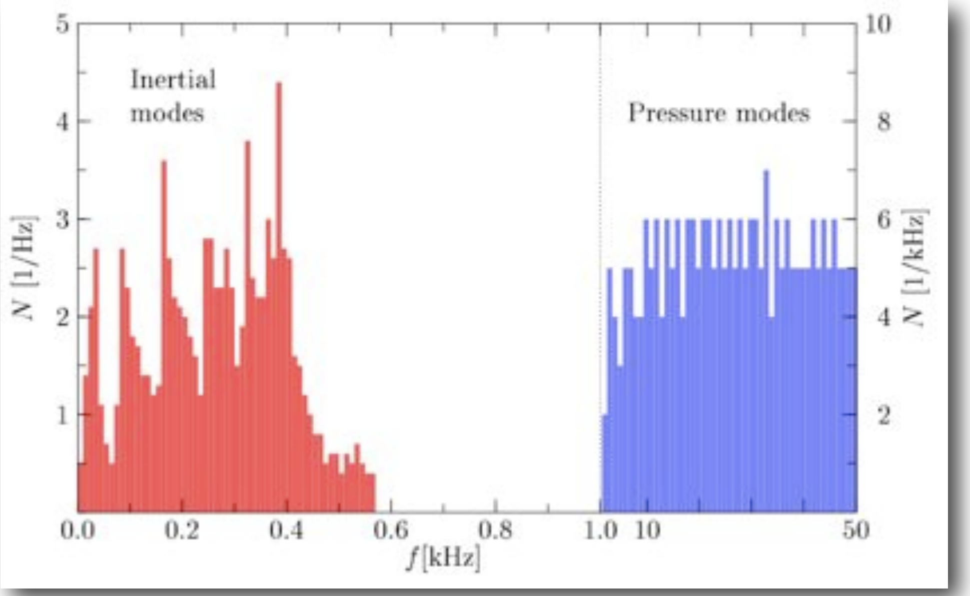
- slow-rotation-approximation: include rotational corrections up to first order in Omega
- Cowling-approximation: neglect all metric perturbations and focus on the fluid motion
- Inverse Cowling-approximation: neglect all fluid perturbations and focus on the spacetime evolution

Solve the equations as eigenvalue problem or by direct numerical integration

# Eigenvalue Problem:

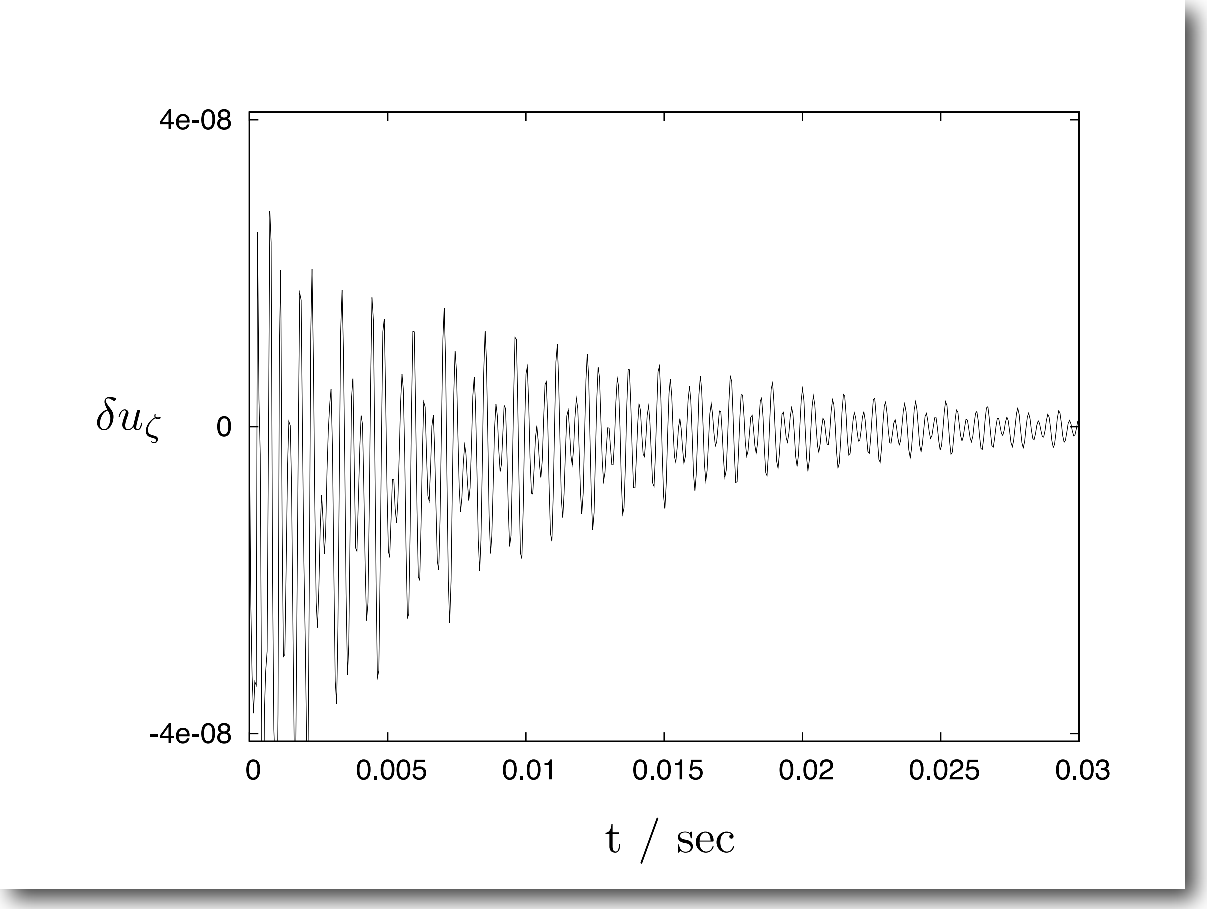
Replace $\partial_t f$ by $i\sigma f$

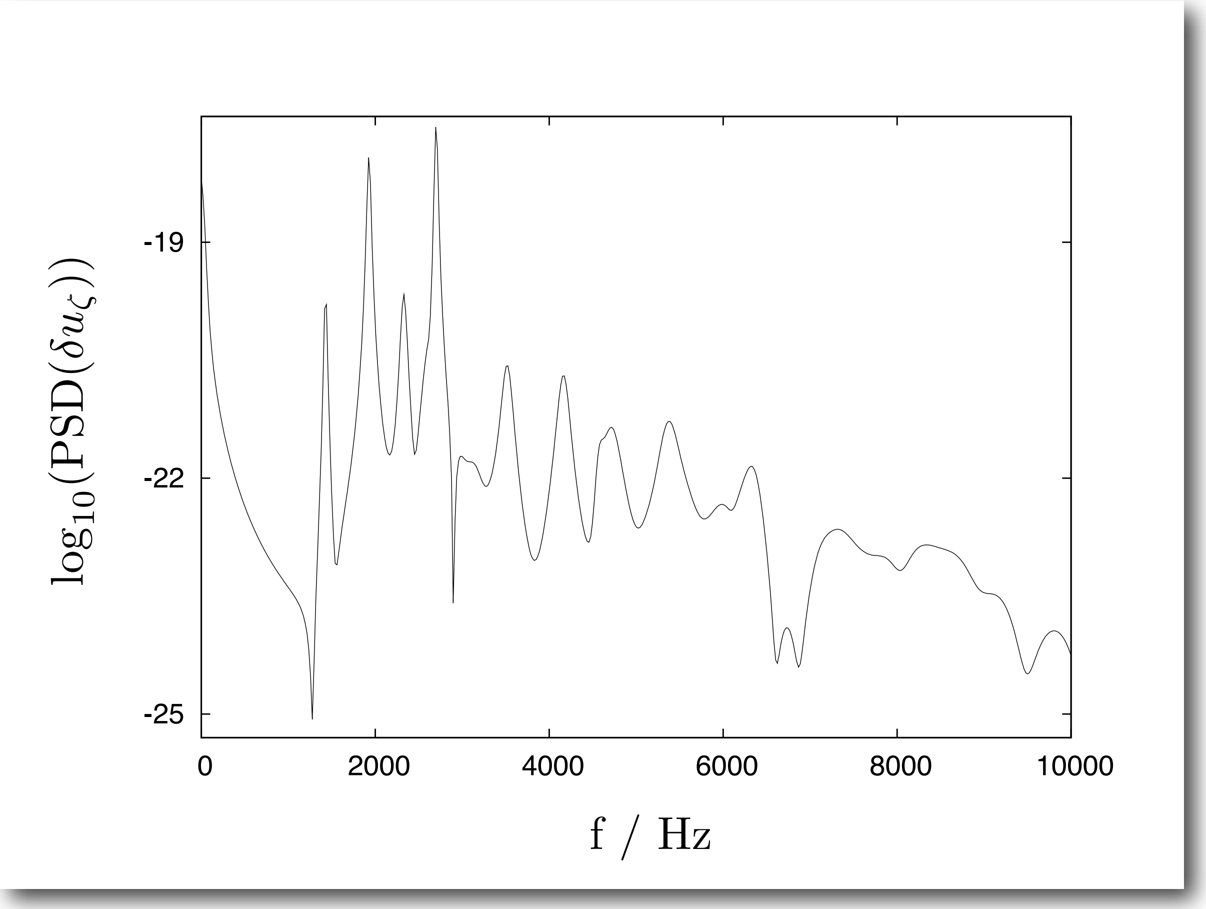
$$\underbrace{\frac{d}{dr}\left(\Gamma P \frac{1}{r^2}\frac{d}{dr}(r^2\xi)\right) - \frac{4}{r}\frac{dP}{dr}\xi + \sigma^2\rho\xi = 0}_{\mathbf{A}\cdot\boldsymbol{\xi} = 0}$$



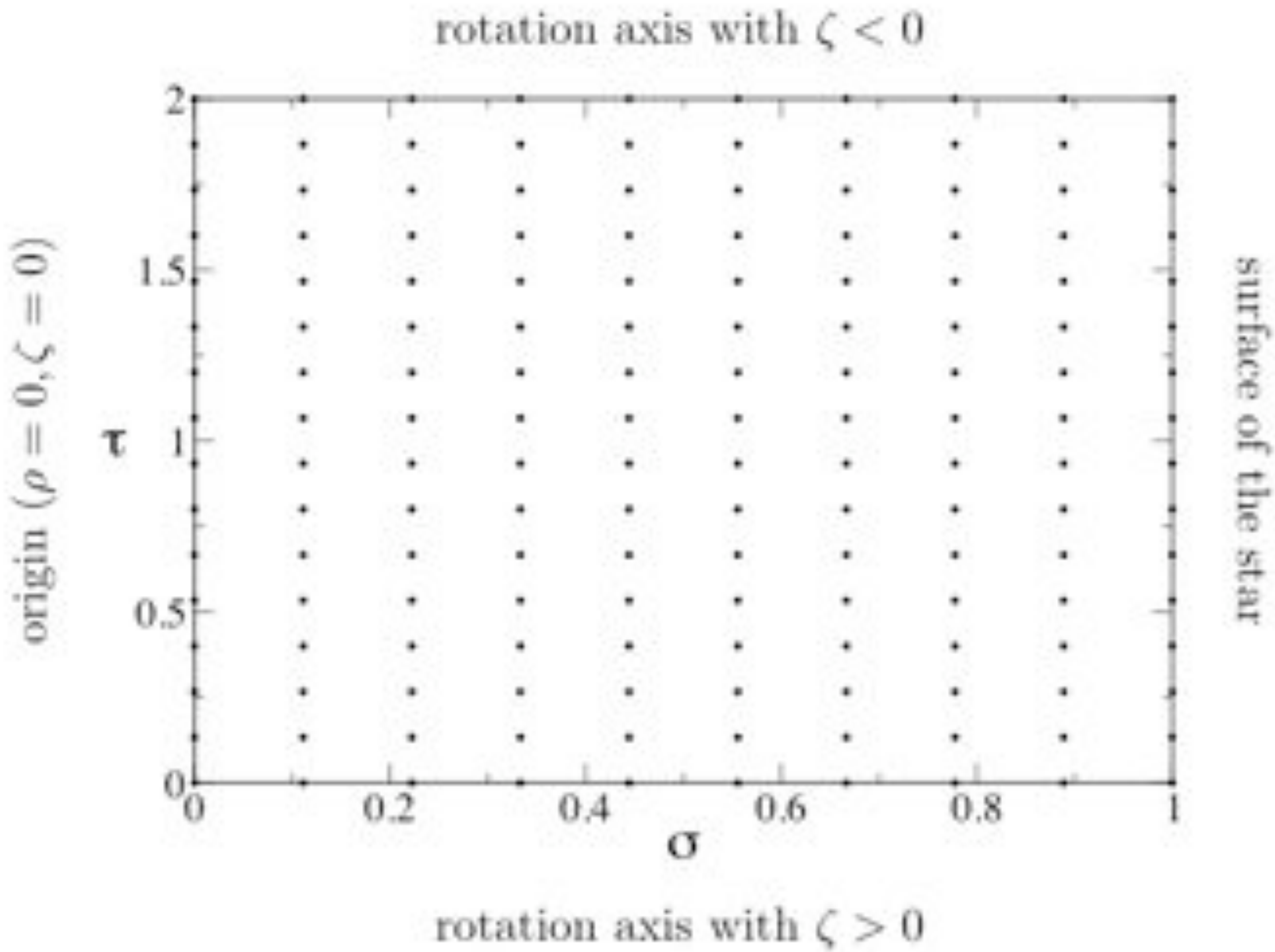

© S. Boutloukos

# Time Evolution:

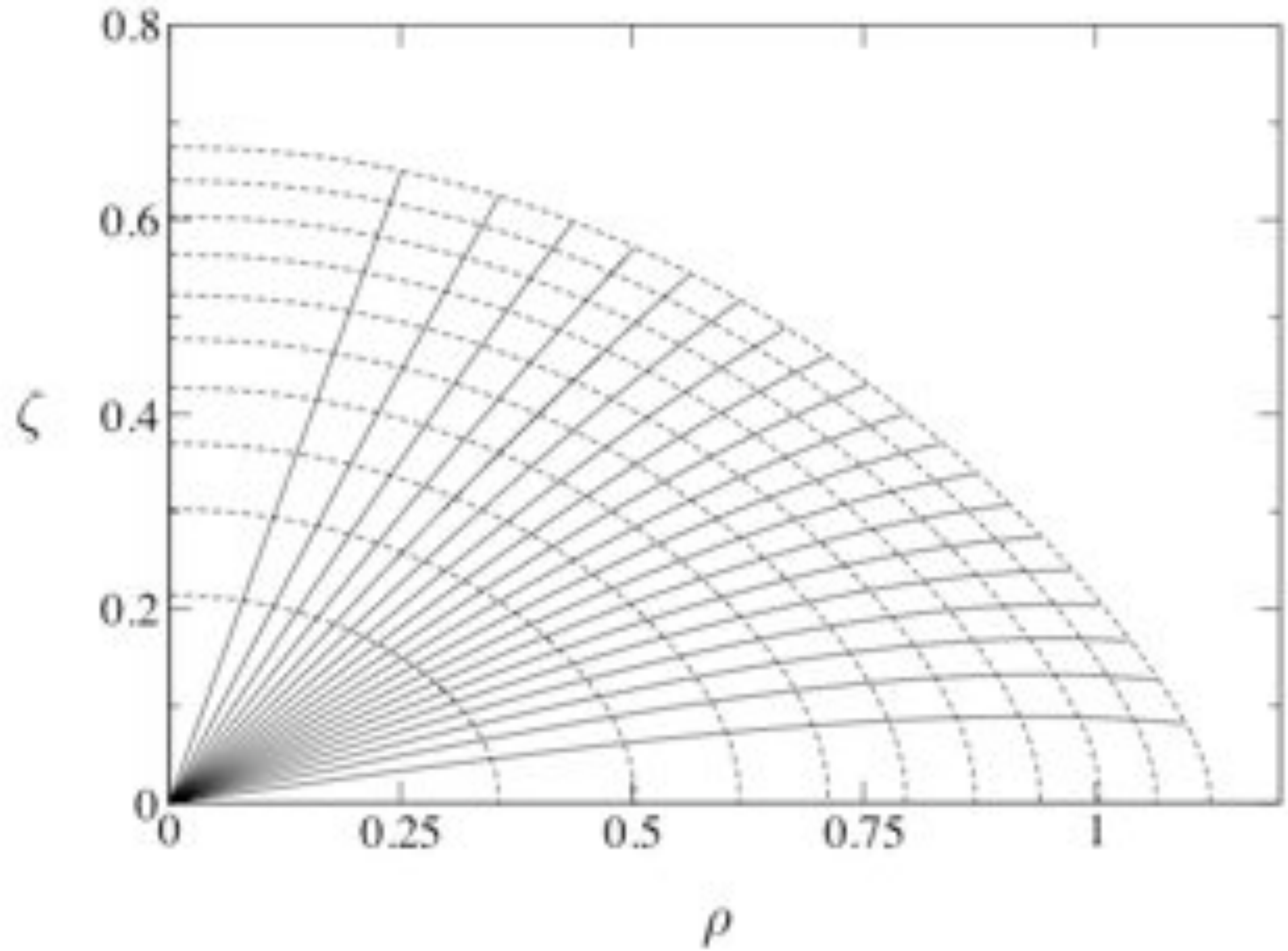


# The numerical arena

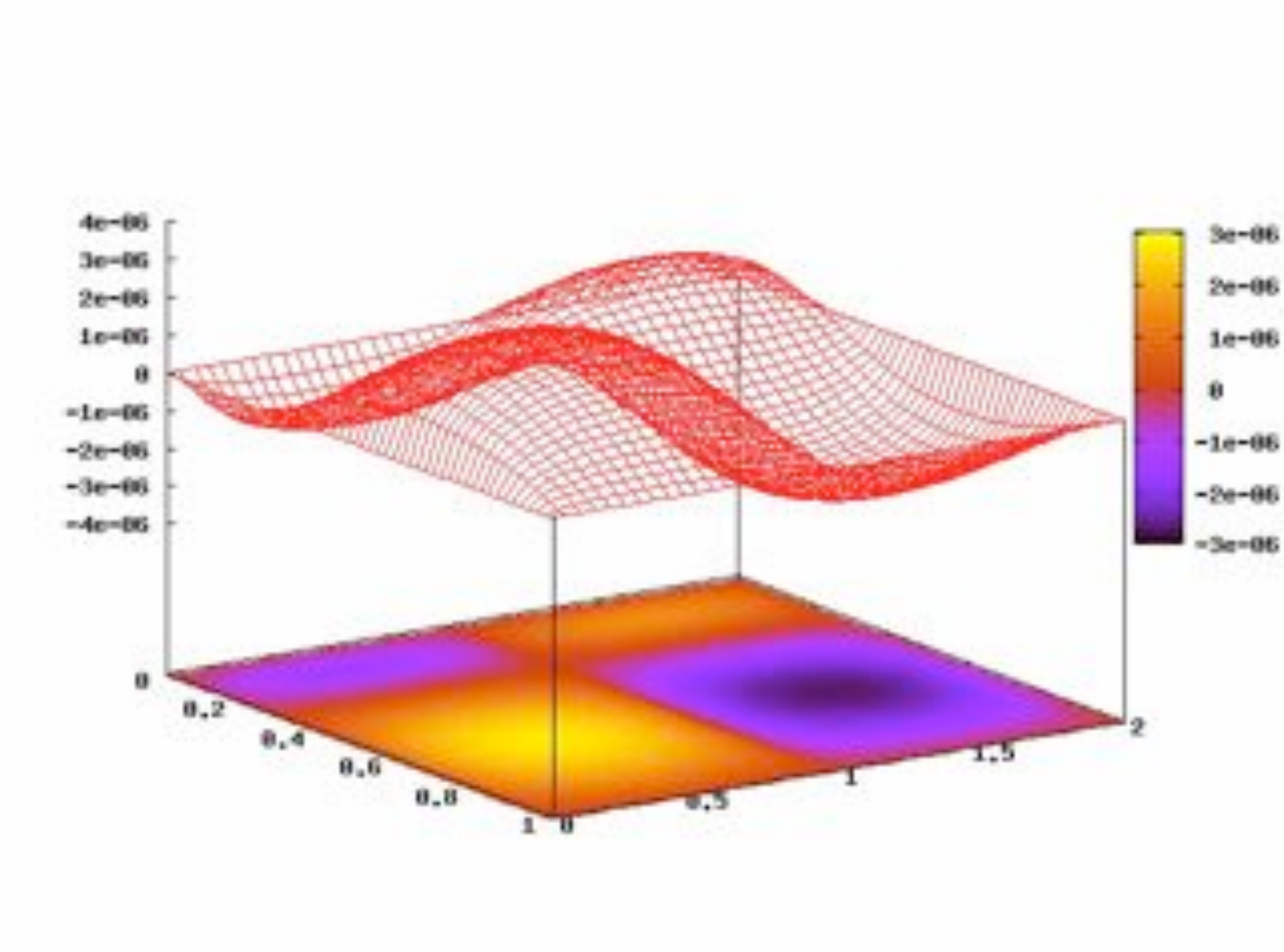


# The numerical arena



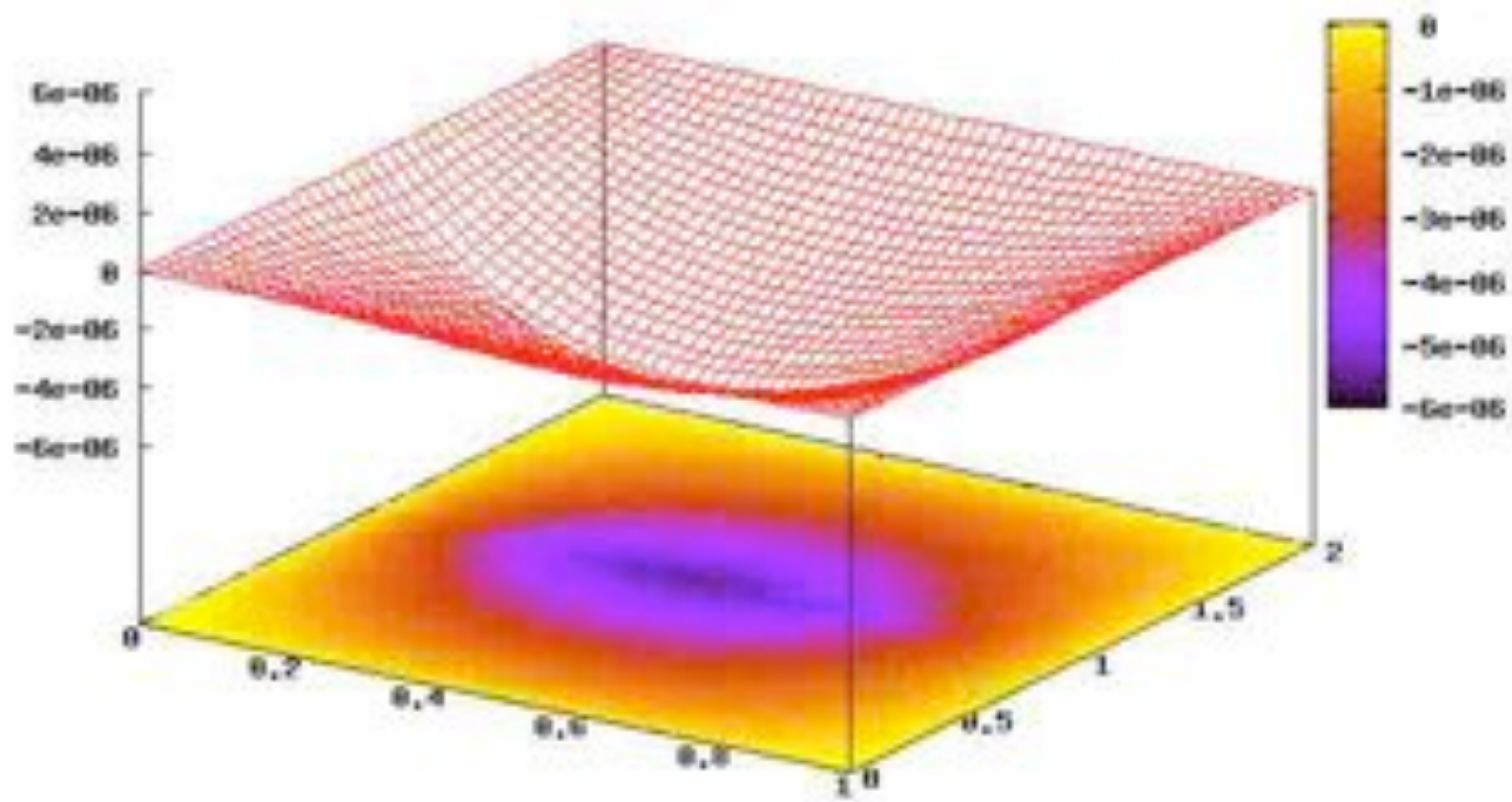
**Demo:** spherically symmetric velocity perturbation on a non-rotating neutron star

time-evolution of the zeta-component...

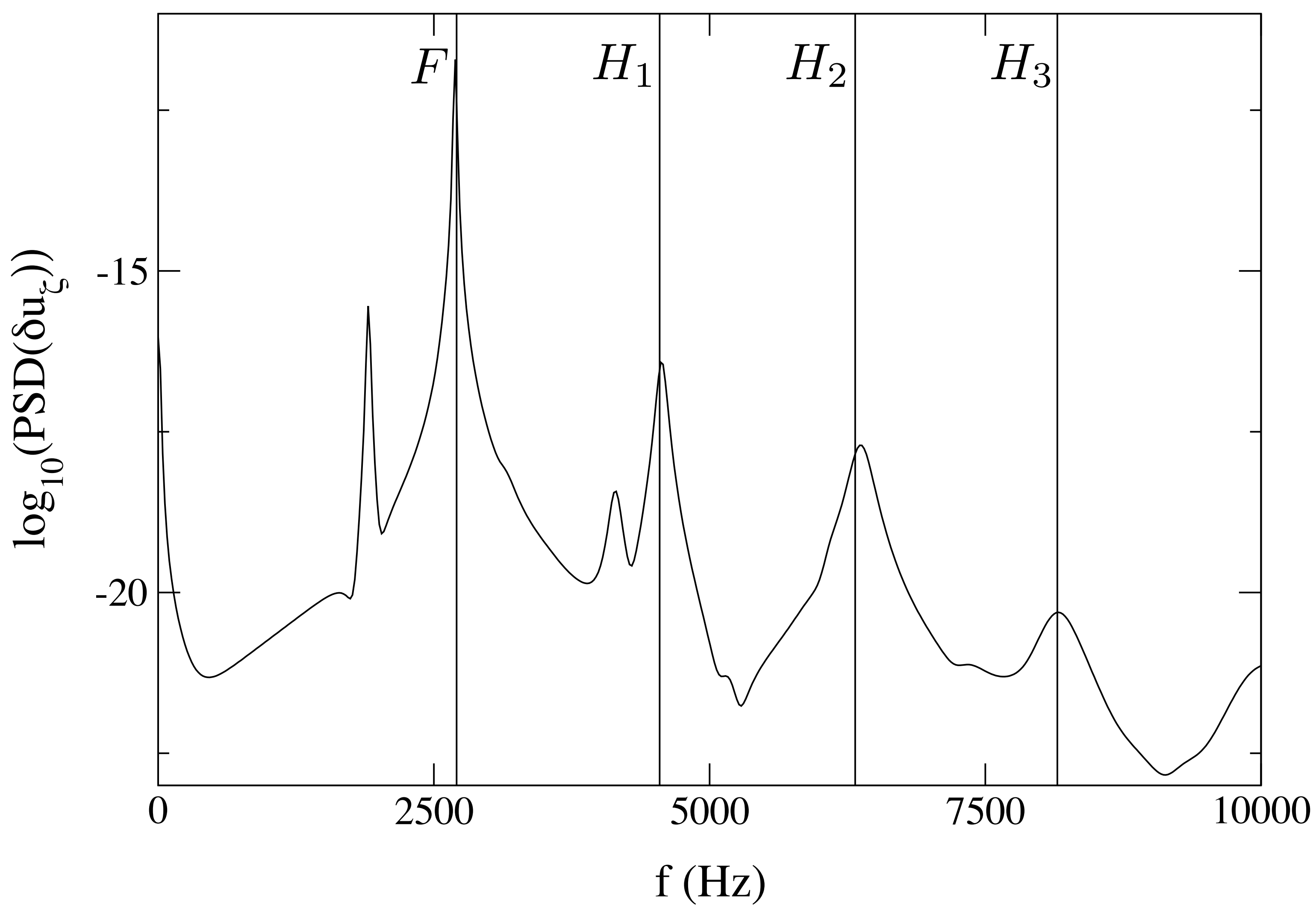


**Demo:** spherically symmetric velocity perturbation on a non-rotating neutron star

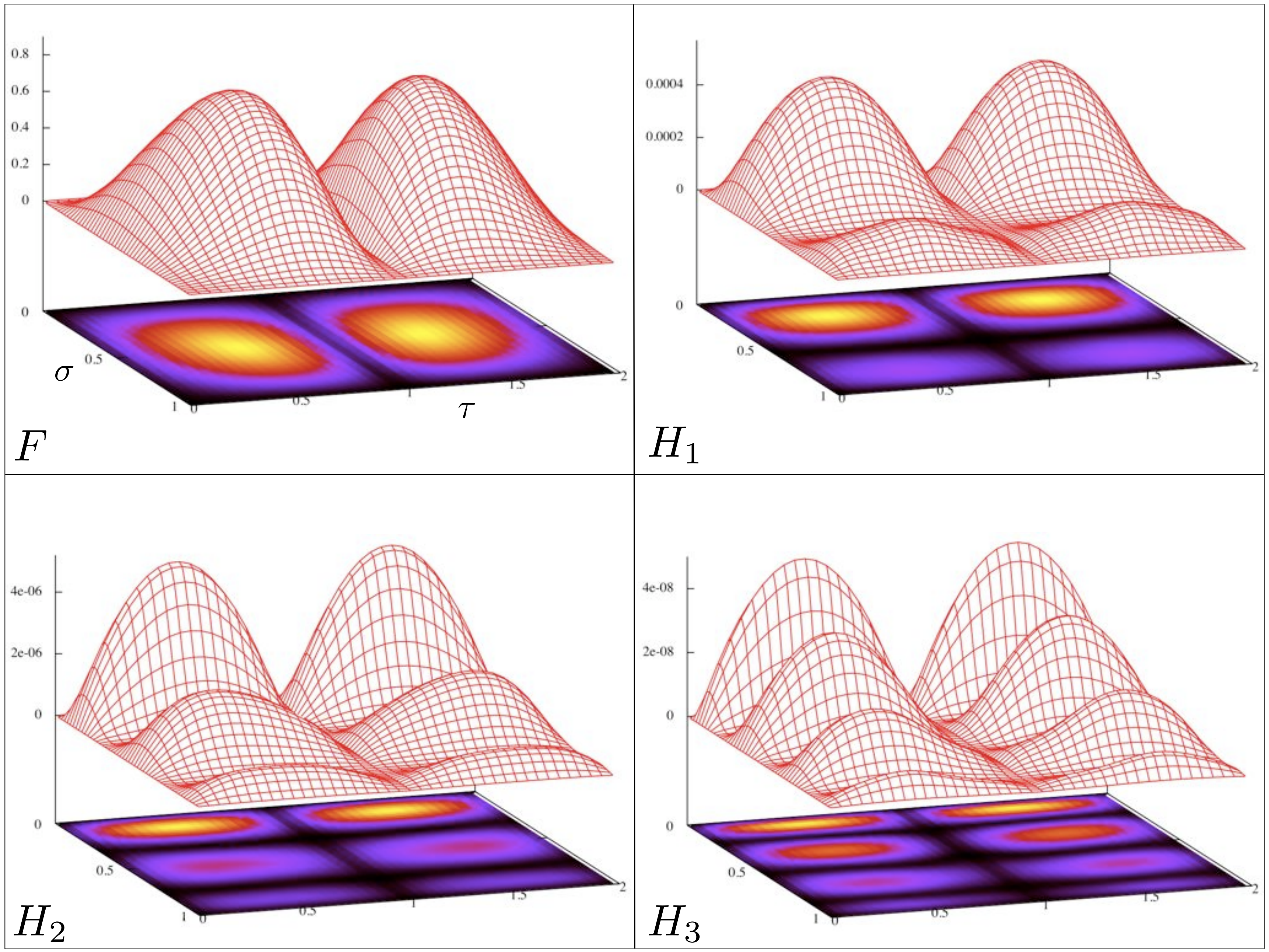
time-evolution of the rho-component...



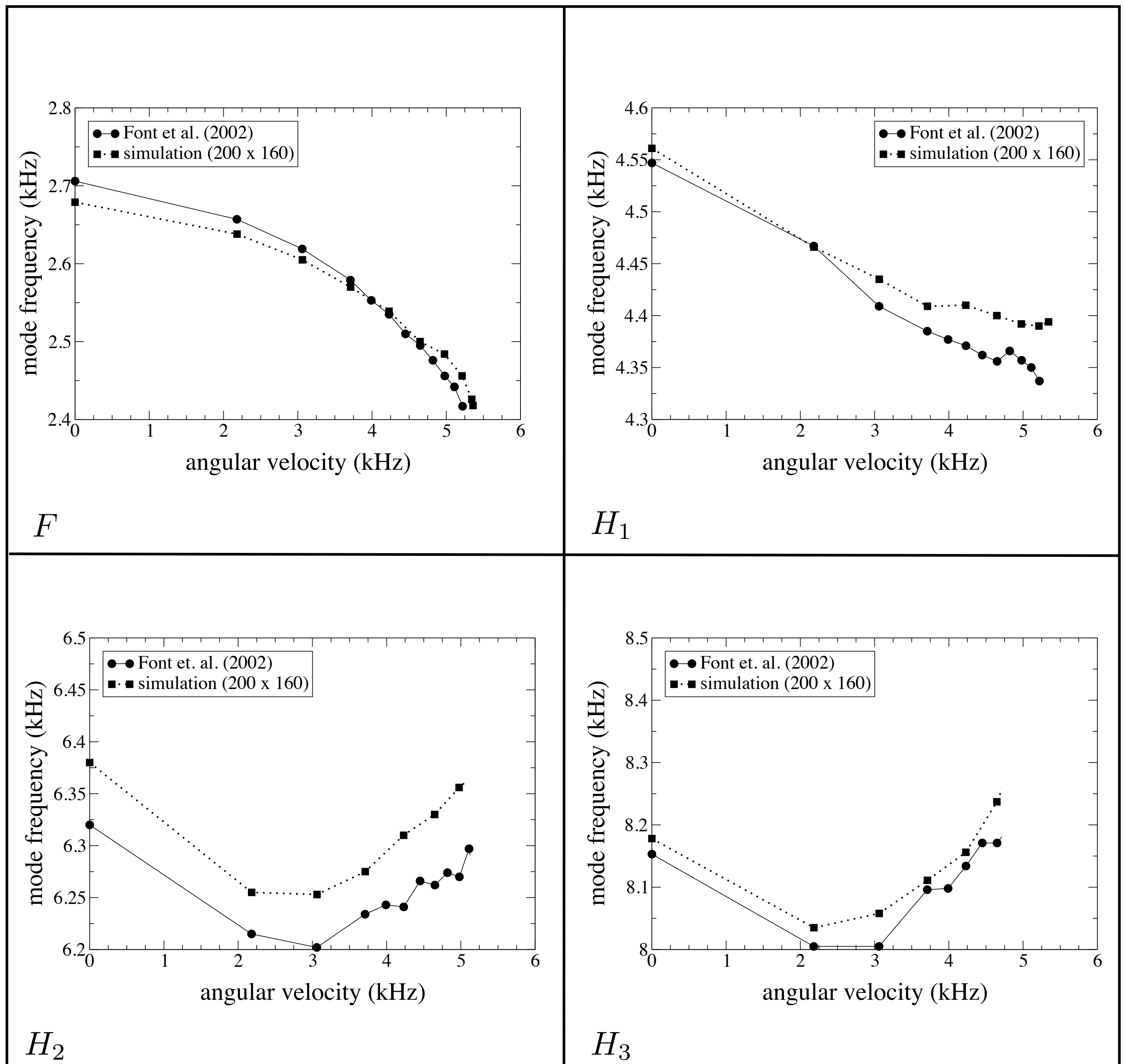
- Frequencies and Eigenfunctions



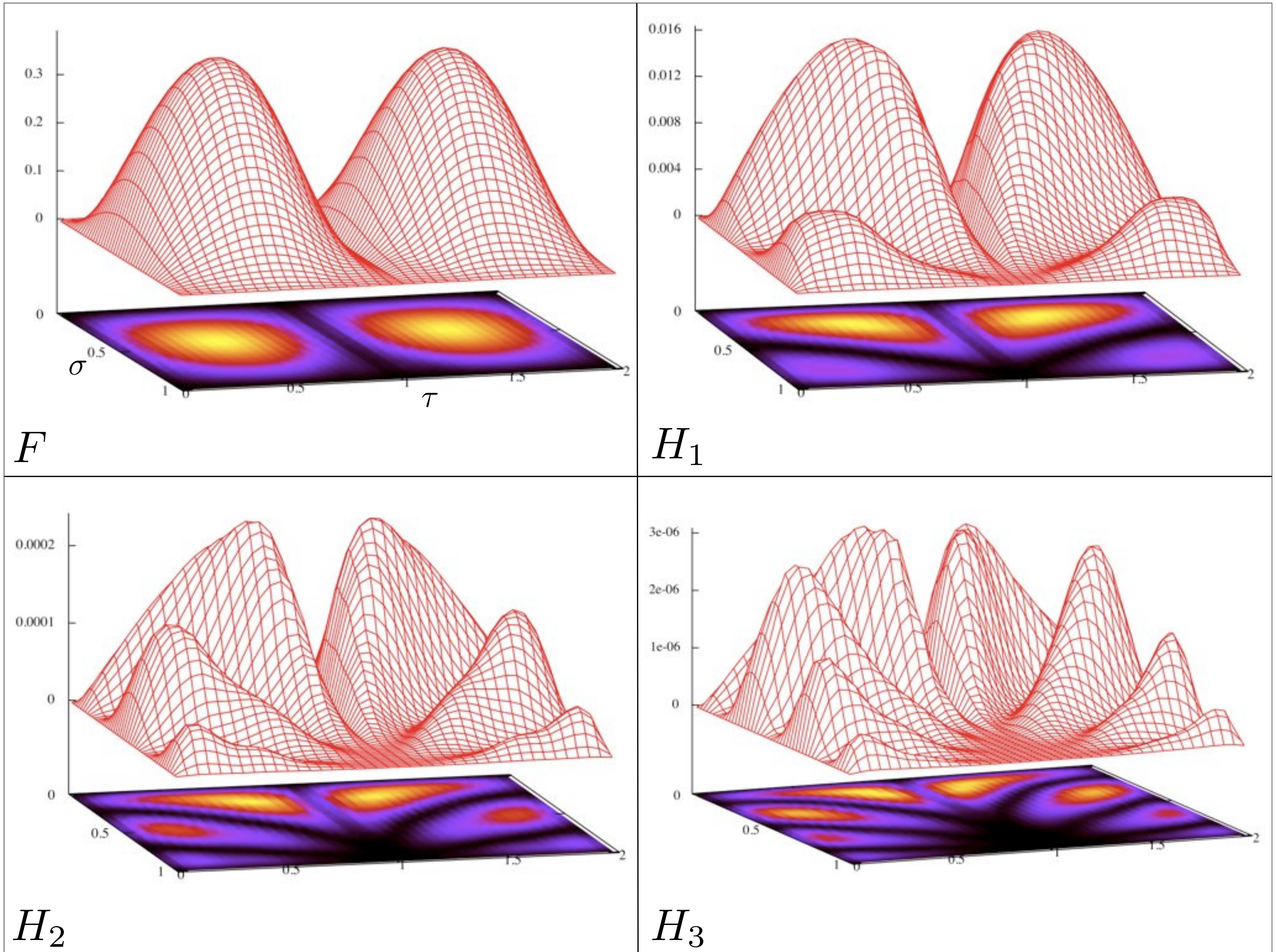
- Frequencies and Eigenfunctions



- Oscillation frequencies and eigenfunctions change once rotation sets in



- Oscillation frequencies and eigenfunctions change once rotation sets in

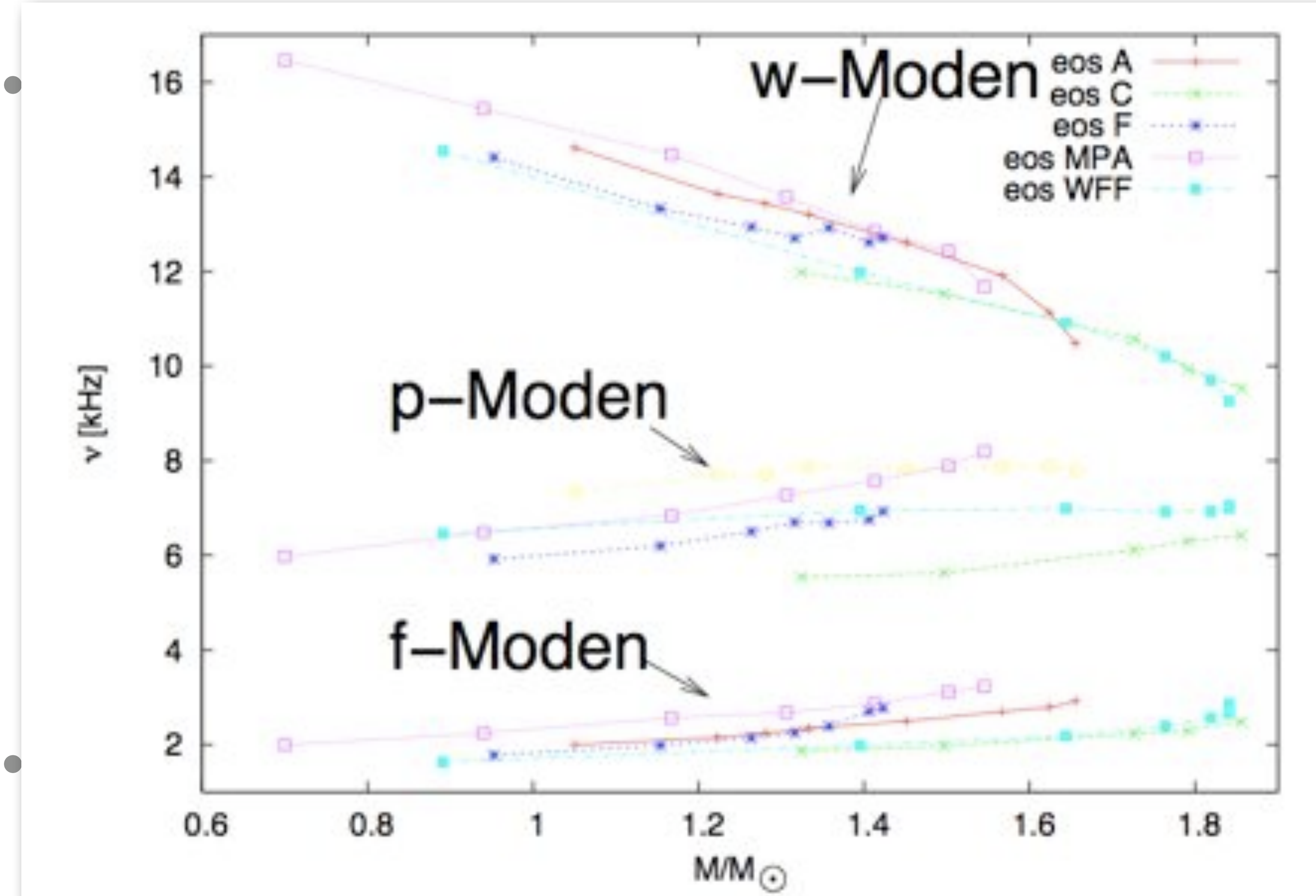


## What's the impact of GR to neutron star oscillations?

- changes in the frequencies of p-, f-, g-, and r-modes when compared to Newtonian results
  - f-, p-modes: $\nu \gtrsim 1.5\ \mathrm{kHz}$
  - g-modes: $\nu \lesssim 500\ \mathrm{Hz}$
  - r-modes: $2\pi\nu \sim \Omega$
- whole new class of (w)ave-modes; oscillations of the spacetime itself with frequencies $\nu \gtrsim 5\ \mathrm{kHz}$

# What's the impact of GR to neutron star oscillations?

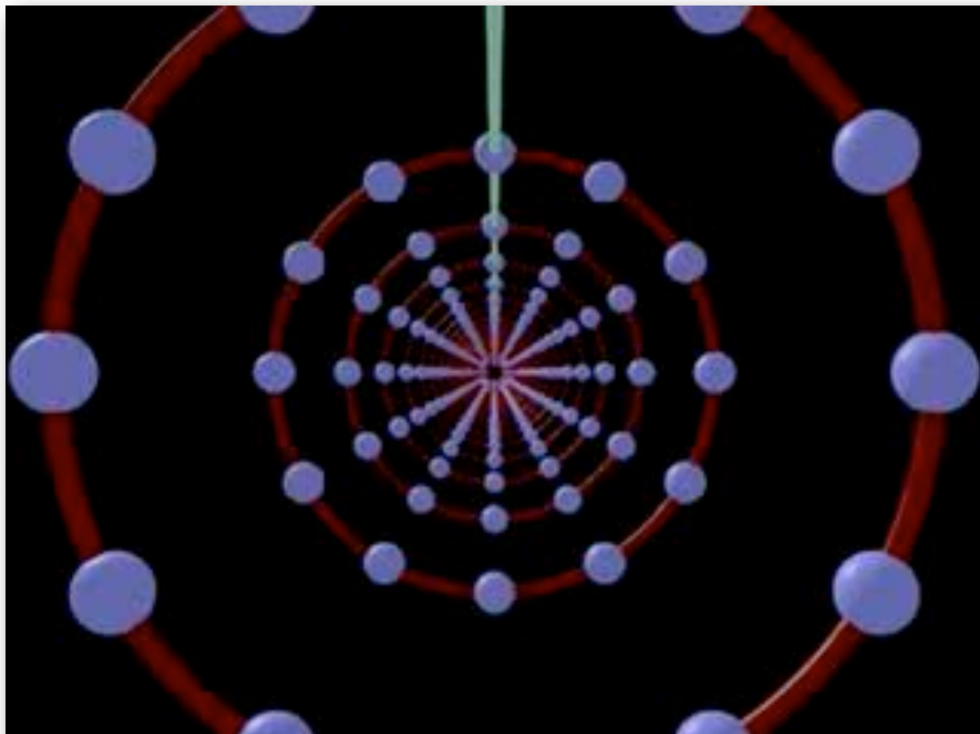
- nen



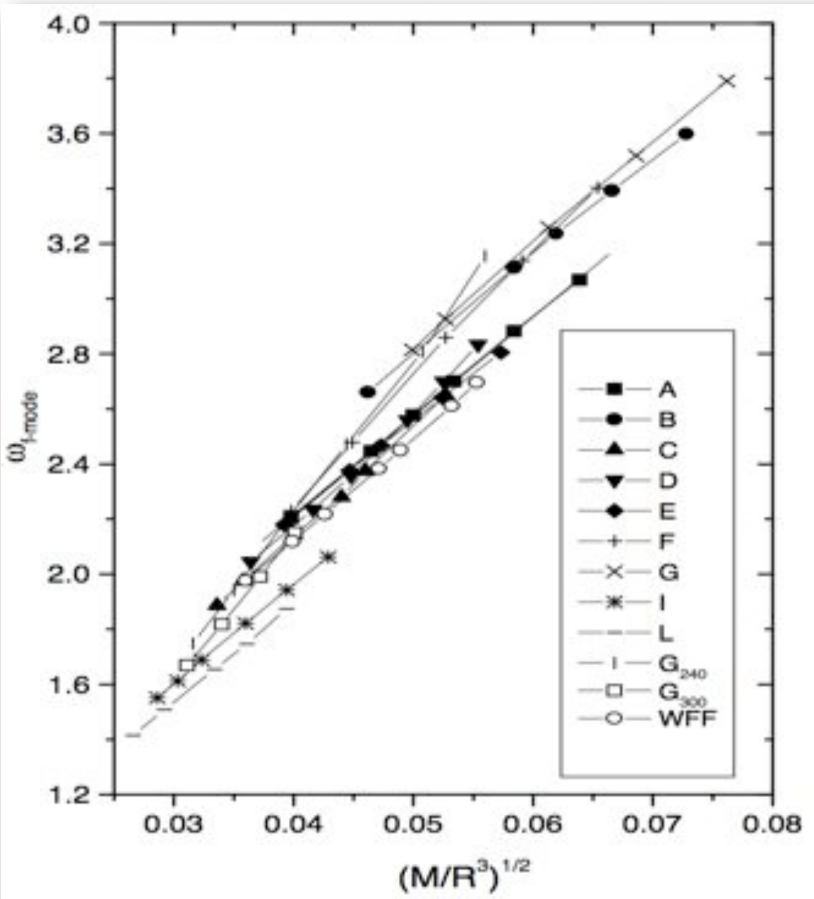

- acetime

# GW Asteroseismology

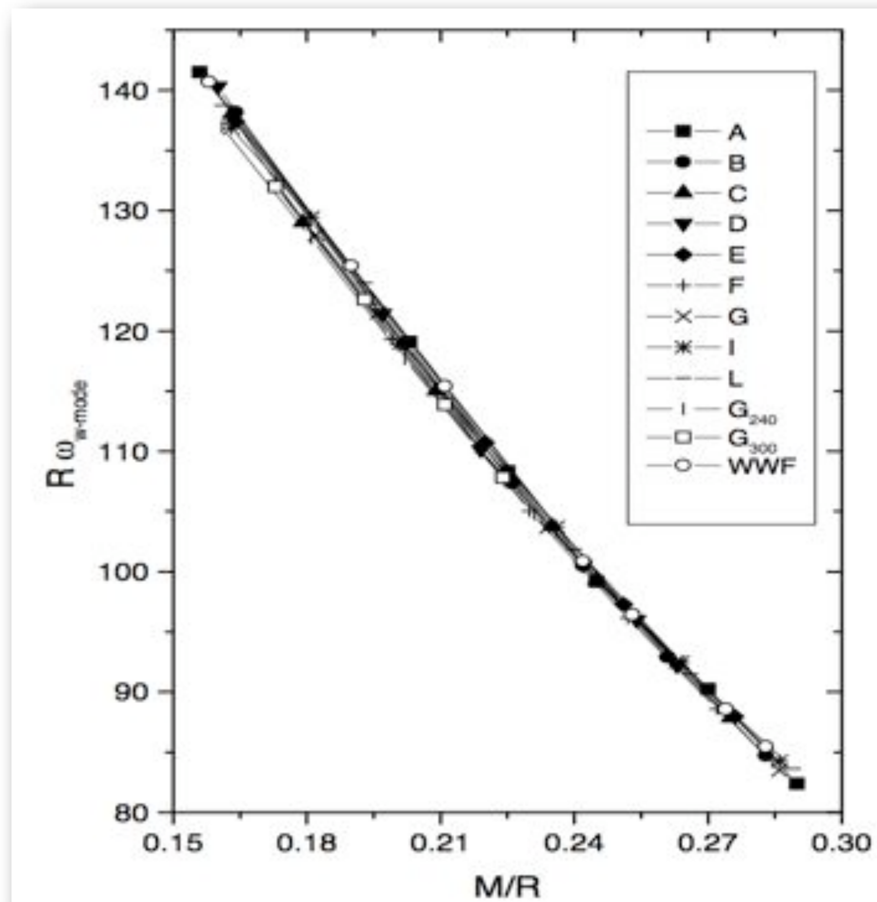
By emitting gravitational radiation, the oscillation pattern can reveal the internal structure of neutron stars: mass, radius, EoS, rotation rate, B-field, ...



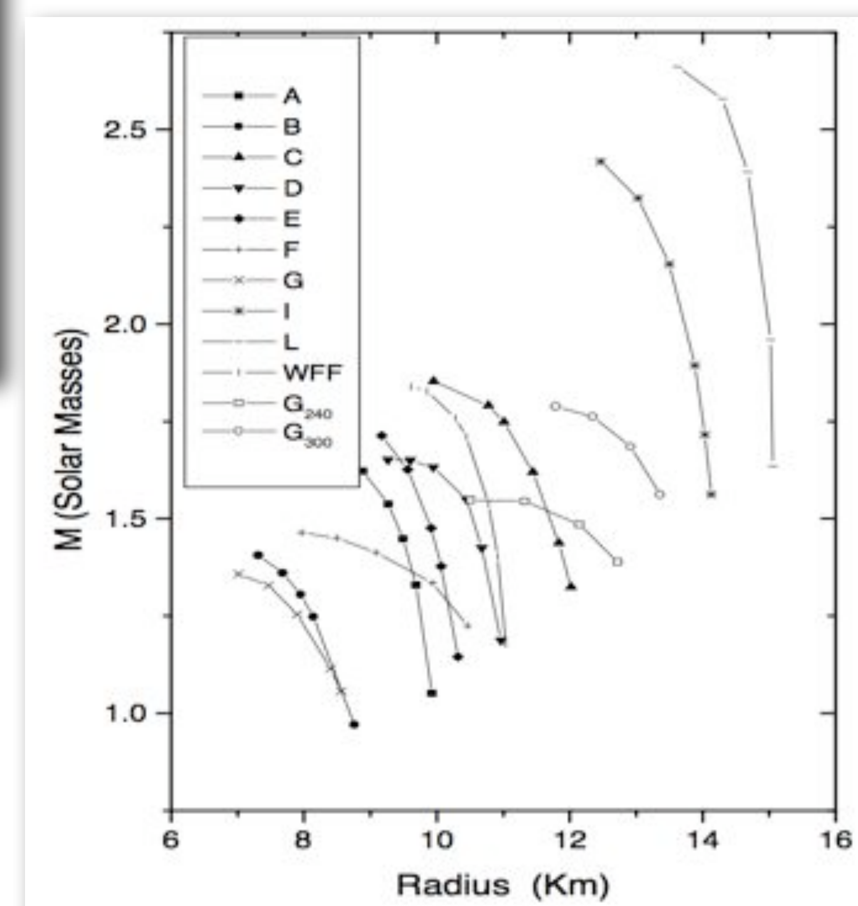
# GW Asteroseismology




+



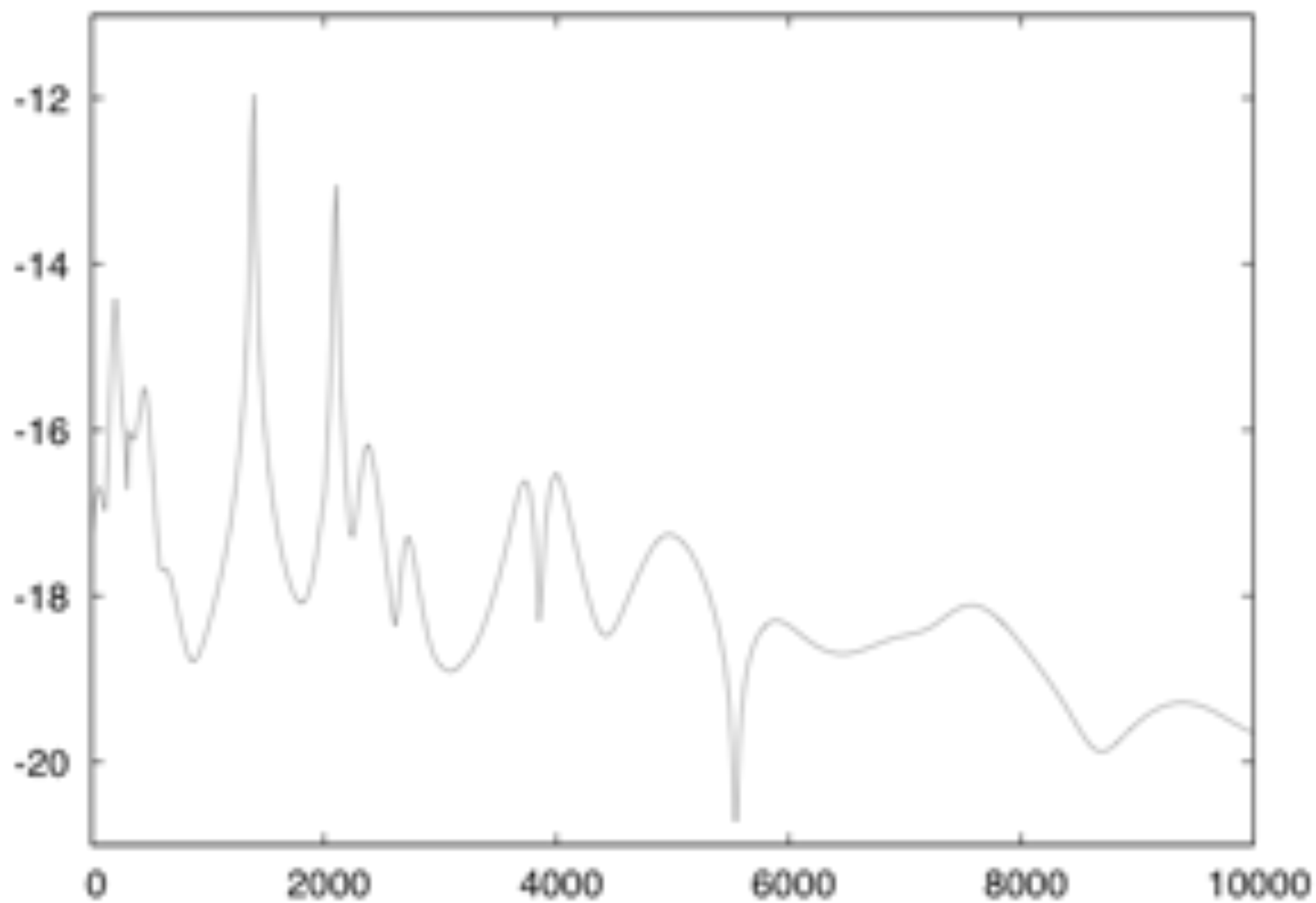

=




fundamental quadrupolar mode
and first w-mode
(no rotation included)

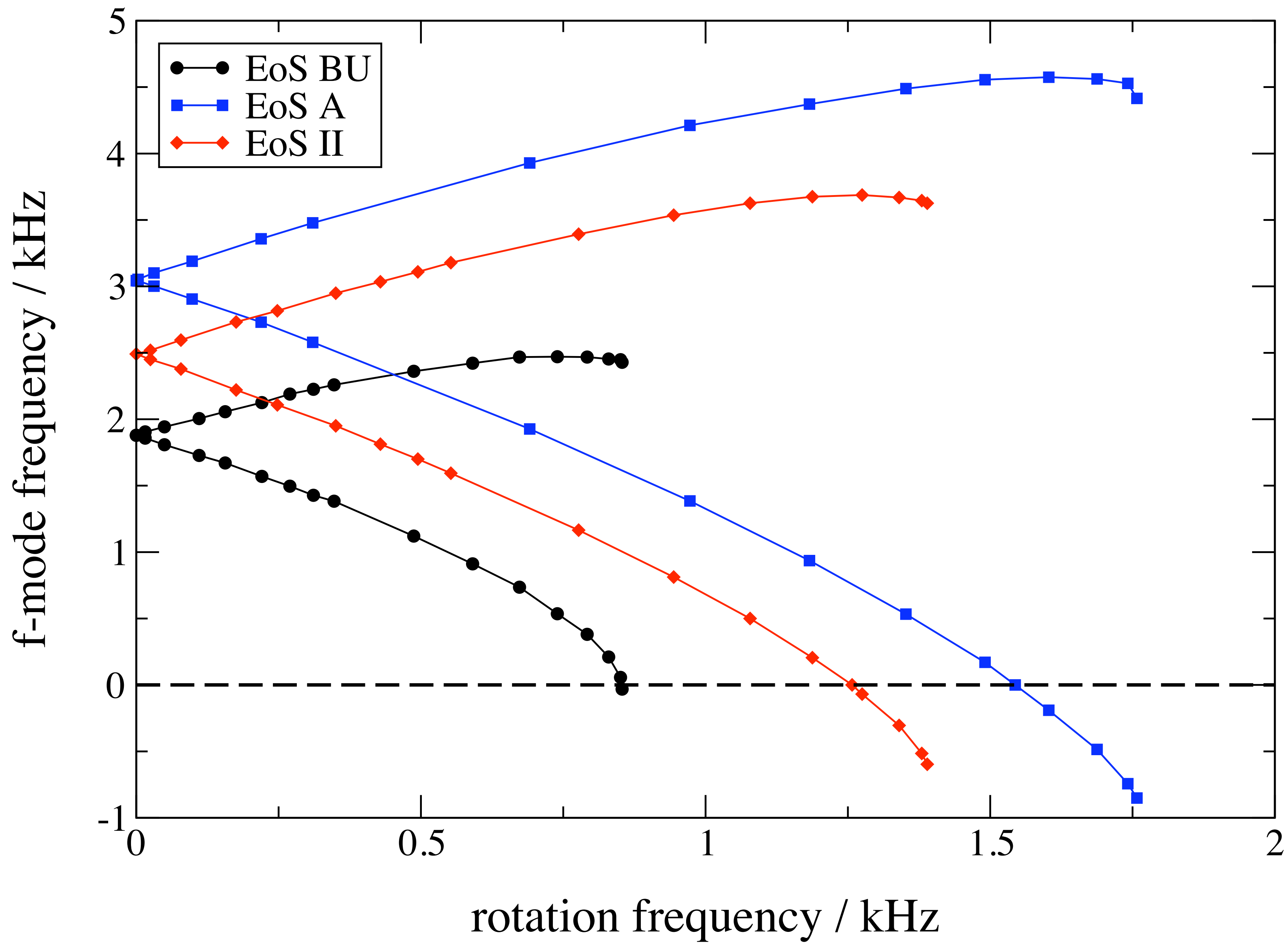
## Rotation splits non-axisymmetric modes

- Degeneracy of pro- and retrograde rotating modes is removed; consider for example *m* = +2/-2




Responsible for rotational instabilities

- How does the fundamental quadrupolar mode change with rotation?



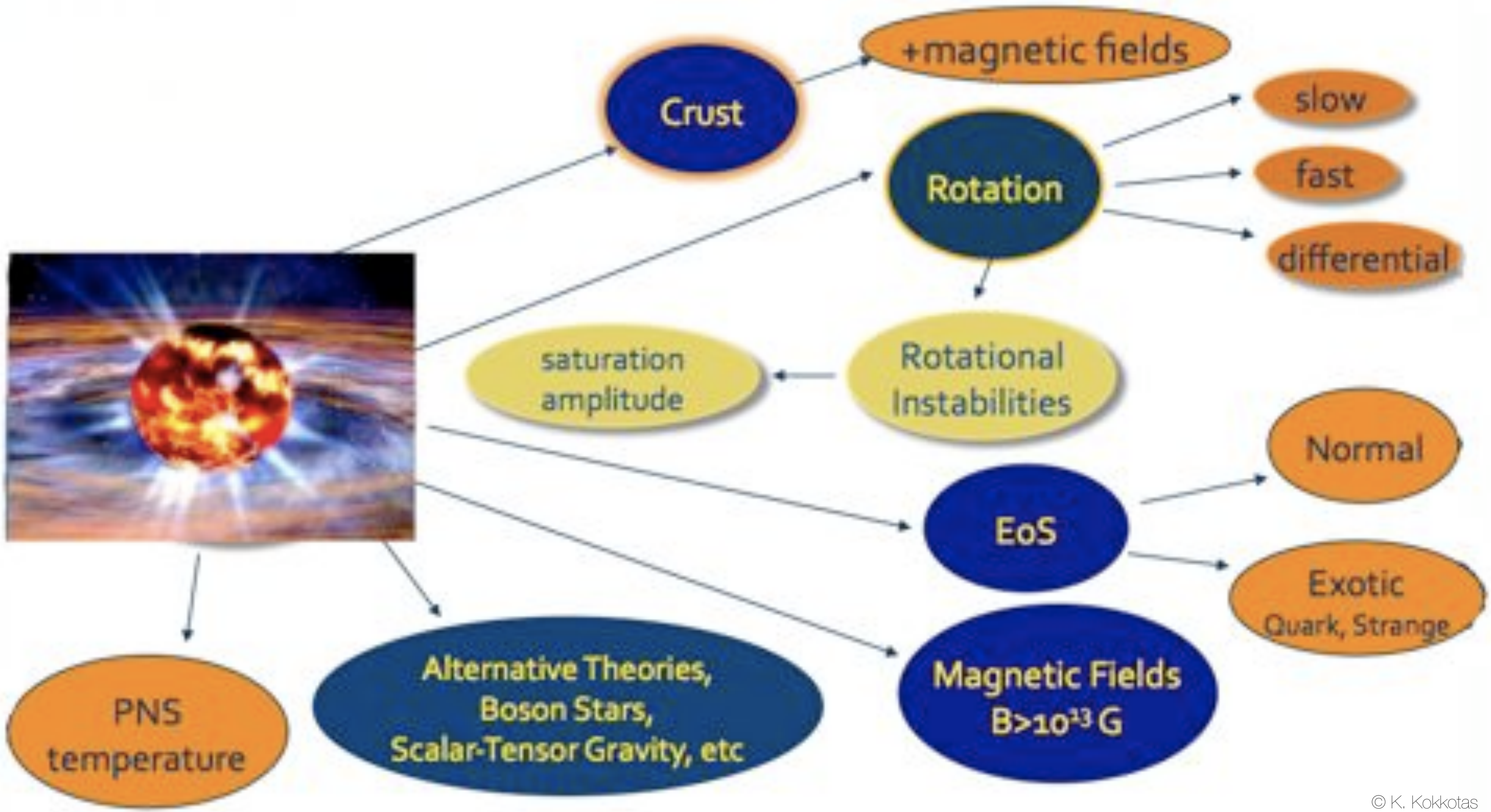
## Some results (mainly Newtonian):

- **f-mode:**
  - The m = 2 mode becomes unstable at $\Omega/\Omega_K > 0.85$
  - differential rotation will affect onset of instability (it happens earlier)
  - Up to 10% of energy/ angular momentum is radiated by GWs

- **r-mode:**
  - GW amplitude depends on saturation
  - mode coupling might not allow for high amplitudes
  - crust, hyperons, magnetic field affect the efficiency of the instability

- **uncertainties:**
  - relativistic growth times
  - nonlinear saturation
  - effect of magnetic fields

# We‘ve just started...




© K. Kokkotas