Dynamics of Quantum Vorticity in a Random Potential

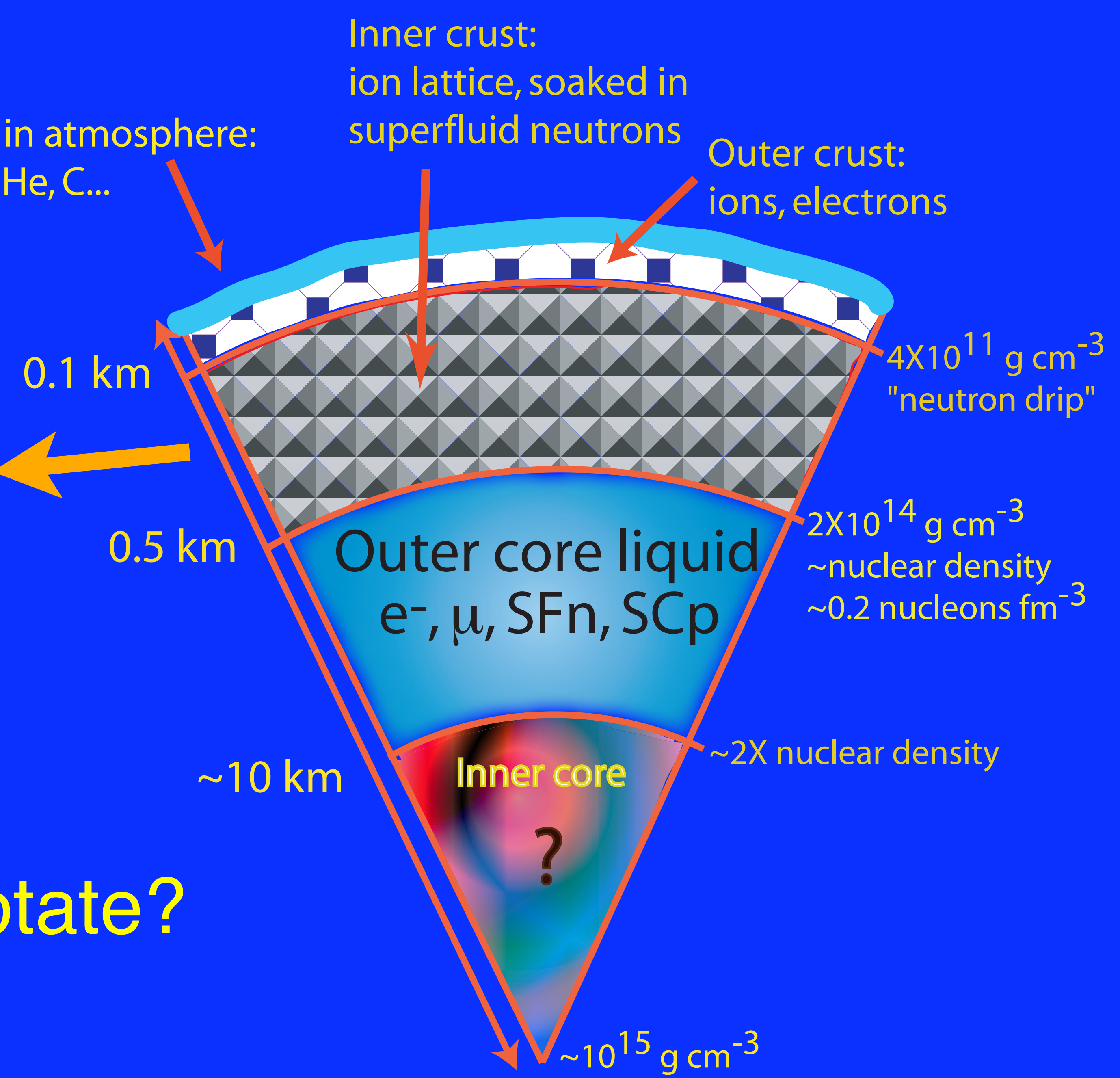
Bennett Link

MONTANA STATE UNIVERSITY

Neutron star consists of distinct superfluids

The inner-crust solid coexists with a *neutron superfluid*.

How does this SF rotate?



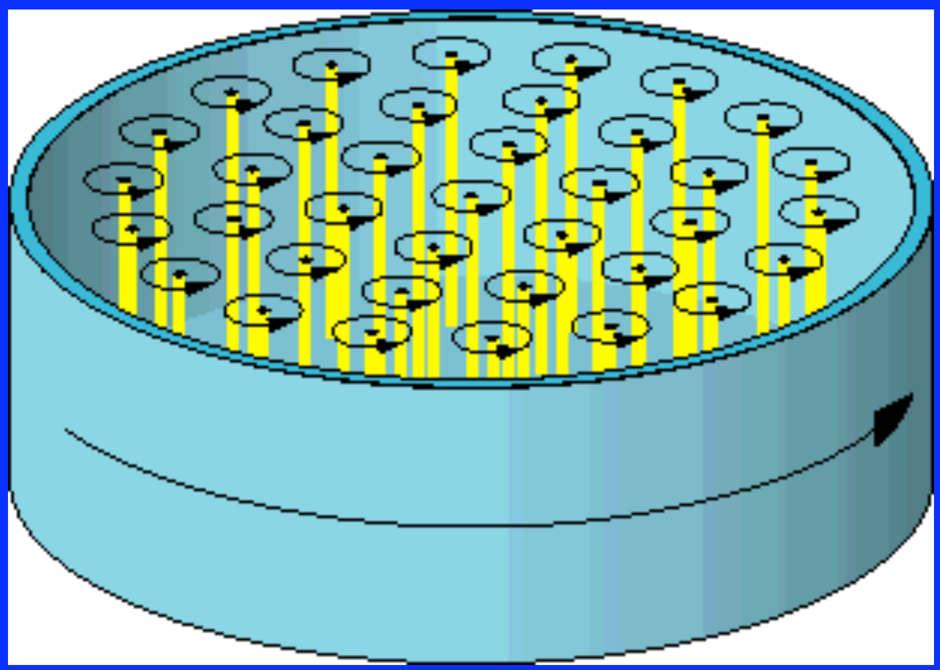
Why worry about SF rotational dynamics?

To develop a theory of NS "seismology"

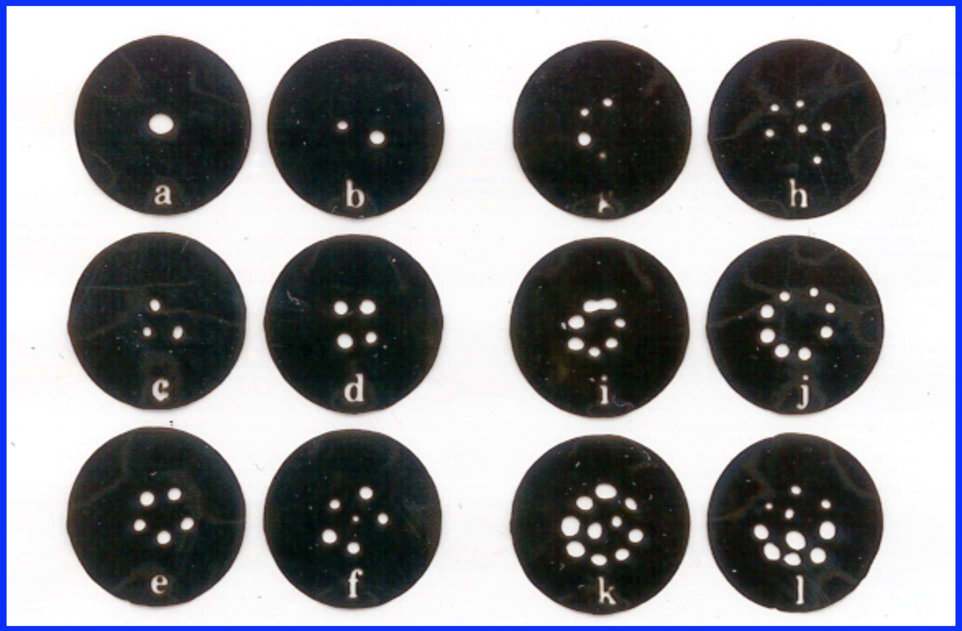
Some observed modes in NSs,
phenomena to explain/understand:

- Spin jumps (glitches).
- Precession ("wobble", nutation).
- Stochastic spin variations (timing noise).
- Crust shear modes excited in magnetar flares.

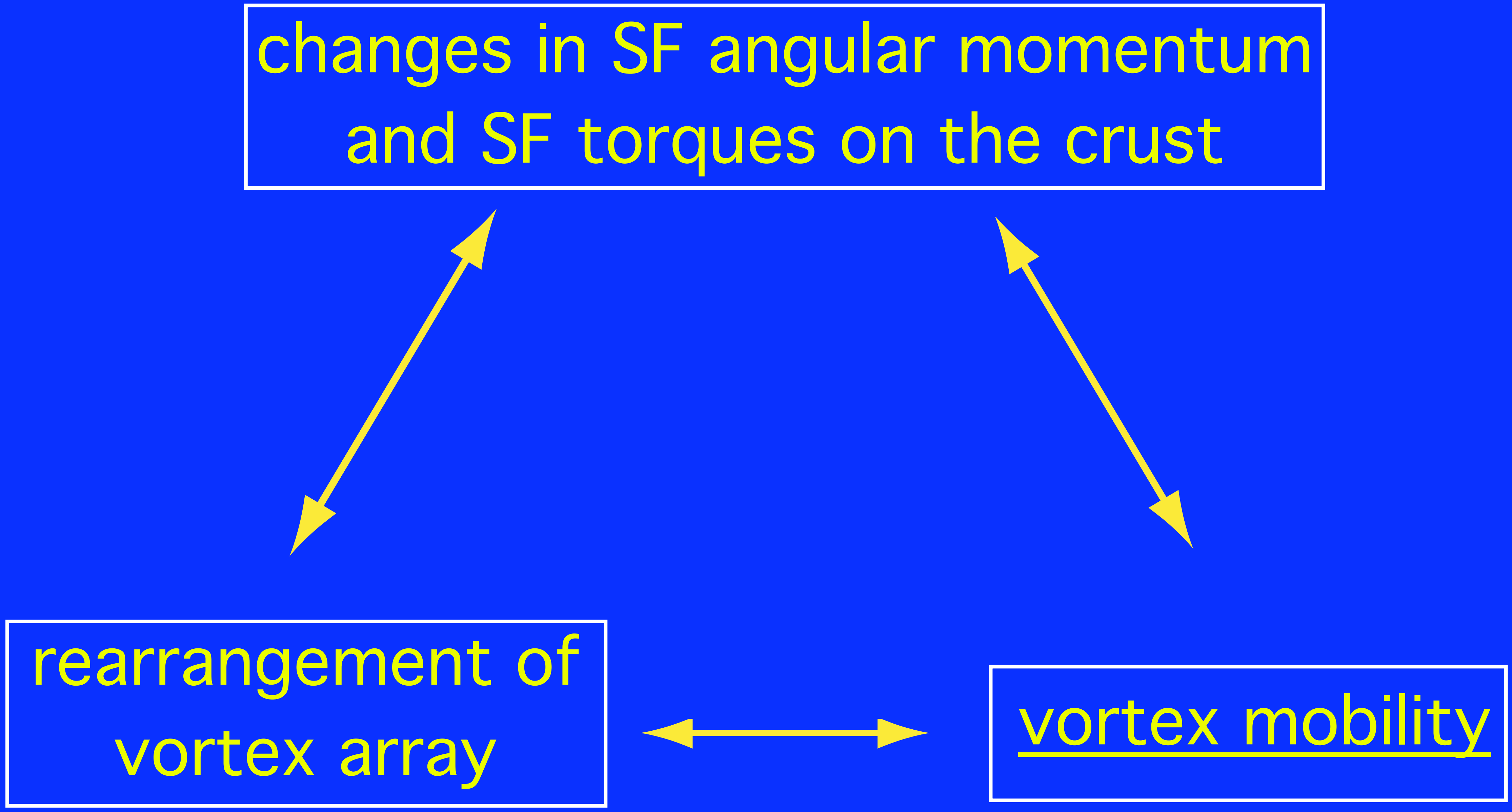
The neutron SF's rotation



Rotating superfluid He



- Distribution of vortices determines SF angular momentum.
- Coupling between container (the crust) and the SF is determined by forces on the vortices.

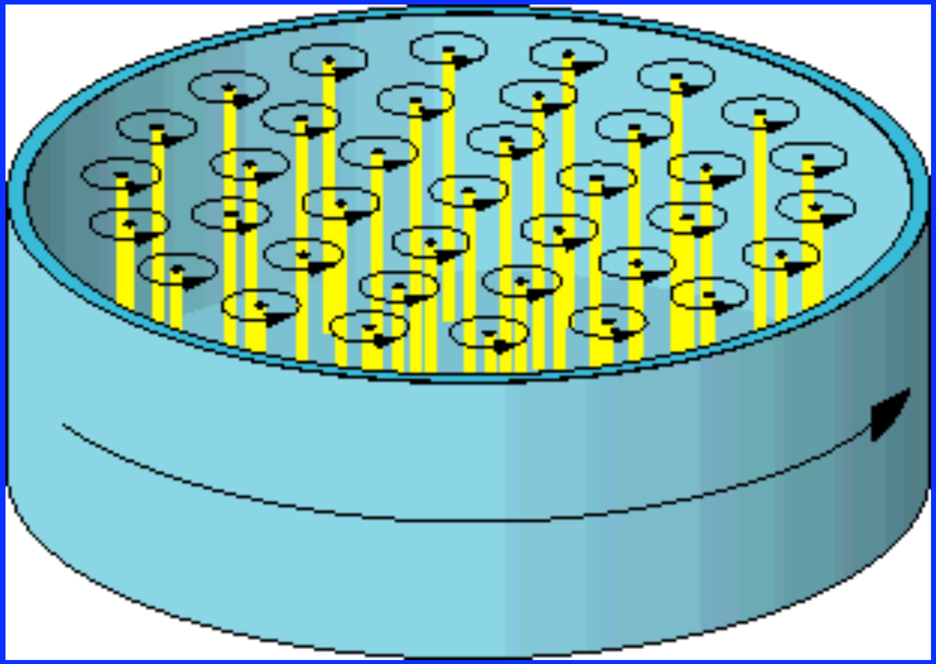


How mobile are the inner-crust vortices in the lattice, and, how is spin dynamics affected?

Dynamical considerations

Two velocities (in the frame of the container):

1. Velocity of the net SF flow (determined by overall vortex distribution)
2. Velocity of a vortex.



Vortex mobility (velocity) is determined by forces...

The forces

- A vortex interacts with nuclei, E_{vn}~1 MeV.
  (Donati & Pizzochero 06; Avvogadro et al. 07)
- A vortex has self-energy (“tension”).
- There is a lift force (Magnus force) on a moving vortex segment.
- There is dissipation force associated with irreversible energy transfer to the lattice.

The forces

- A vortex interacts with nuclei, E_{vn}~1 MeV.

  (Donati & Pizzochero 06; Avvogadro et al. 07)

- A vortex has self-energy (“tension”).
- There is a lift force (Magnus force) on a moving vortex segment.
- There is dissipation force associated with irreversible energy transfer to the lattice.

Vortex/nucleus interaction due to density dependence of the SF gap

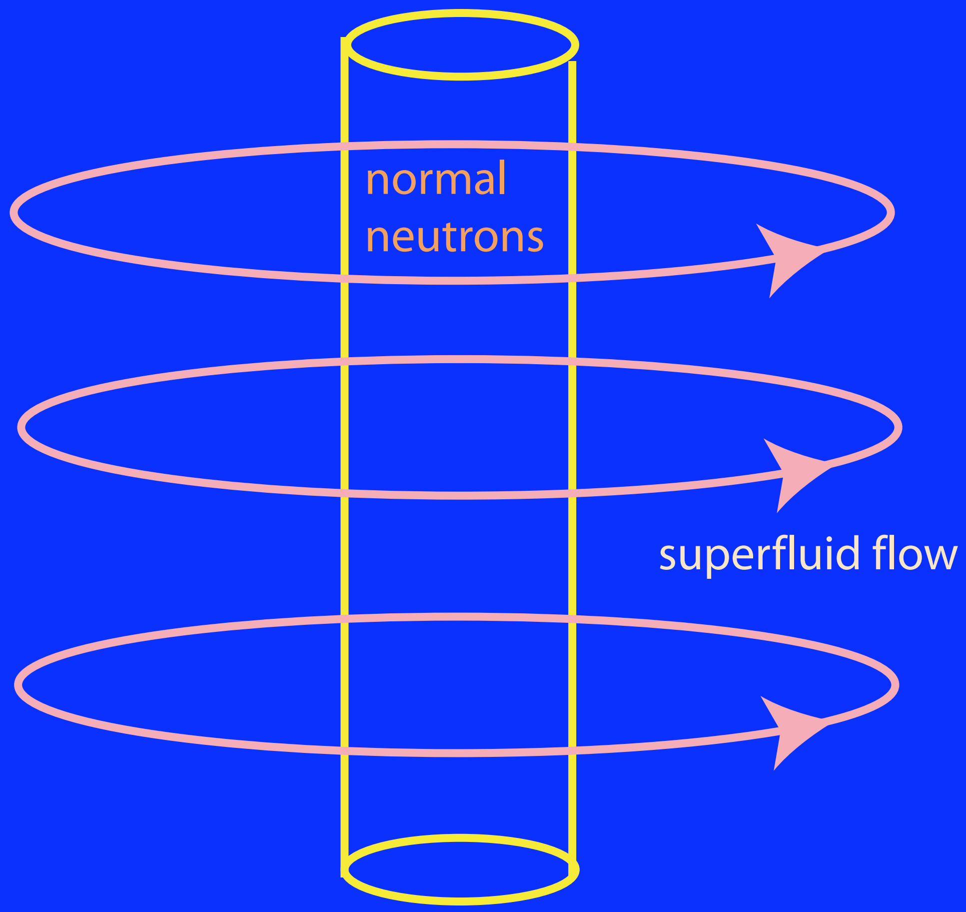
The forces

- A vortex interacts with nuclei, $E_{vn} \sim 1$ MeV.

  (Donati & Pizzochero 06; Avvogadro et al. 07)

- A vortex has self-energy (“tension”).
- There is a lift force (Magnus force) on a moving vortex segment.
- There is dissipation force associated with irreversible energy transfer to the lattice.

A vortex has a large self-energy



$$\oint v_s \cdot \mathbf{d}l = \kappa \equiv h/2m_n$$

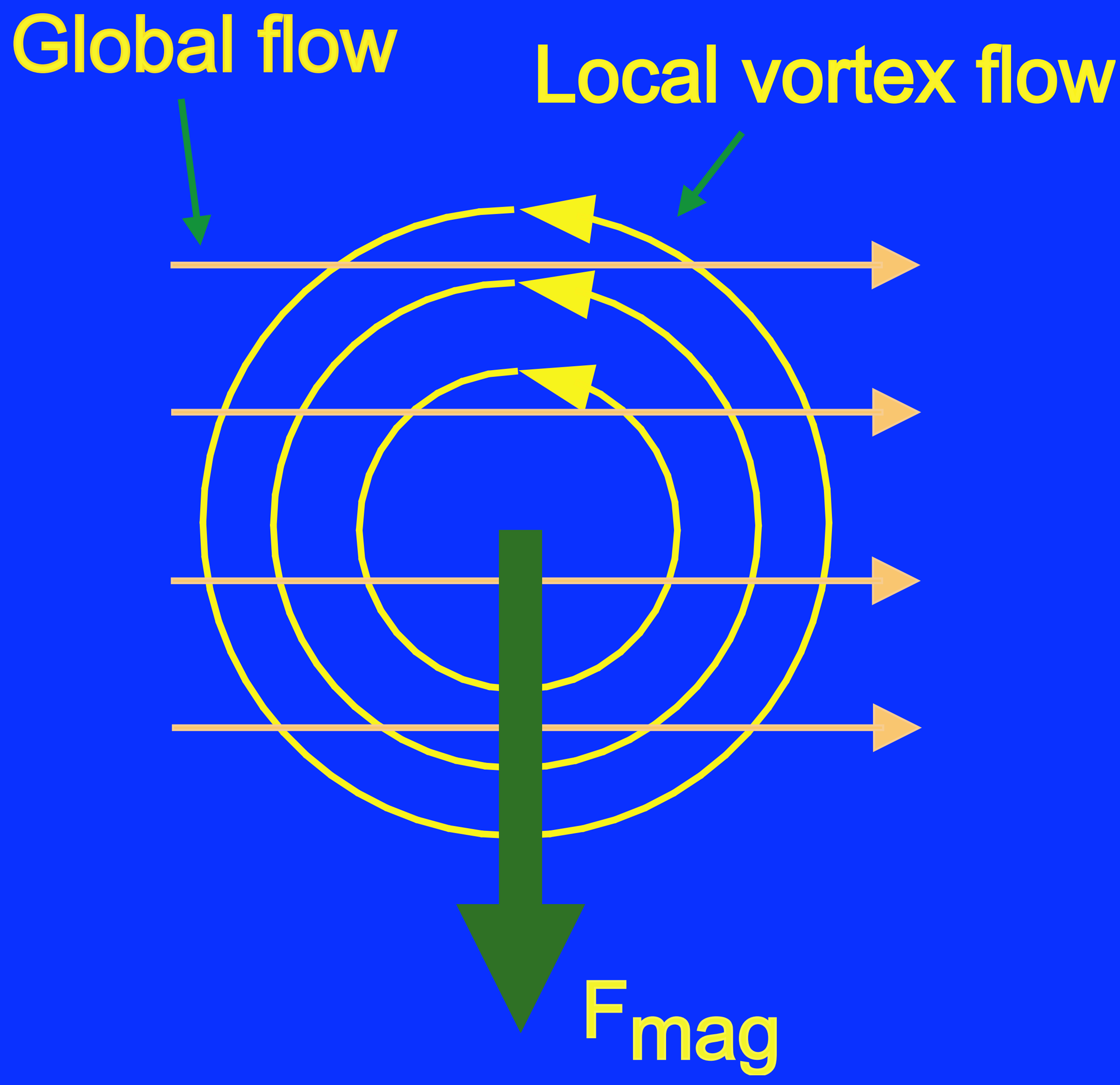
"Tension"

$$\frac{E_k}{L} \equiv T_v \simeq \; 10 \text{ MeV fm}^{-1}$$

The forces

- A vortex interacts with nuclei, E_{vn}~1 MeV.
  (Donati & Pizzochero 06; Avvogadro et al. 07)
- A vortex has self-energy (“tension”).
- <u>There is a lift force (Magnus force) on a moving vortex segment.</u>
- There is dissipation force associated with irreversible energy transfer to the lattice.

The Magnus force

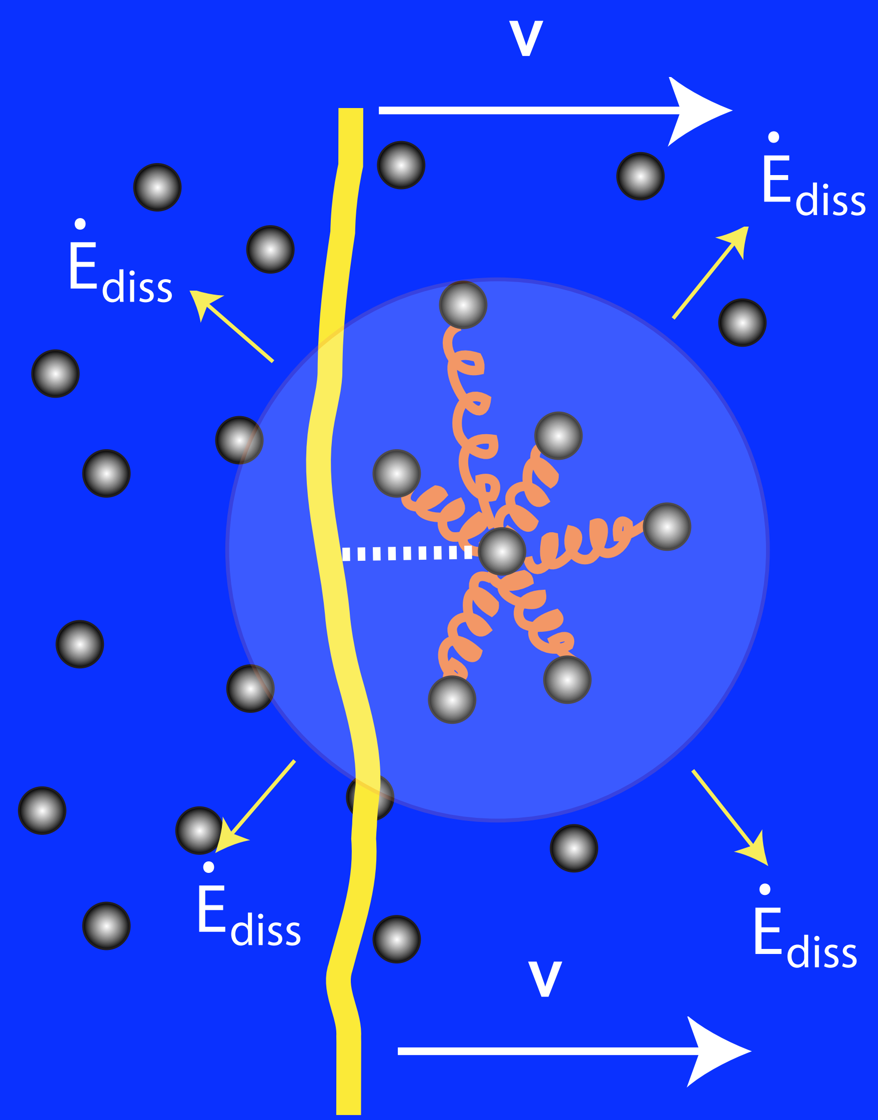


The forces

- A vortex interacts with nuclei, E_{vn}~1 MeV.

  (Donati & Pizzochero 06; Avvogadro et al. 07)
- A vortex has self-energy ("tension").
- There is a lift force (Magnus force) on a moving vortex segment.
- <u>There is dissipation force associated with irreversible energy transfer to the lattice.</u>

Vortex motion couples to phonons



Equations of motion

(Sonin 87; Link 08)

$$T_v \frac{\partial^2 \mathbf{u}}{\partial z^2} + \rho_s \boldsymbol{\kappa} \times \left(\frac{\partial \mathbf{u}}{\partial t} - \mathbf{v_s} \right) + \mathbf{f_0} - \eta \frac{\partial \mathbf{u}}{\partial t} = 0$$

vortex displacement

ambient SF flow

drag term

tension

SF density

vorticity vector

non-dissipative lattice force

Properties

$$T_v \frac{\partial^2 \mathbf{u}}{\partial z^2} + \rho_s \boldsymbol{\kappa} \times \left(\frac{\partial \mathbf{u}}{\partial t} - \mathbf{v_s} \right) + \mathbf{f_0} - \eta \frac{\partial \mathbf{u}}{\partial t} = 0$$

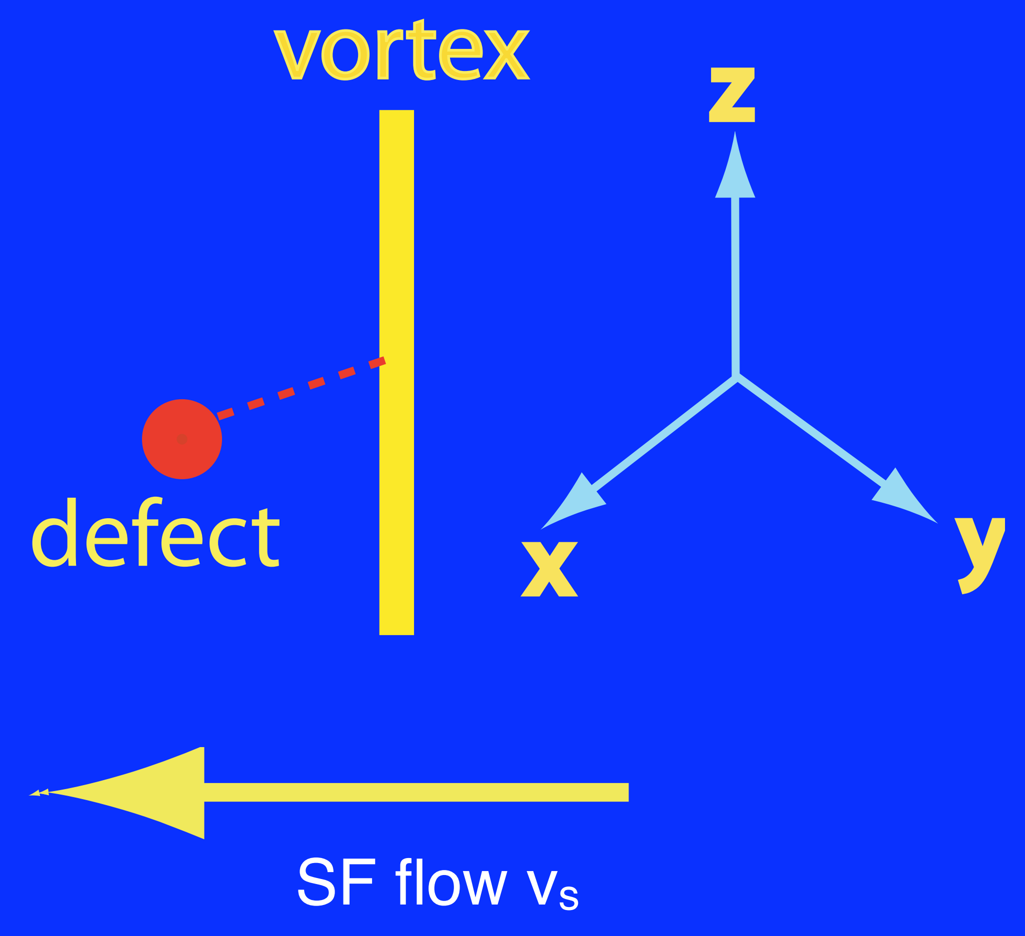
- Diffusive equation. No inertial term. Vortex supports helical waves, “kelvin modes”.
- Vortex moves transverse to an applied force.
- Velocity (not acceleration) is determined by applied forces. State of motion depends on position.

How does a vortex move through the lattice?

Simpler problem:

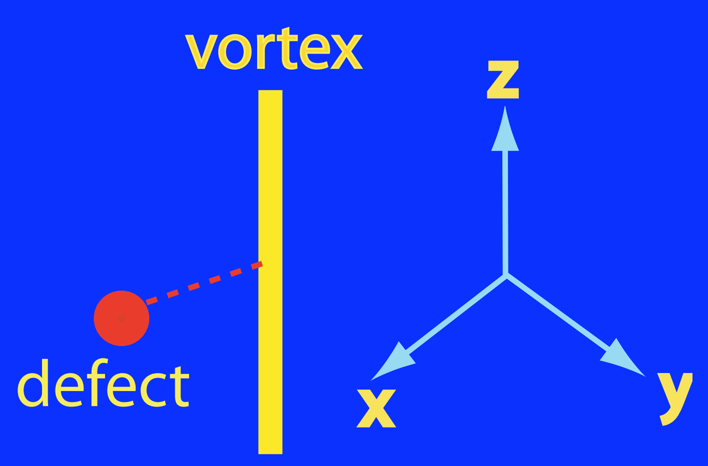
(A. Sedrakian, 95)

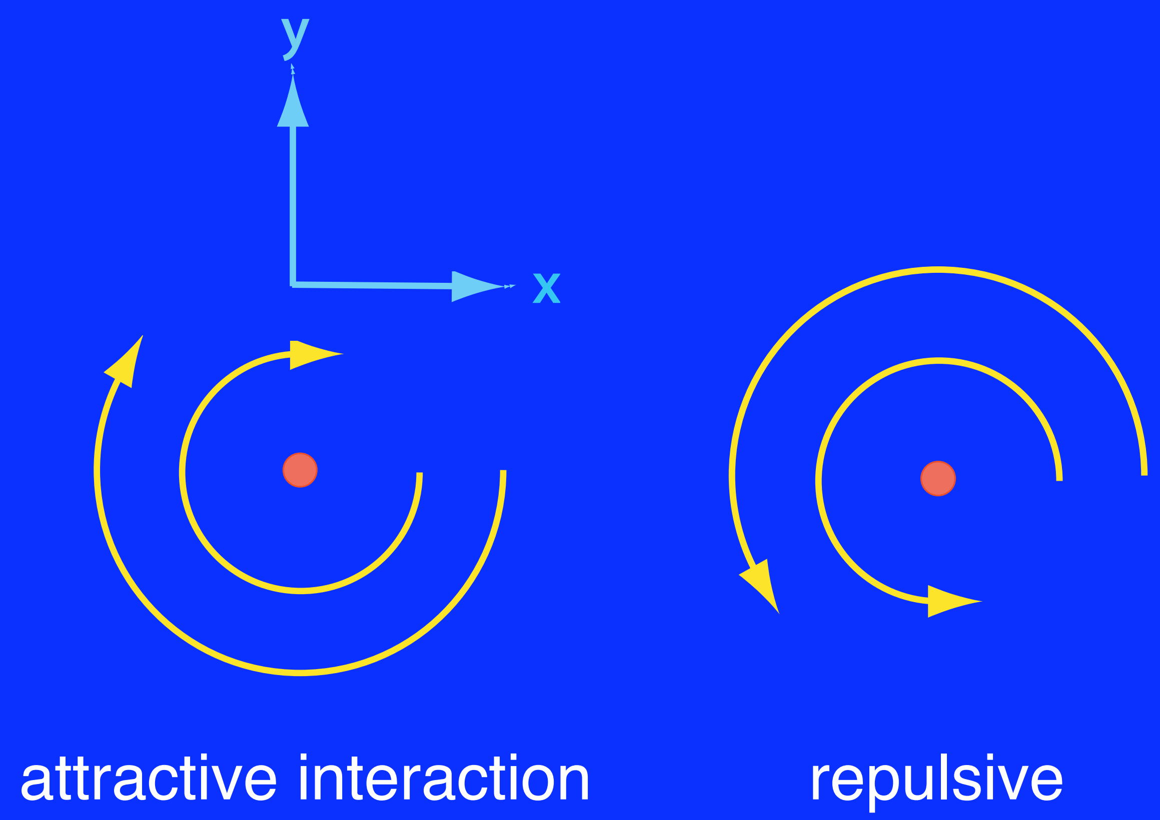
- Straight vortex segment.
- Single, point potential.



Motion of a segment

- Zero dissipation.
- SF flow = 0.

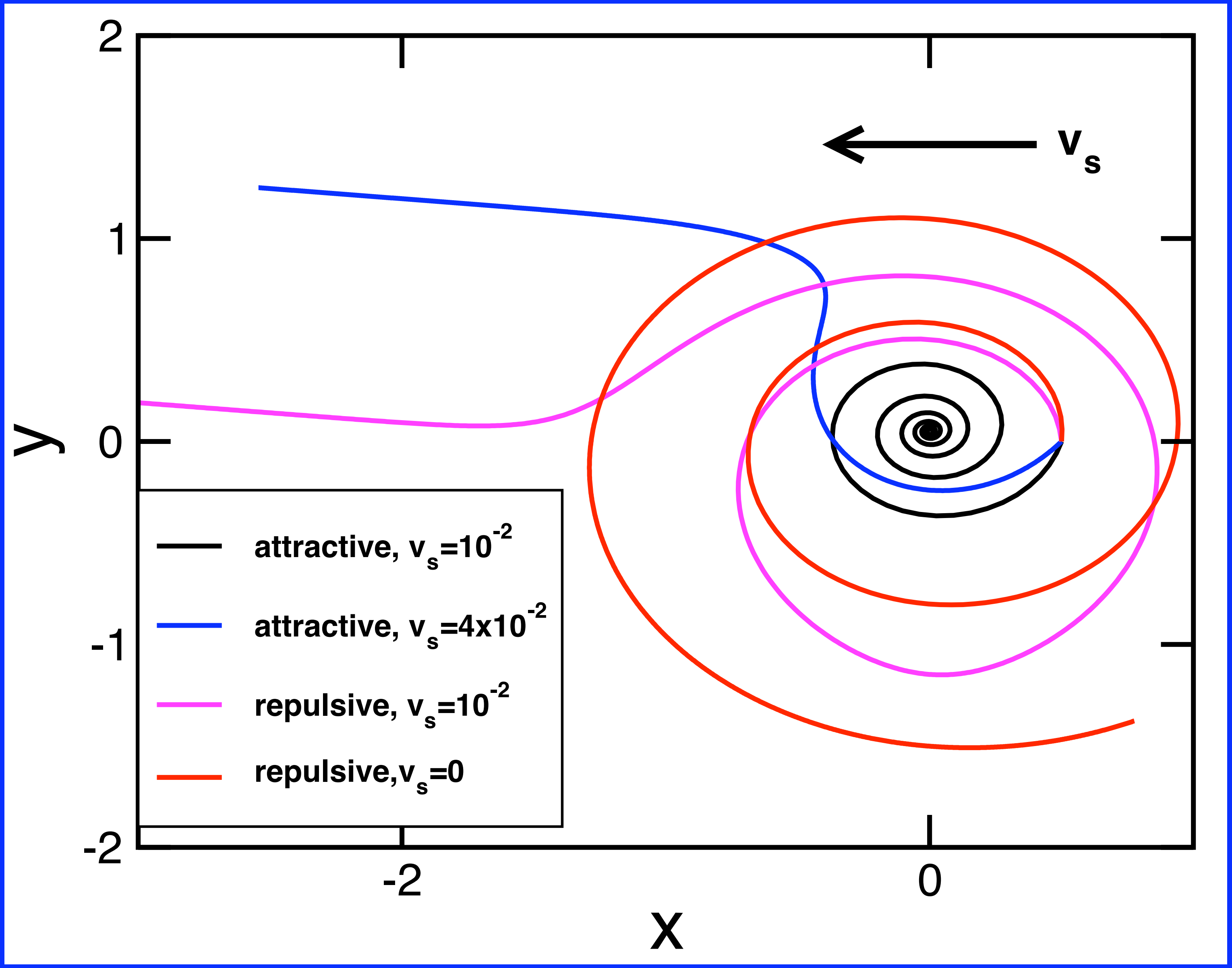




Vortex segment is trapped in an orbit

Motion of a segment

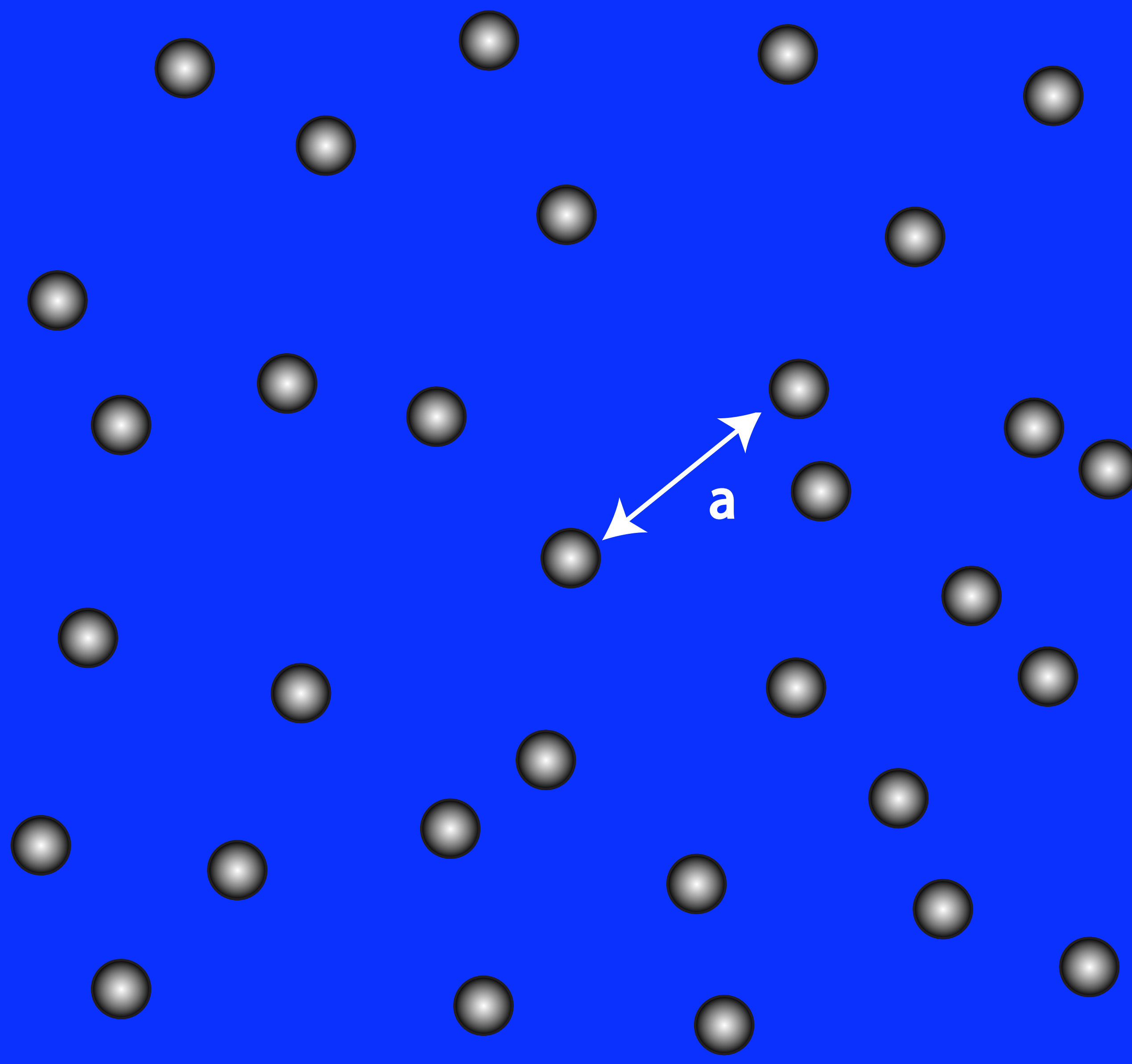
- with dissipation and
- ambient SF flow



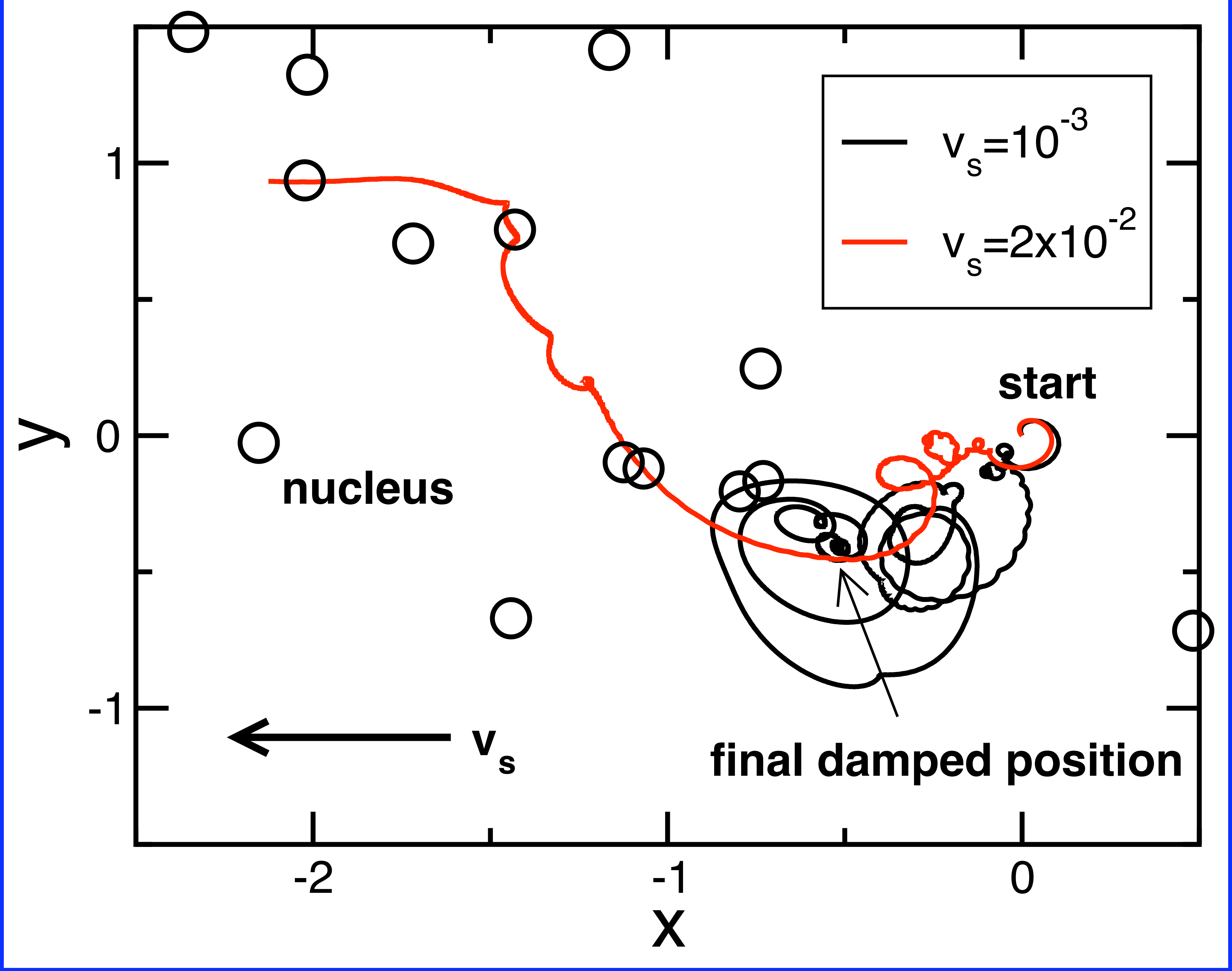
How does a vortex move through the lattice?

The nuclear lattice is probably amorphous

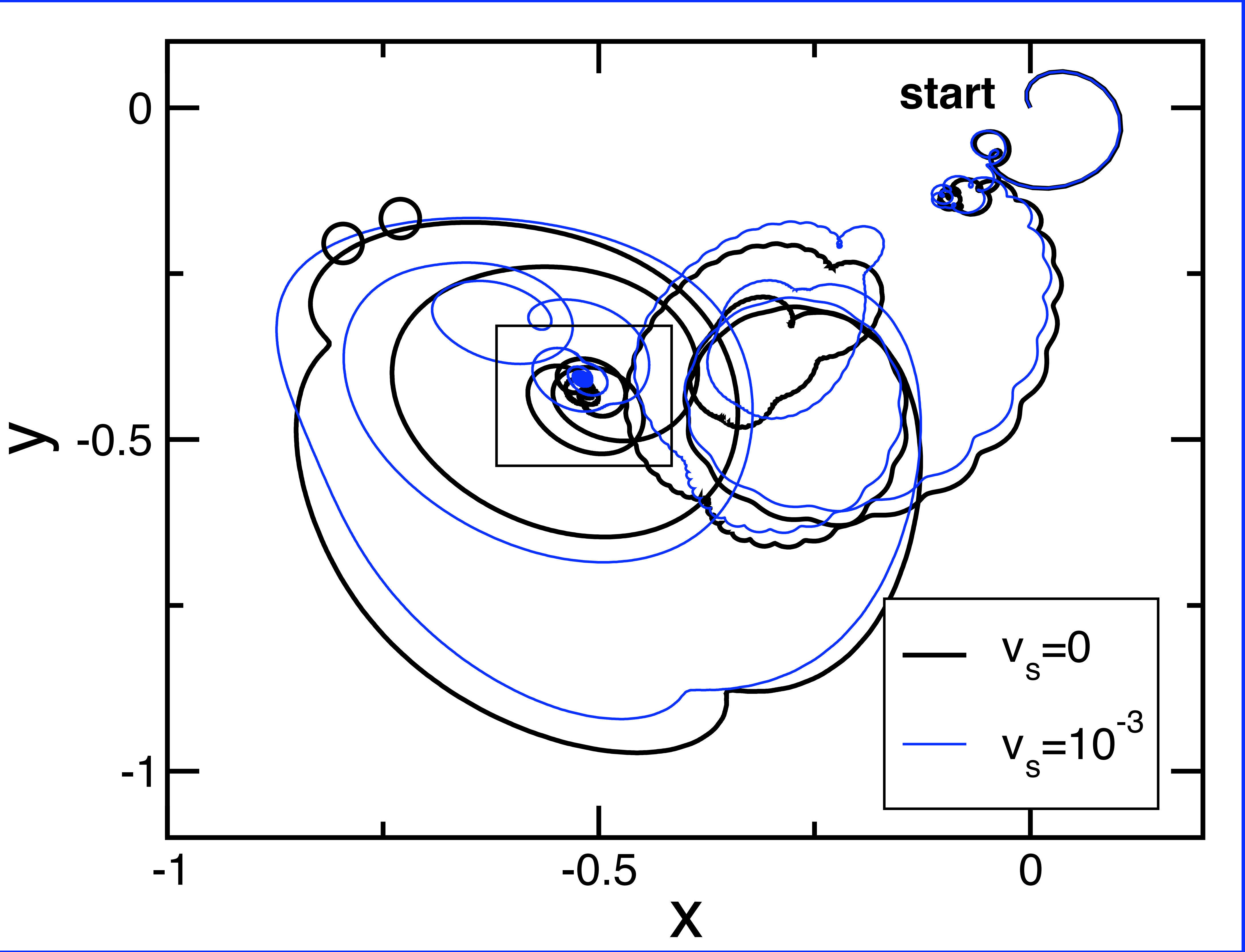
(Jones 01)



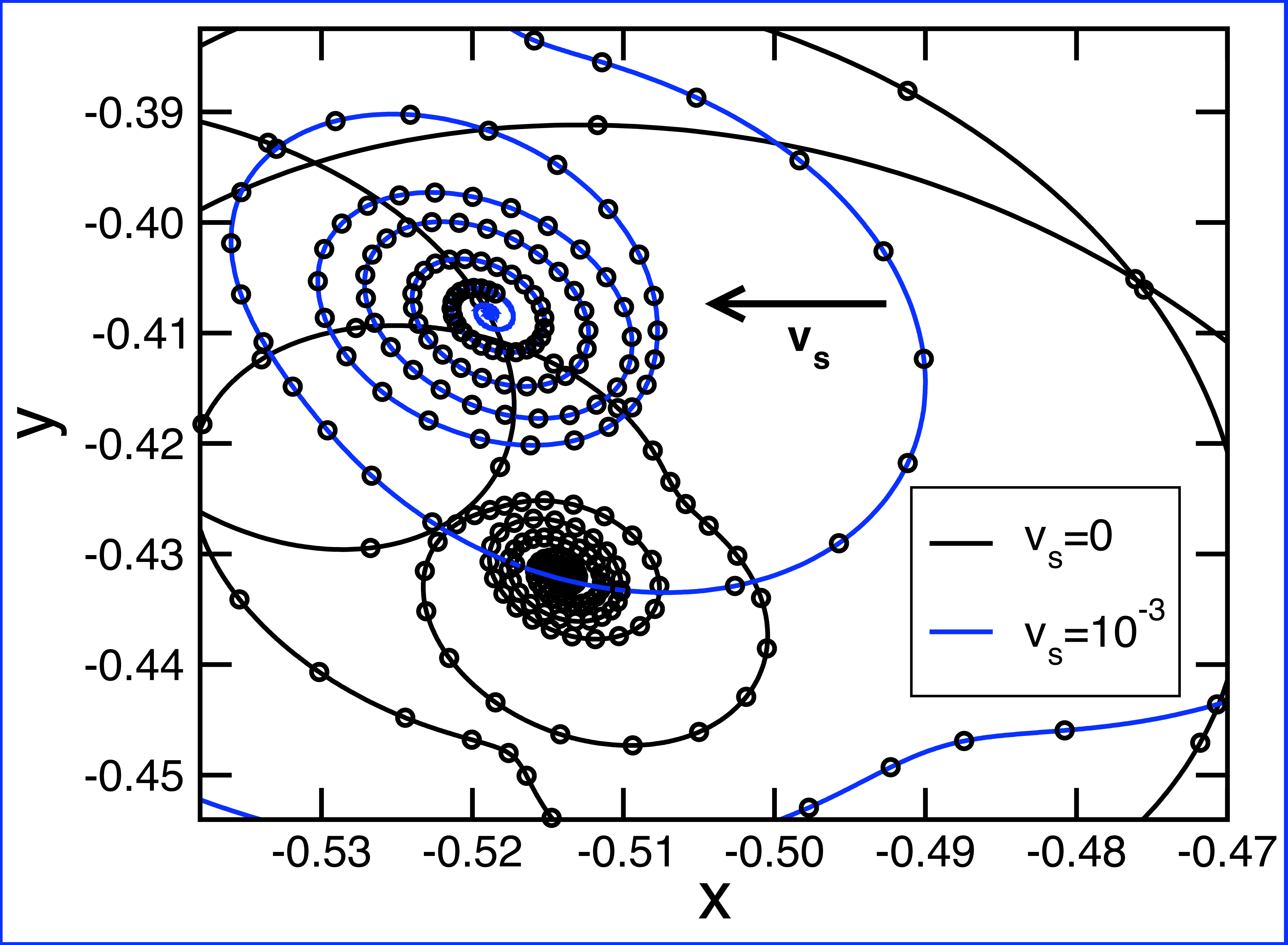
Motion in random lattice with dissipation: example segment



Pinning at low v_s

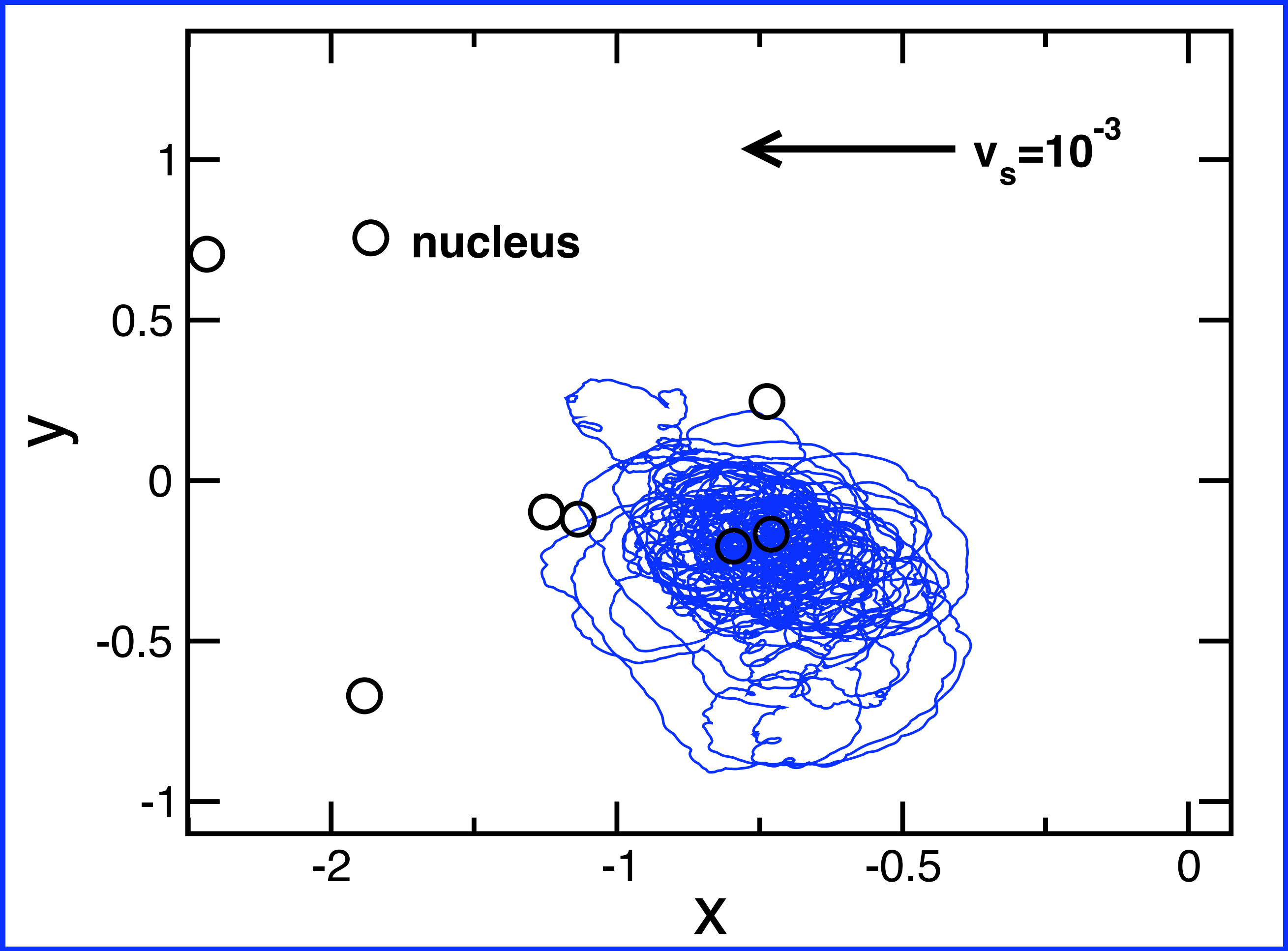


Pinning at low v_s (detail)

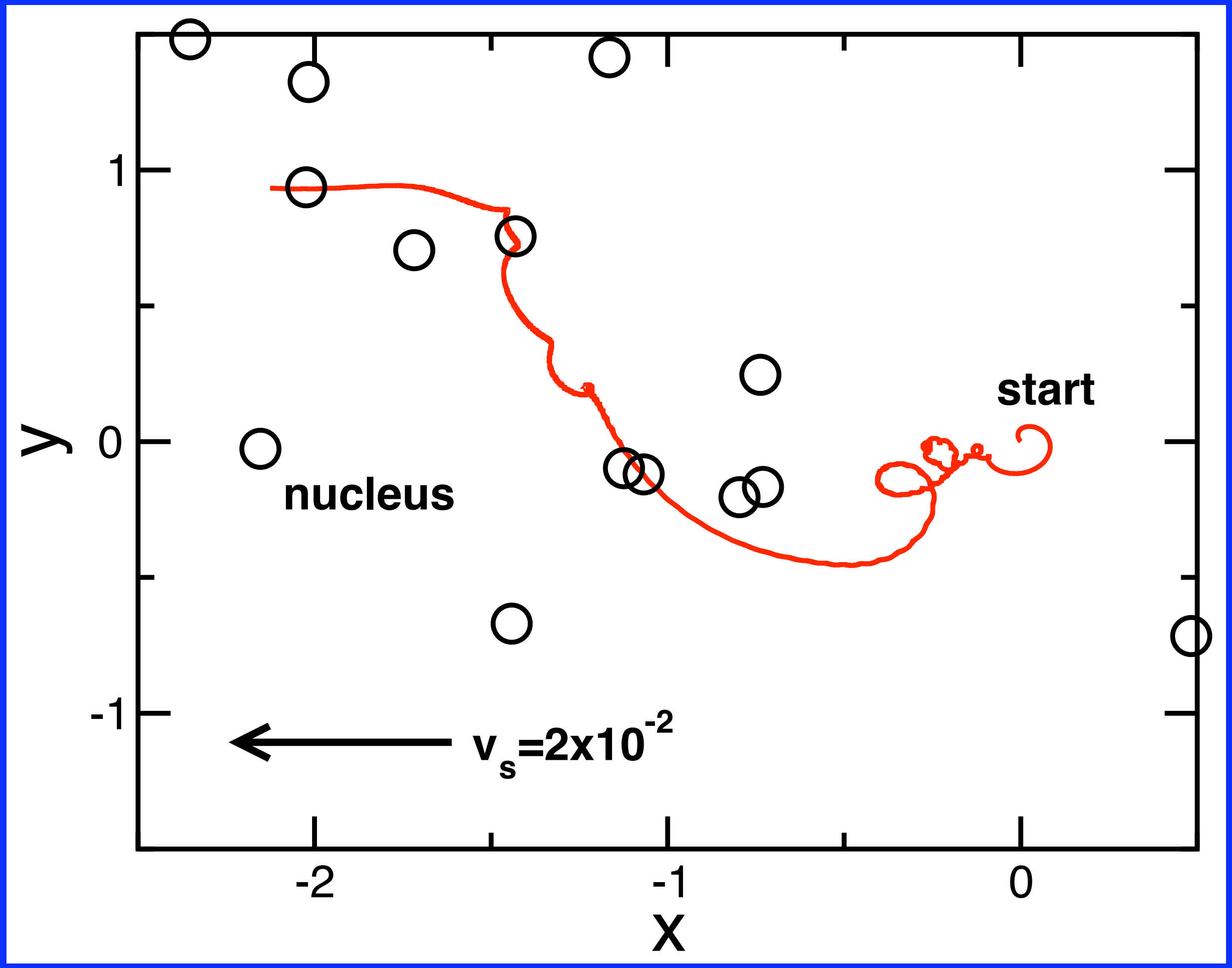


Motion with no dissipation

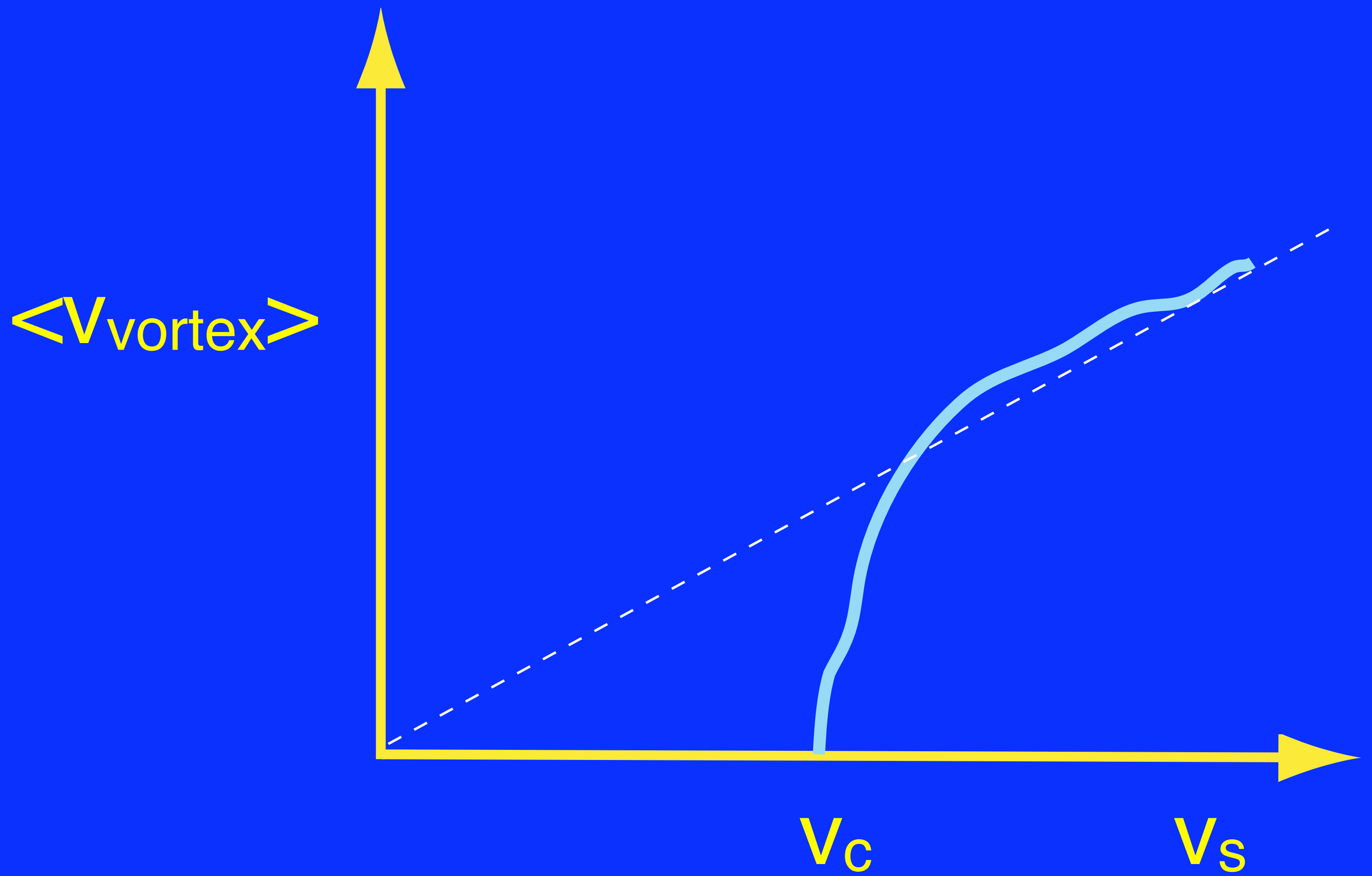
Below critical v_s



Above critical v_s



Existence of a transition



There is no translatory motion below a critical velocity,
even for zero drag.

The critical velocity

$$v_c \sim \frac{1}{\rho_s \kappa T_v^{1/2}} \left(\frac{E_{vn}}{a} \right)^{3/2} \sim \boxed{10^6 - 10^7 \text{ cm s}^{-1}}$$

$$\begin{aligned} E_{vn} &= \text{ vortex-nucleus interaction energy} \\ a &= \text{ nuclear spacing} \\ T_v &= \text{ vortex tension} \\ \rho_s &= \text{ SF mass density} \\ \kappa &= \text{ vorticity quantum} \end{aligned}$$

Critical velocities this large (10^6-10^7 cm s^{-1}) can account for glitches

(Link & Cutler 02)

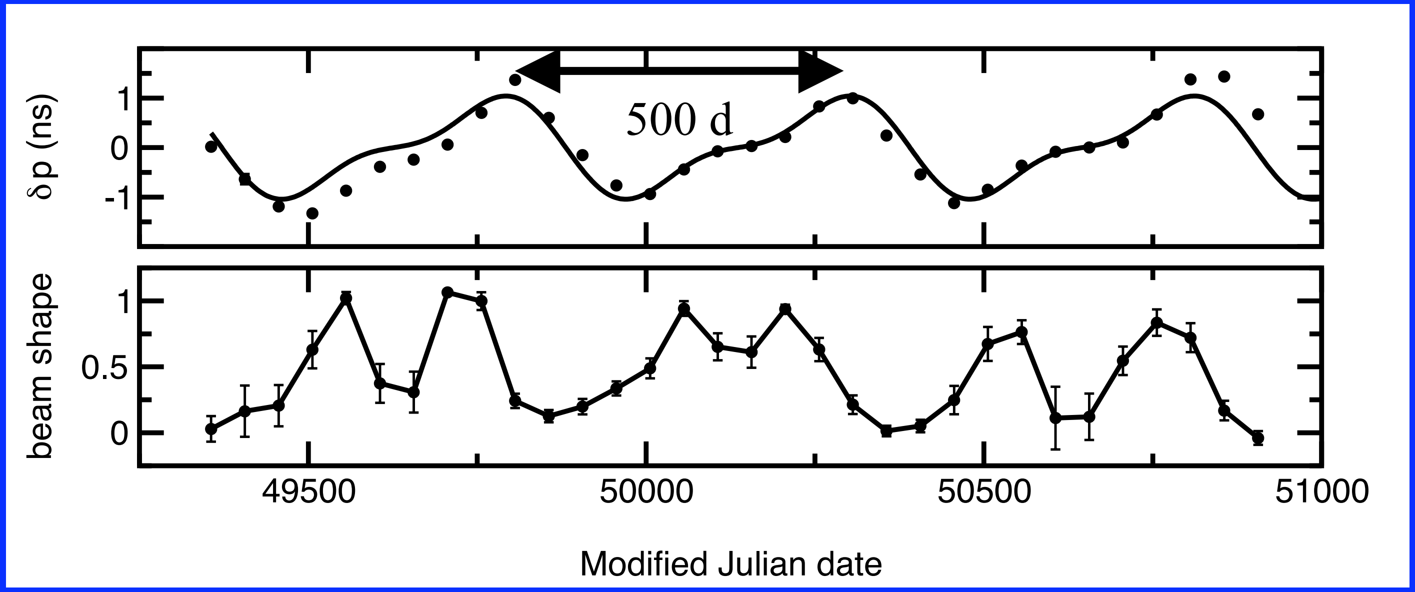
...but now there is another problem...

Some isolated neutron stars that appear to be precessing

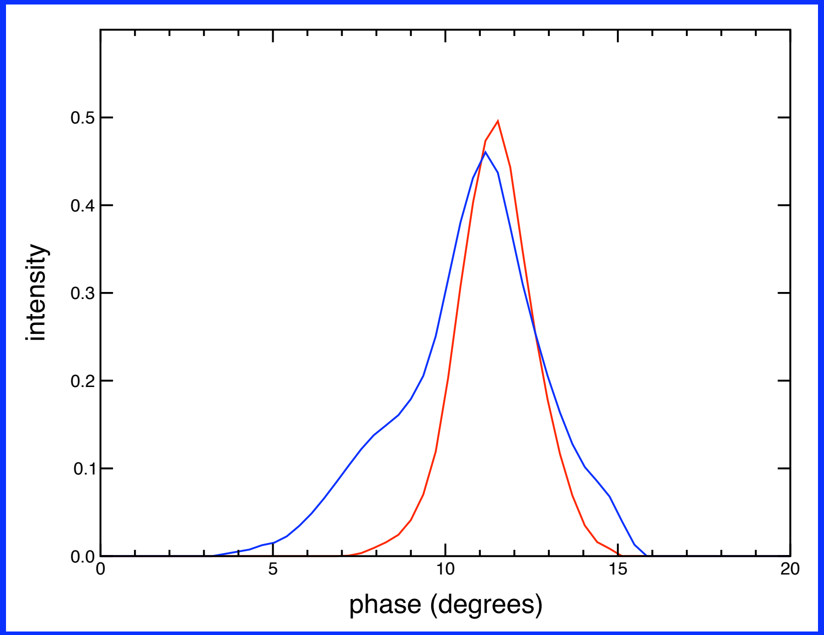
- PSR 1828-11 (Stairs, Lyne, Shemar 00)
  <u>period of ~500 d.</u>
- PSR B1642-03 (Shabanova, Lyne, Urama 01)
  <u>period of ~3 yr.</u>
- RX J0720.4 (Haberl et al. 06)
  <u>period of ~7 yr.</u>

Evidence for precession: B1828-11

(Stairs et al. 00)

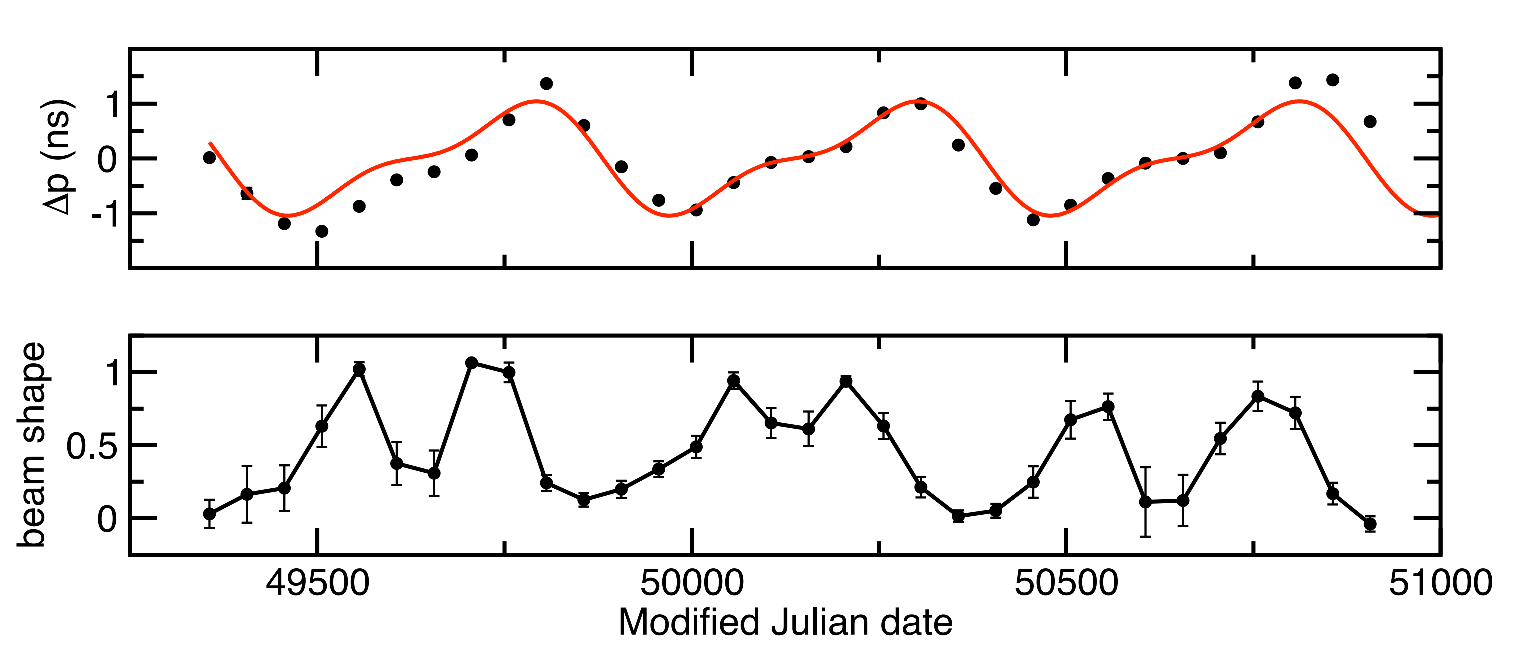


- Periodic but non-sinusoidal ⇒ harmonics.
- Strong periodicities at 511 and 256 d.
- Correlated changes in beam width.



Rigid-body modeling

(Link & Epstein 01; Akgun, Wasserman & Link 06)

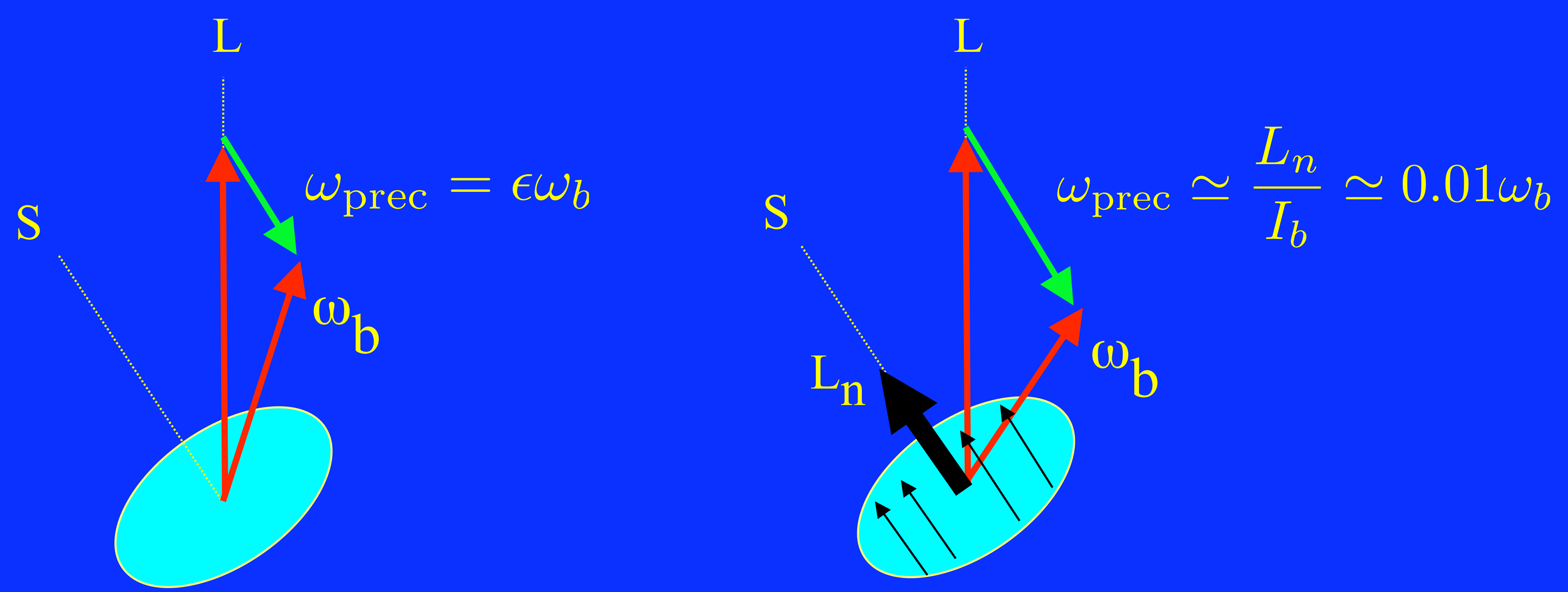


Wobble angle α = 3 deg
Dipole angle χ = various
Precession period = 511 d
Harmonic at 256 d.

- Triaxiality (expected) gives a wide parameter space.
- Non-linearity in the torque or triaxiality produces the harmonic.

With immobilized vortices, precession is very fast

(Shaham 77; A. Sedrakian et al. 99; Link & Cutler 02)



Without pinning

With pinning (gyroscopic effect)

PROBLEM: <u>The precession frequency would be 10^6 times faster than observed.</u>

Conclusions

- Non-dissipative lattice force immobilizes inner-crust vortices.
- Inconsistent with long-period precession (in simplified, rigid-body hydrodynamics).
- Need a better understanding of the modes of rotating quantum liquid mixtures.

"Bert's inner core is spinning on its own."

His inner core is spinning...

How does a neutron star do it?