Magnetorotational core-collapse supernovae.

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Outline

- Introduction

- Numerical method (implicit Lagrangian scheme)

- Magnetorotational(MR) mechanism of supernova explosion.

- Core-collapse simulations.

- MR supernova with quadrupole field.

- Magnetorotational instability(MRI) in MR supernova.

- MR supernova with dipole field, jet formation.

- MR supernova with different core masses and rotation rates.

- 3D features of MR supernova simulations

- Mirror symmetry violation of the magnetic field in rotating stars – one-sided jets, kicks.

- Conclusions.

Supernova classification

- Type I – no hydrogen in SN spectra.
- Тип II – hydrogen lines in SN spectra.
- Type Ia – light curves are similar (thermonuclear supernovae)
- Type Ib – no hydrogen, but light curves resemble SN type II light curves.
- Type Ic – no helium lines, light curves resemble SN type II light curves.
- There is more detailed SN classification.

Introduction (continued)
Stars on late evolution stages. Gravitational collapse.

- Loss of stability. Averaged adiabatic index decrease 5/3 -> 4/3
- Iron dissociation.
- Collapse.
- Matter neutronization.
- Density in the center reaches nuclear values ($2.8 \cdot 10^{14}$ g/sm^3).
- Neutrino cooling (URCA processes).
- The matter becomes nontransparent for neutrino.
- Formation of protoneutron star, radius 10-20 km.
- If the mass of the star > 30-40M$_{sun}$ ->black hole can be formed.
- Supernova explosion -> star brightness increases by 9-10 orders.
- Maximal supernova luminosity ~ galaxy luminosity.
- Supernova explosion energy ~ 10^{51}erg.

Intoduction (continued)
Core collapse supernova explosions mechanisms

Spherically symmetrical model (neutrino deposition, shock wave formation).
(simulations by *Colgate, White; Ivanova, Imshennik, Nadezhin*).

Shock stalls at 100-200km. No explosion.

Observations show: supernova explosions are asymmetrical (SN 1987A).

Neutrino convection and hydrodynamic instabilities:

Neutrino convection *inside protoneutron star* -> increase of the energy of radiated neutrino. Does not help to explosions.

Neutrino convection *after shock front* (detailed simulations does not lead to the supernova explosion *Mueller, Janka*)

Standing Accretion Shock Instability (SASI) (Blondin, Mezzacappa, Janka, Yamada…). Self-consistent simulations do not give explosion with sufficient level of confidence.

Acoustic supernova (Barrows) neutron star oscillations. Energy is too small for explosions. (papers by K.Sato group, our results)

Magnetorotational supernova (Bisnovatyi-Kogan,1970). Rotation +magnetic field

Operator-difference scheme
developed by N.V.Ardeljan et al. (Moscow State University)

Lagrangian, implicit, triangular grid with rezoning, completely conservative

Method of **basic** operators (Samarskii) – grid analogs of basic differential operators:

GRAD(scalar) (differential) ~ GRAD(scalar) (grid analog)
DIV(vector) (differential) ~ DIV(vector) (grid analog)
CURL(vector) (differential) ~ CURL(vector) (grid analog)
GRAD(vector) (differential) ~ GRAD(vector) (grid analog)
DIV(tensor) (differential) ~ DIV(tensor) (grid analog)

Implicit scheme. Time step restrictions are weaker for implicit schemes (no CFL condition).

The scheme is Lagrangian=> conservation of angular momentum.

The stability of the method was explored in
Ardeljan & Kosmachevskii Comp. and Math. Modelling 1995, 6, 209 and references therein

General scheme for the IMPLICIT MHD numerical method

External iterations (Newton method)

Intermediate iterations

Dynamical group
ρ, v

Magnetic group
B,E,σ

Energy group
E,T,p

Internal iterations

Internal iterations

Internal iterations

Equation of state

j-level ⟶ j+1-level ⟶ j+2-level
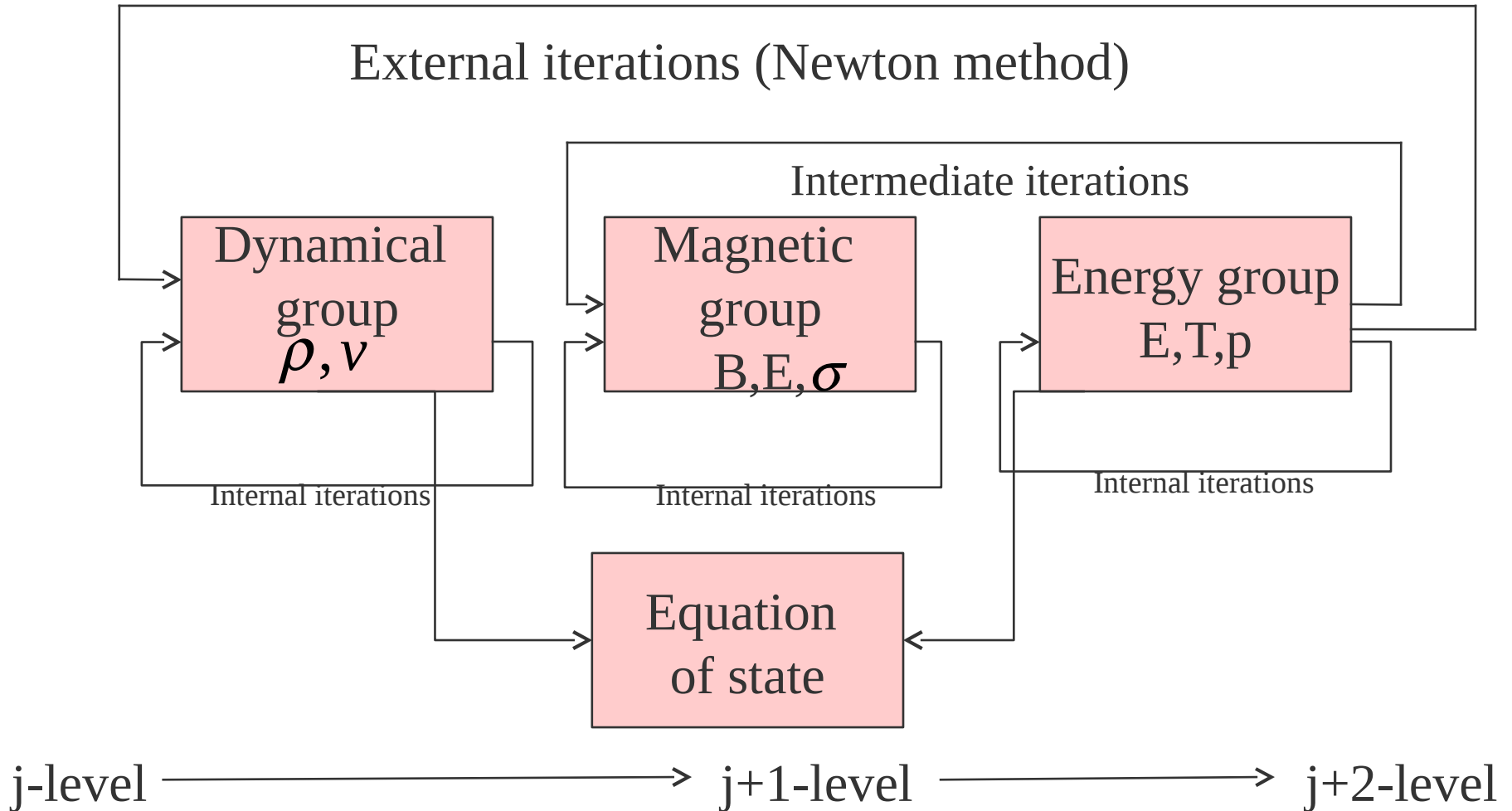
Operators are selfconjugated ⟹ Sparse matrices are symmetrical and positively defined

Matrices are inversed by conjugate gradients method (iterative).

Grid reconstruction
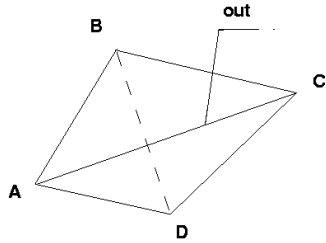
Elementary reconstruction: BD connection is introduced instead of AC connection. The total number of the knots and the cells in the grid is not changed.
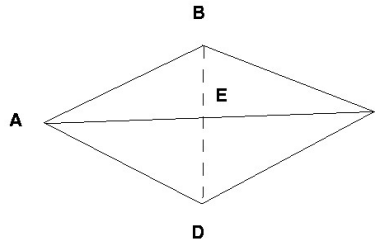
Addition a knot at the middle of the connection: the knot E is added to the existing knots ABCD on the middle of the BD connection, 2 connections AE and EC appear and the total number of cells is increased by 2 cells.
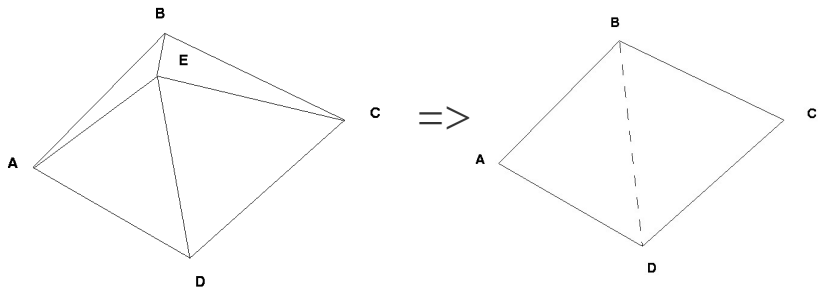
Removal a knot: the knot E is removed from the grid and the total number of the cells is decreased by 2 cells

Interpolation of grid functions on a new grid structure (local):

Should be made in *conservative* way. Conditional minimization of special functionals.

Numerical method testing

The method was tested on the following tests:

1. Collapse of a dust cloud without pressure
2. Decomposition of discontinuity problem
3. Spherical stationary solution with force-free magnetic field,
4. MHD piston problem,

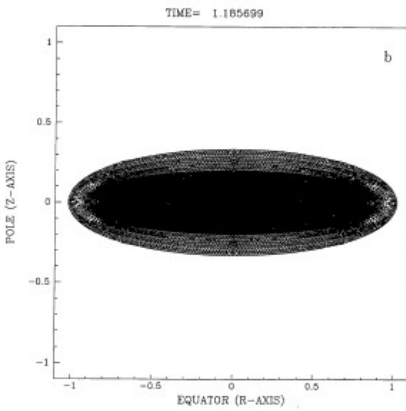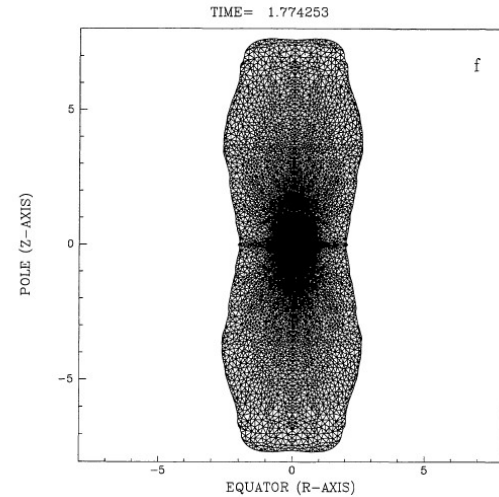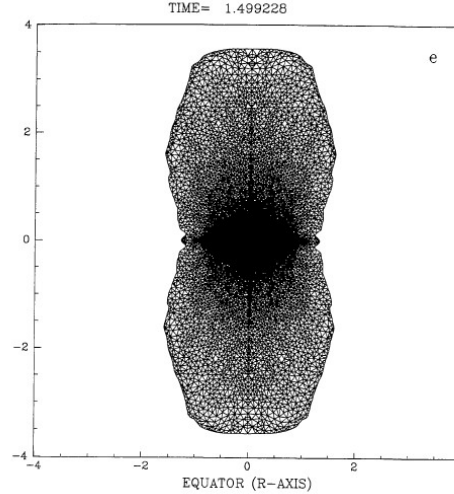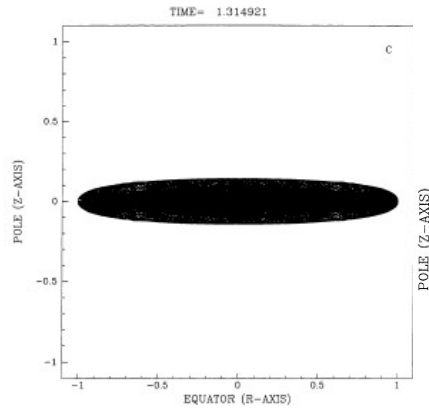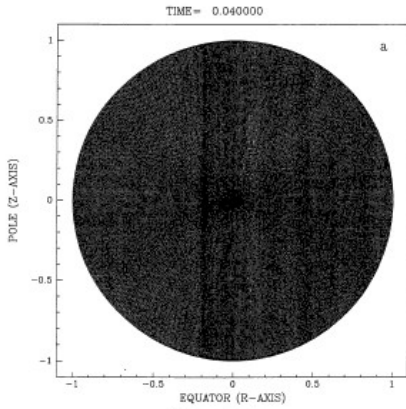# Example of calculation with the triangular grid
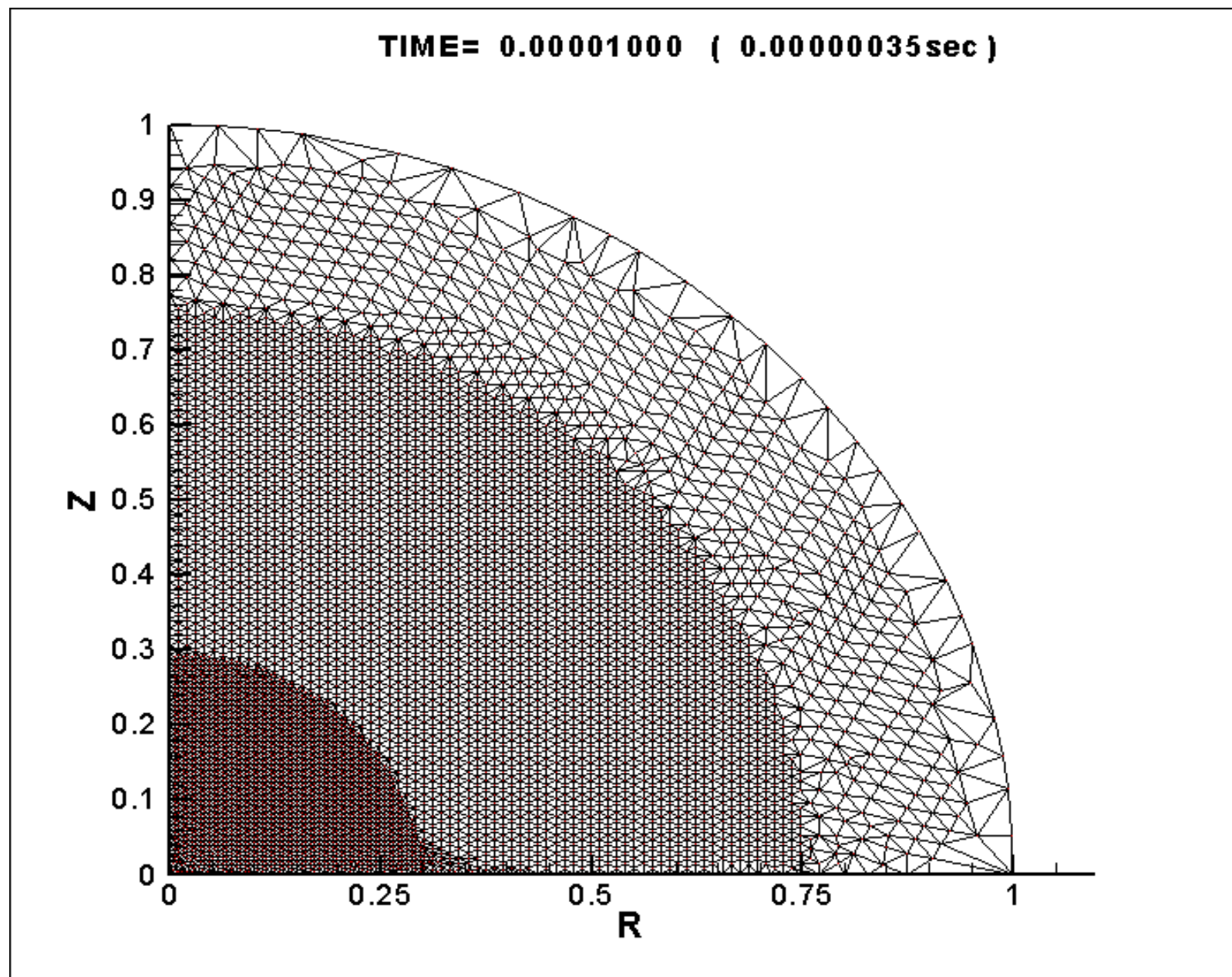
Example of the triangular grid
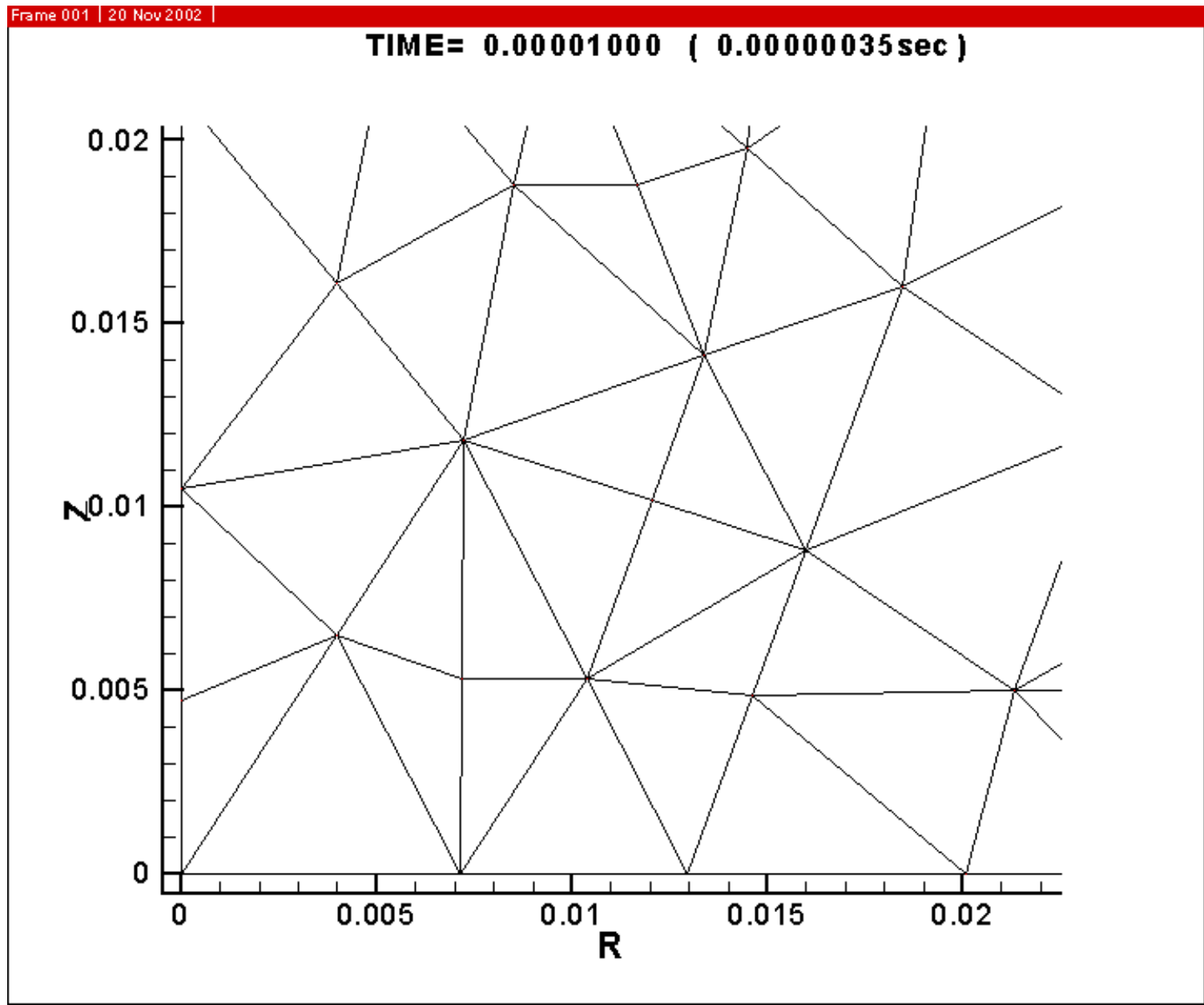
Collapse of rapidly rotating cold protostellar cloud

A&Ass 1996,115, 573
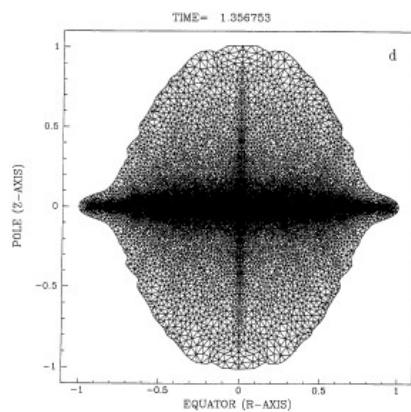
$$\alpha = E_{\text{in}}/|E_{\text{gr}}| = 0.00425, \quad \beta = E_{\text{rot}}/|E_{\text{gr}}| = 0.324,$$

Magnetorotational mechanism for the supernova explosion Bisnovatyi-Kogan (1970)(original article was submitted: September 3, 1969)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion

First 2D calculations: LeBlanck&Wilson (1970))(original article was submitted: September 25, 1969) ->too large initial magnetic fields. $E_{mag0} \sim E_{grav} \Rightarrow$ axial jet

Bisnovatyi-Kogan et al 1976, Meier et al. 1976, Ardeljan et al.1979, Mueller & Hillebrandt 1979, Symbalisty 1984, Ardeljan et al. 2000, Wheeler et al. 2002, 2005, Yamada & Sawai 2004, Kotake et al. 2004, 2005, 2006, Burrows et al.2007, Sawai, Kotake, Yamada 2008…

It is popular topic now!

The realistic values of the magnetic field are: $E_{mag} << E_{grav}$ ($E_{mag}/E_{grav} = 10^{-8} - 10^{-12}$)

Small initial magnetic field -is the main difficulty for the numerical simulations.

The hydrodynamic time scale is much smaller than the magnetic field amplification time scale (*if magnetorotational instability is neglected*).

Explicit difference schemes can not be applied. (CFL restriction on the time-step).

Implicit schemes should be used.

Basic equations: MHD +self-gravitation, infinite conductivity:

$$\begin{cases} \dfrac{dx}{dt} = \mathbf{u}, \dfrac{d\rho}{dt} + \rho \,\mathrm{div}\,\mathbf{u} = 0, \\[2ex] \rho \dfrac{du}{dt} = -\mathrm{grad}\left(p + \dfrac{\mathbf{H} \times \mathbf{H}}{8\pi} \right) + \dfrac{1}{4\pi}\mathrm{div}(\mathbf{H} \otimes \mathbf{H}) - \rho\,\mathrm{grad}\Phi \\[2ex] \rho \dfrac{d\varepsilon}{dt} + p\,\mathrm{div}\,\mathbf{u} + \rho F(\rho,T) = 0,\ p = P(\rho,T), \varepsilon = \mathrm{E}(\rho,T), \\[2ex] \Delta\Phi = 4\pi \mathrm{G}\rho, \\[2ex] \rho \dfrac{d}{dt}\left(\dfrac{\mathbf{H}}{\rho} \right) = \mathbf{H} \times \nabla \mathbf{u}. \end{cases}$$

Additional condition $\mathrm{div}\mathbf{H}=0$

Axis symmetry $\left(\dfrac{\partial}{\partial\phi} = 0 \right)$ and equatorial symmetry (z=0) are supposed.

$$\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + \mathbf{u} \times \nabla, \mathbf{x} = (\mathrm{r}, \varphi, \mathrm{z}), \mathbf{u} - \text{velocity}, \rho - \text{density}, p - \text{pressure},$$

$\mathbf{H}$ − magnetic field, Φ − gravitational potential, ε − internal energy,

G − gravitational constant.

Boundary conditions

Axial symmetry

$$r = 0 : u_r = u_\phi = H_r = H_\phi = \mathrm{rot}_r \mathbf{H} = \mathrm{rot}_\phi \mathbf{H} = 0,$$

Equatorial symmetry

$$u_z = H_z = 0, \qquad \text{Quadrupole field}$$

$z = 0 :$ or

$$u_z = \frac{\partial B_z}{\partial z} = 0, \qquad \text{Dipole field}$$

Outer boundary: $P = \rho = T = H_\phi = 0, \mathbf{H}_{\mathrm{poloidal}} = \mathbf{H}_q$

(from Biot-Savart law)

Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas

.

Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.

Equations of state (approximation of tables)

$$P \equiv P(\rho, T) = P_0(\rho) + \Re T \rho + \frac{T^4 \sigma}{3}$$

$$P_0(\rho) = \begin{cases} P_0^{(1)} = b_1 \rho^{5/3}(1 + c_1 \rho^{1/3}), \ for \rho \leq \rho_1 \\ P_0^{(k)} = a \cdot 10^{b_k (\lg \rho - 8.419)^{c_k}} \ for \rho_{(k-1)} \leq \rho \leq \rho_k, k = \overline{2,6}. \end{cases} \quad \varepsilon_0(\rho) = \int_0^\rho \frac{P_0(x)}{x^2} dx.$$

$$\varepsilon = \varepsilon(\rho, T) = \varepsilon_0(\rho) + \frac{3}{2} \Re T + \frac{\sigma T^4}{\rho} + \varepsilon_{Fe}(\rho, T), \ \varepsilon_{Fe}(\rho, T) = \frac{E_{b,Fe}}{A_{m_p}} \left(\frac{T - T_{0,Fe}}{T_{1,Fe} - T_{0,Fe}} \right), \quad \textbf{Fe –dissociation}$$

Neutrino losses: **URCA processes, pair annihilation, photo production of neutrino, plasma neutrino**

URCA: $f(\rho, \overline{T}) = 1.3 \cdot 10^9 \chi(\overline{T}) / [1 + (7.1 \cdot 10^{-5} \rho / \overline{T}^3)^{2/5}] erg \cdot g^{-1} \cdot c^{-1}$

$$\lambda(T) = \begin{cases} 1, \overline{T} < 7, \\ 664.31 + 51.024(\overline{T} - 20), 7 \leq \overline{T} \leq 20, \\ 664.31, \overline{T} > 20, \end{cases} \qquad \overline{T} = T \cdot 10^{-9.}$$

Approximation of tables from Ivanova, Imshennik, Nadyozhin,1969

$$F(\rho, T) = f(\rho, T) e^{-\tau_v} \qquad \text{neutrino diffusion}$$

Initial state

$M = 1.2042 \times M_{sun}$, spherically symmetrical stationary state, initial angular velocity 2.519 (1/sec)

Initial temperature distribution $\quad T = \delta\rho^{2/3}$
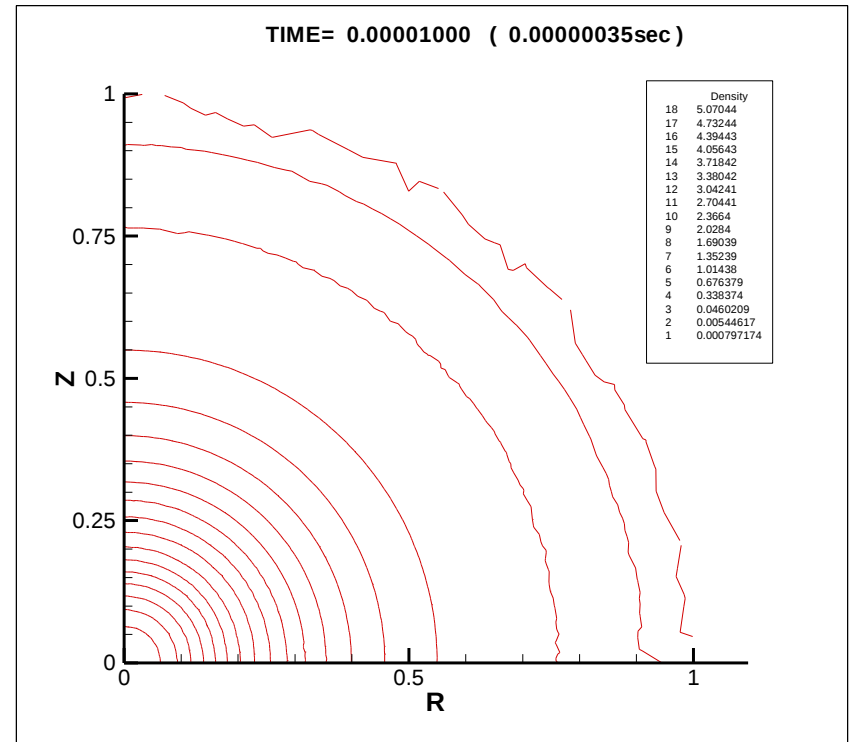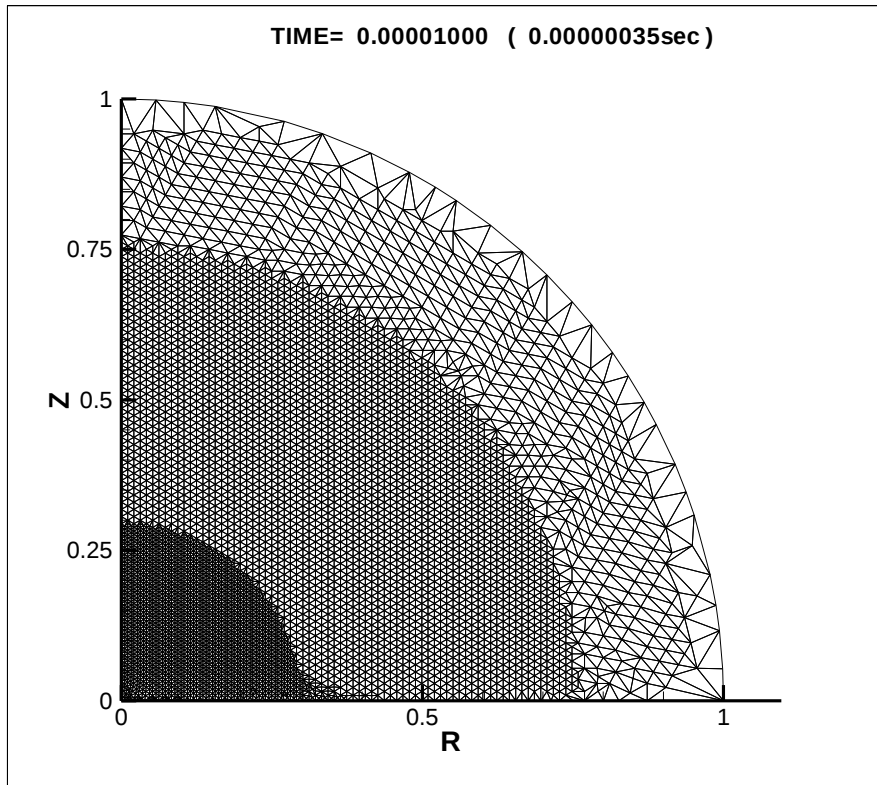
$$\frac{E^{rot}}{E^{grav}} = 0.571\% \quad \frac{E^{int}}{E^{grav}} = 72.7\%$$

Maximal compression state
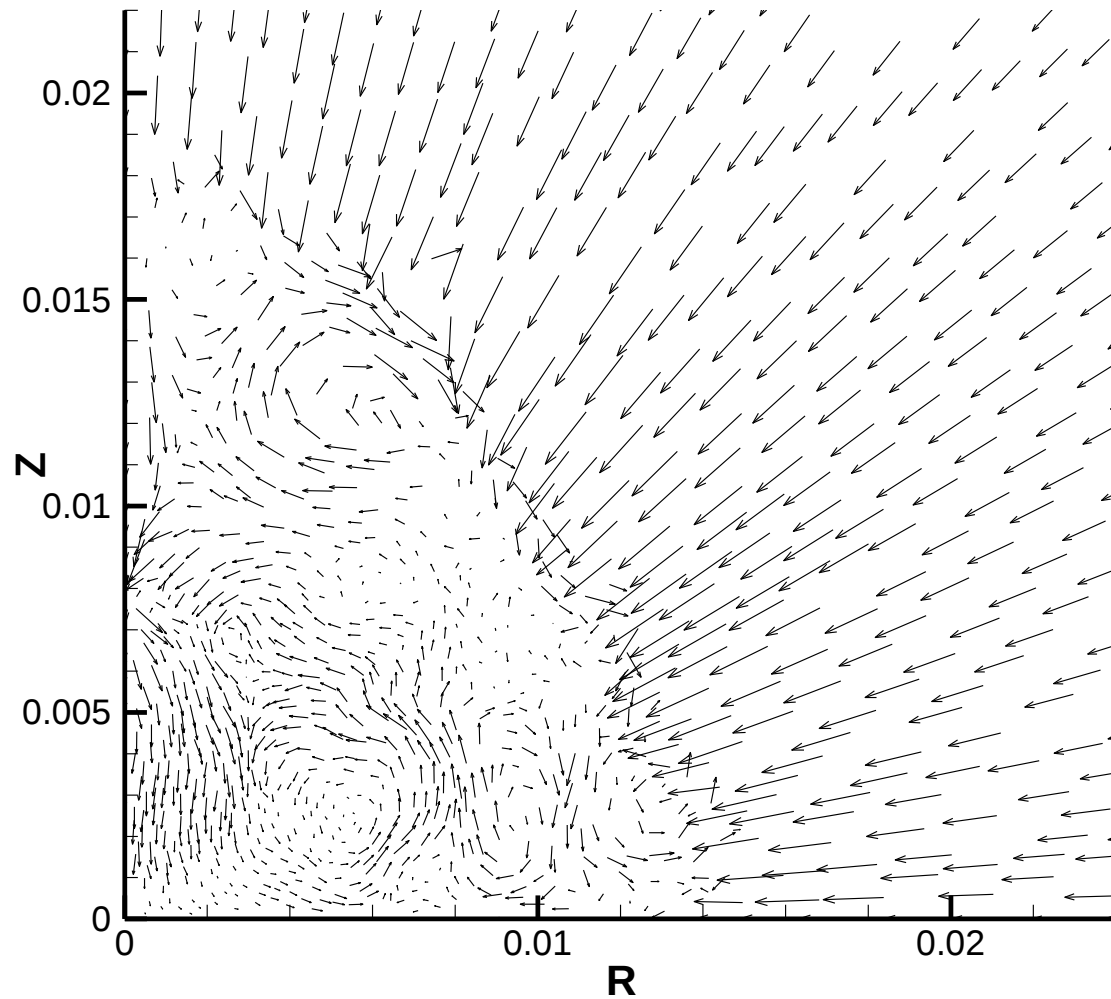
Max. density = $2.5 \cdot 10^{14} \text{g/cm}^3$

Neutron star formation in the center and formation of the shock wave

TIME= 4.12450792 (0.14246372sec)

«0.01»~10km

Mixing

TIME= 5.29132543 (0.18276651sec)
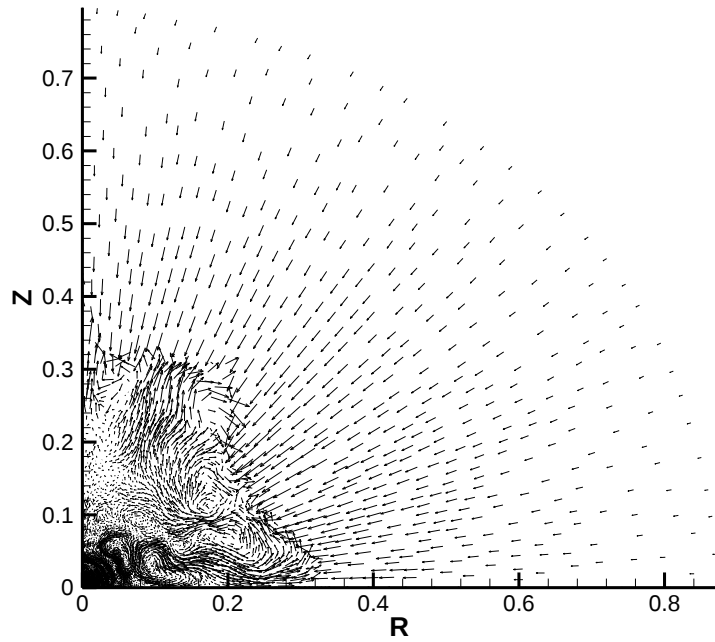
TIME= 5.29132543 (0.18276651sec)
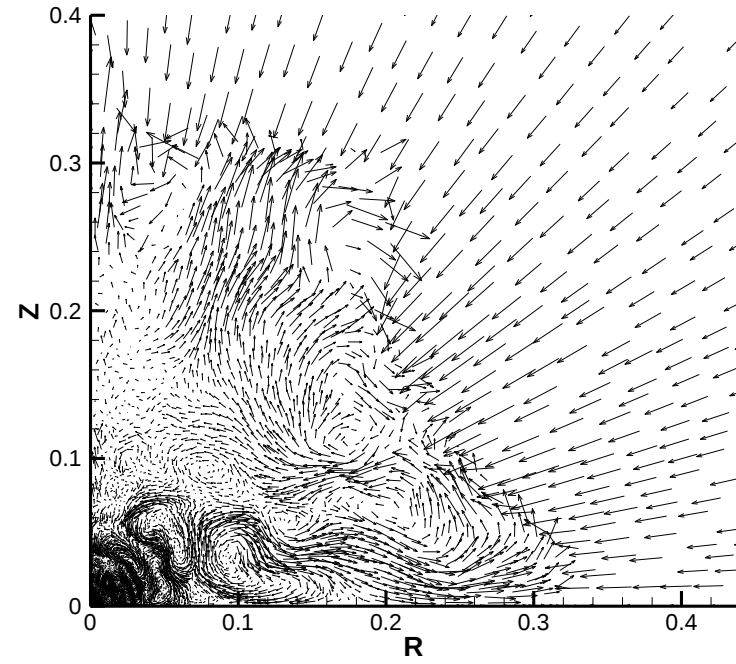
Bounce shock wave does not produce SN explosion :(

Angular velocity (central part of the computational domain). Rotation is differential.

TIME= 4.15163360 (0.14340067sec)
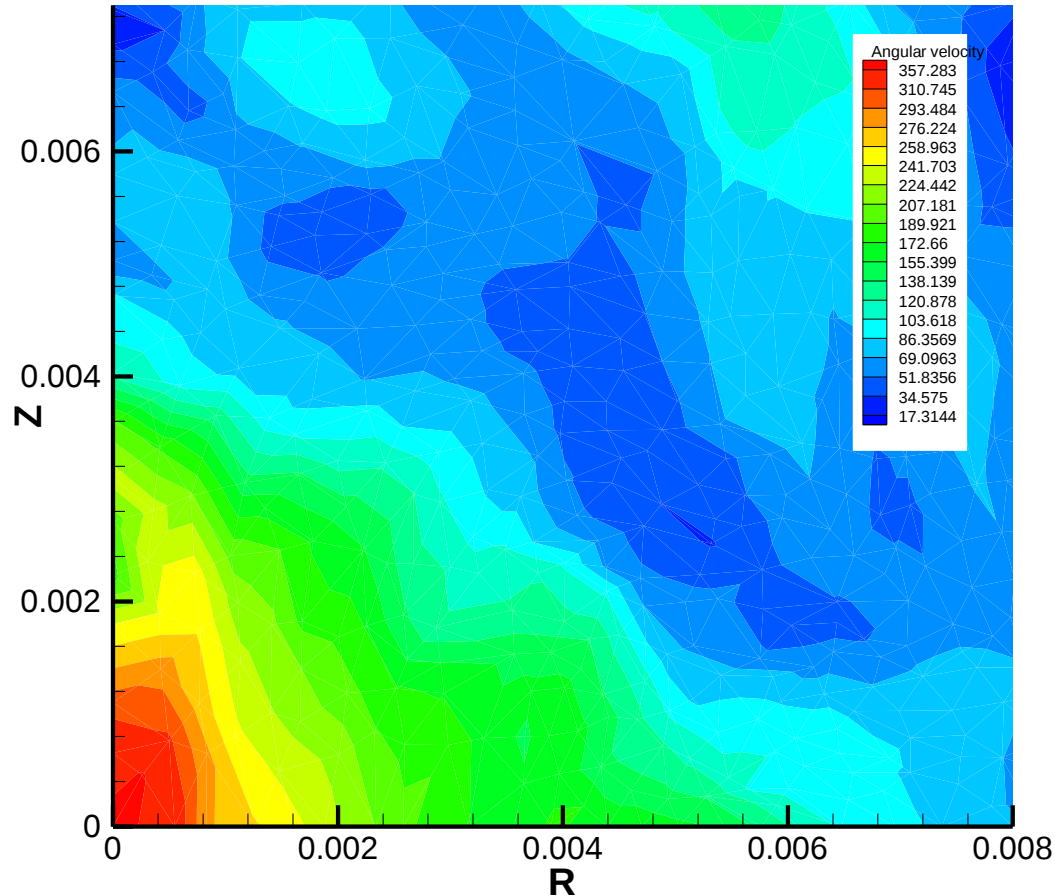
Distribution of the angular velocity

The period of rotation of the young neutron star is about 0.001- 0.003 sec

Rapidly rotating

pre-SN

Crab pulsar P=33ms –

rapid rotation at birth

Core collapse

in binaries

Initial magnetic field

Z

Outer boundary
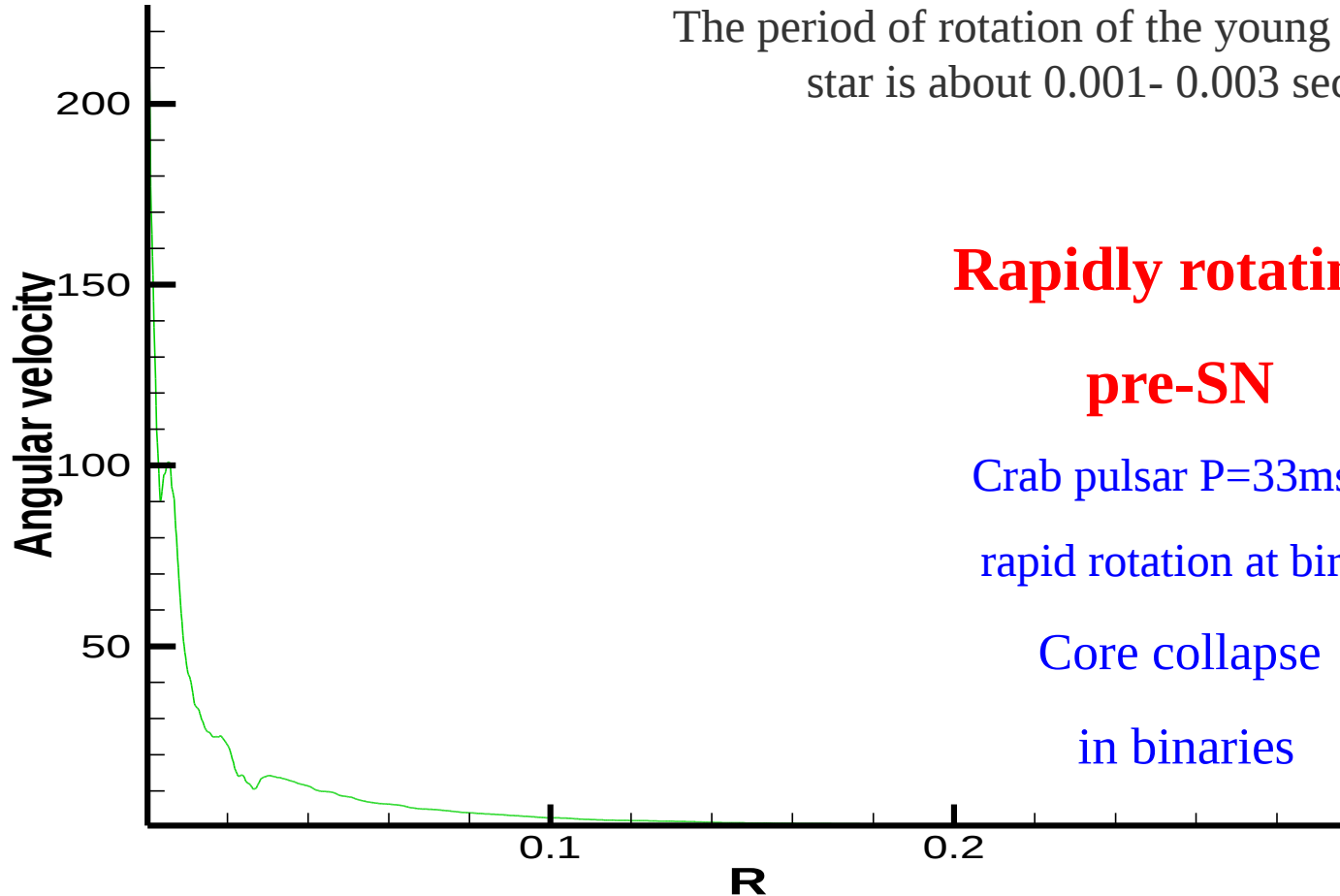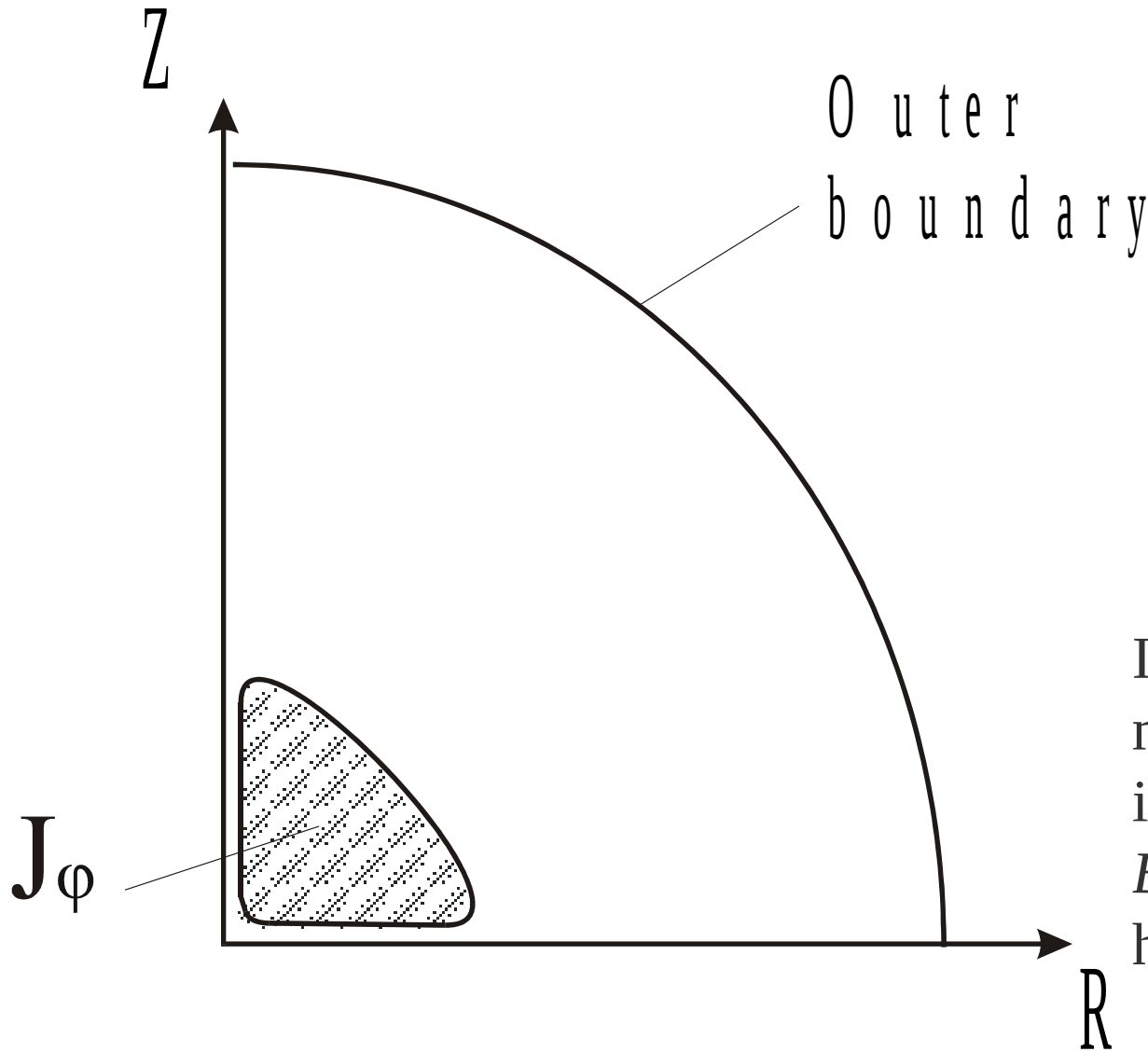
$$\mathbf{H} = \frac{1}{c} \int \frac{\mathbf{J} \times \mathbf{R}}{R^3} d\mathrm{v},$$

Biot-Savart law

$$J_\phi -> (H_r, H_z)$$

J_φ

Initial poloidal magnetic field is divergence free, *BUT* not balanced. We have to balance it.

R

Initial magnetic field –quadrupole-like symmetry

Ardeljan, Bisnovatyi-Kogan, SM, MNRAS 2005, 359, 333
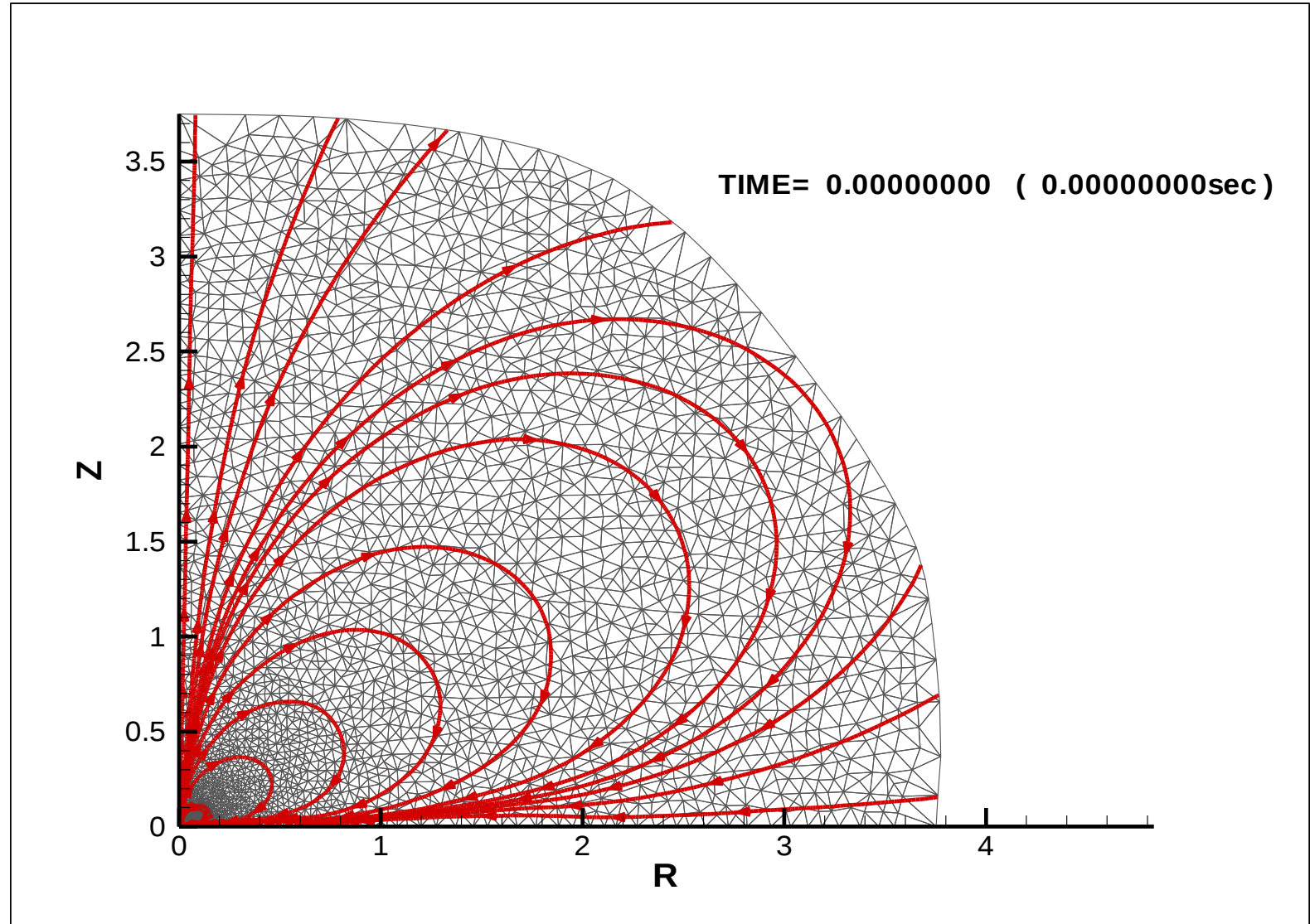
Toroidal magnetic field amplification.

pink – maximum_1 of Hf^2   blue – maximum_2 of Hf^2
Maximal values of Hf=$2.5 \cdot 10(16)$G

TIME= 0.00000779 (0.00000027sec)
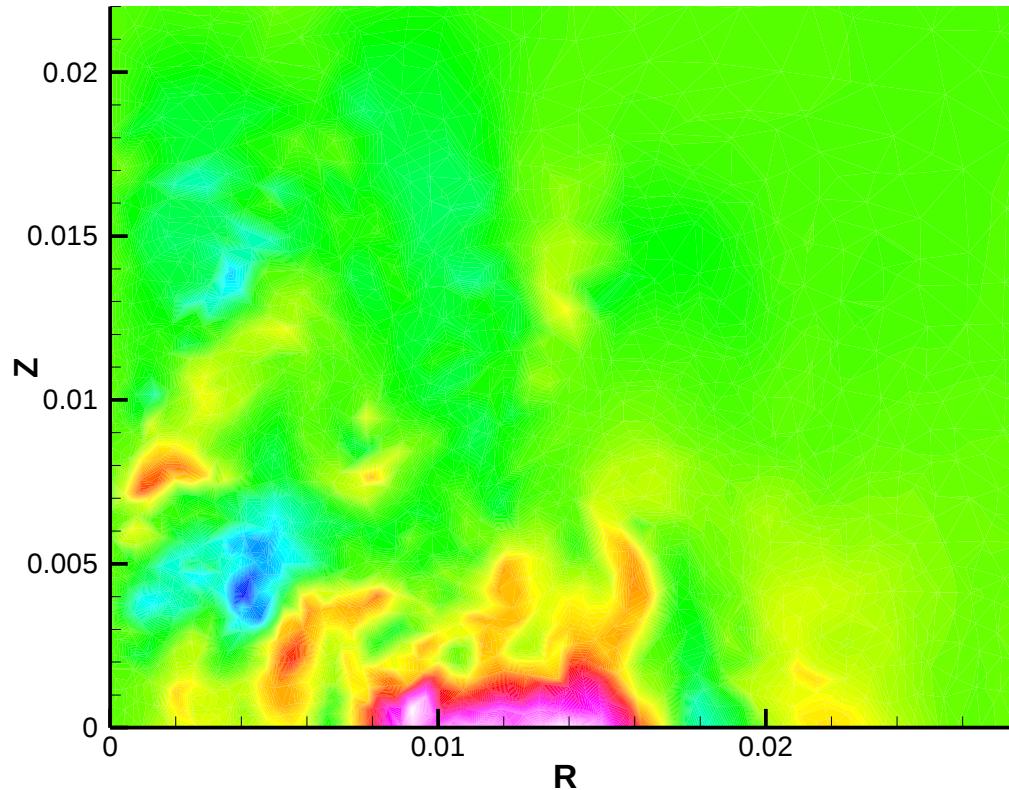
After SN explosion at the border of neutron star H=$2 \cdot 10^{14}$G

Temperature and velocity field

TIME= 0.00000779 (0.00000027sec)
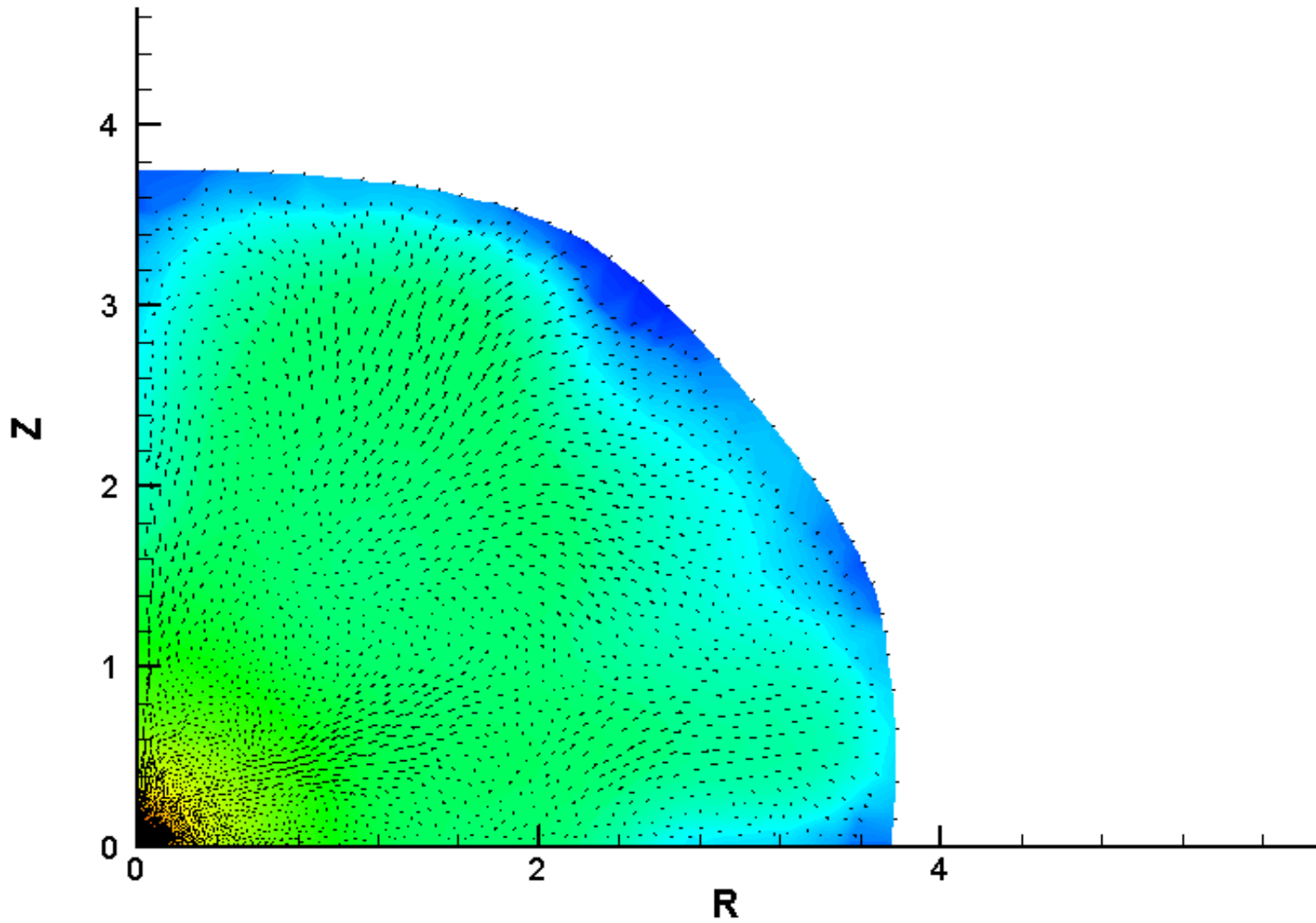
Specific angular momentum rV_{φ}

TIME= 0.00000779 (0.00000027 sec)
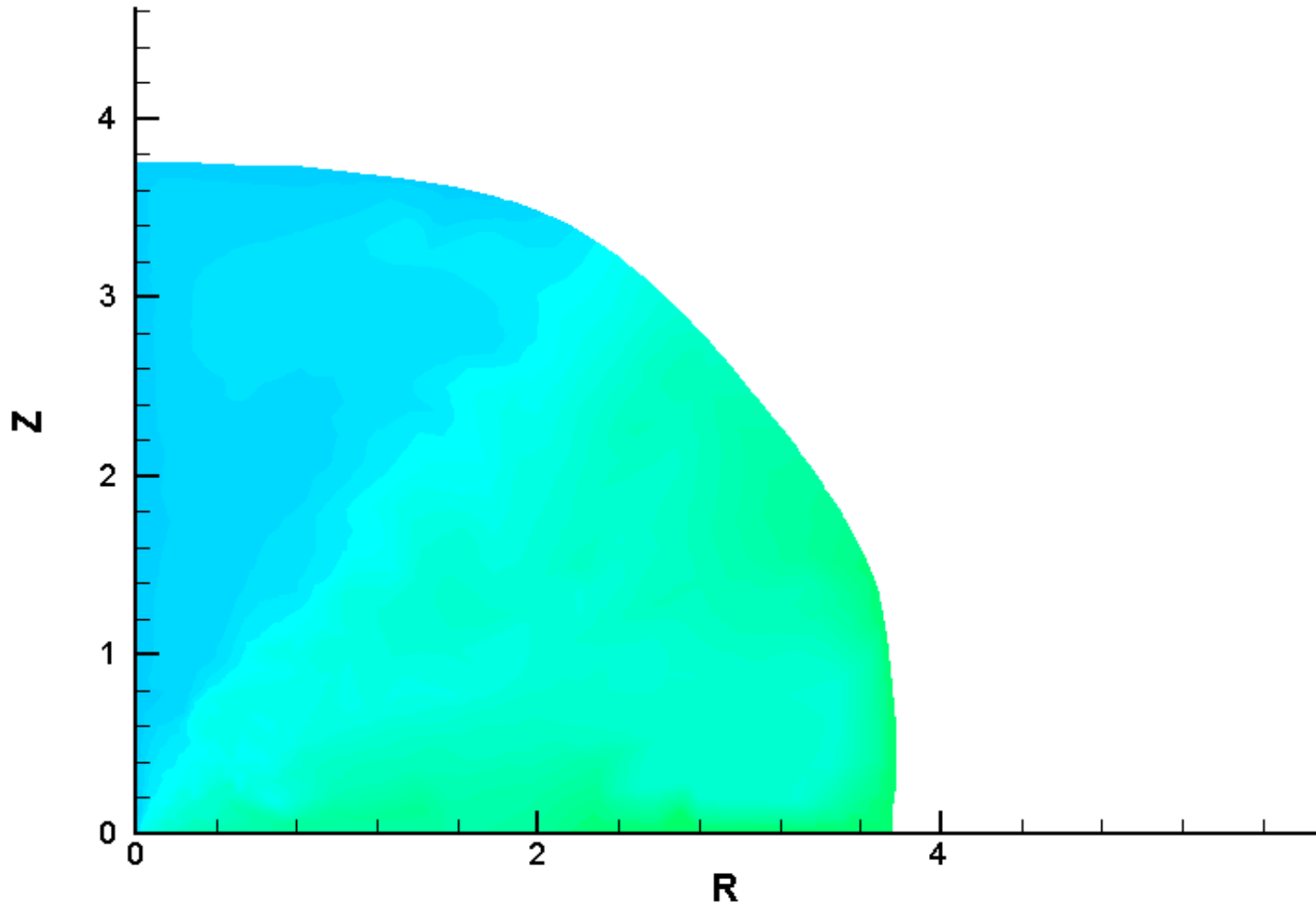
Time evolution of different types of energies
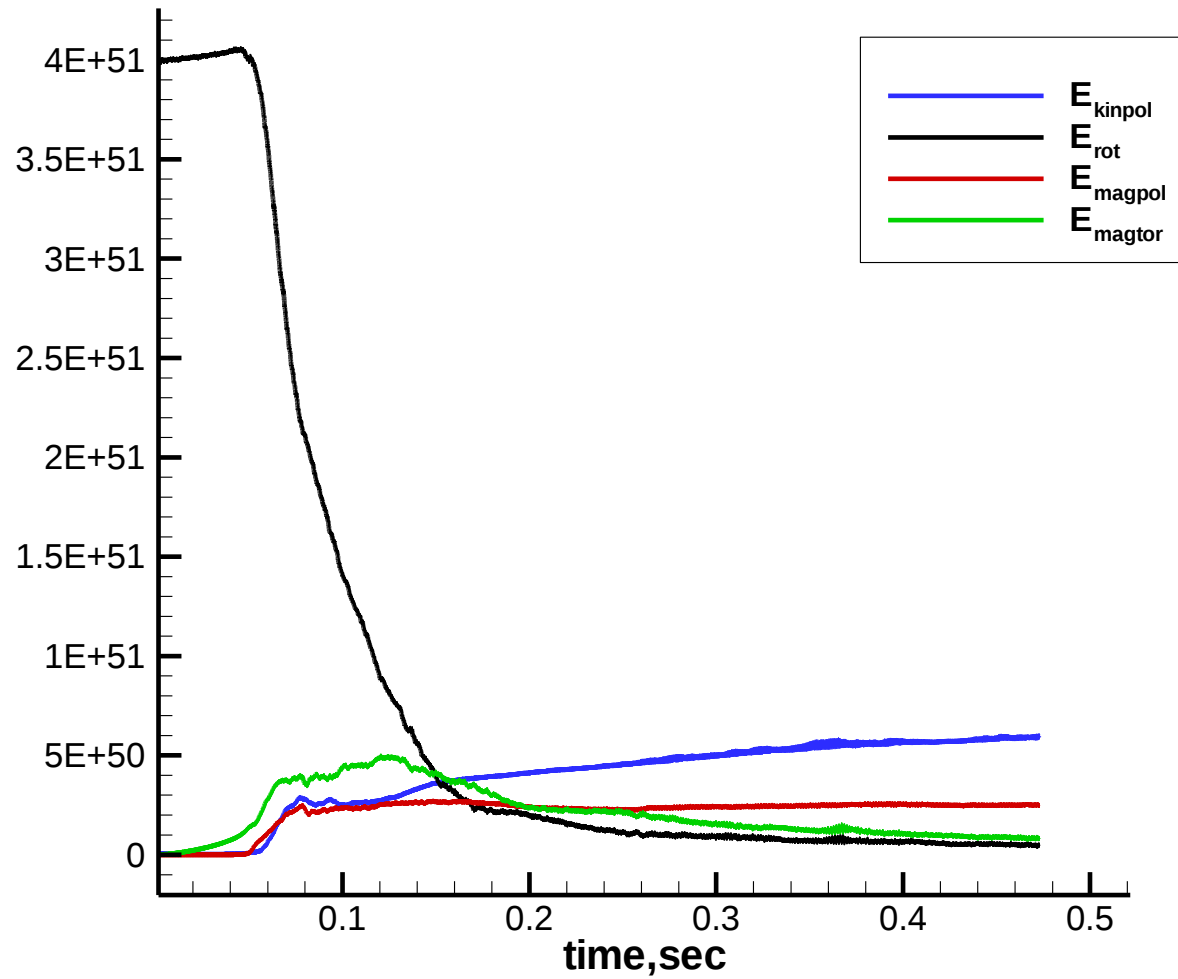
Time evolution of the energies

Gravitational energy
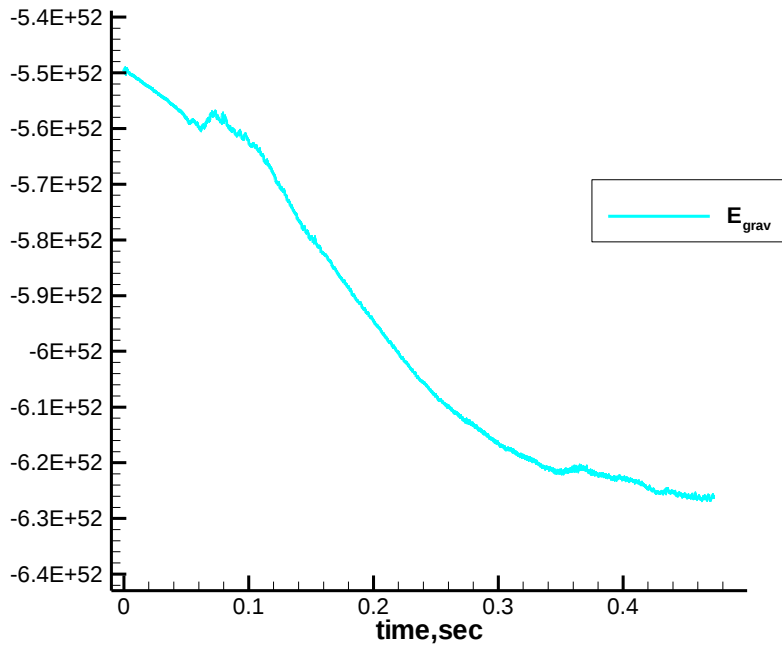
Internal energy
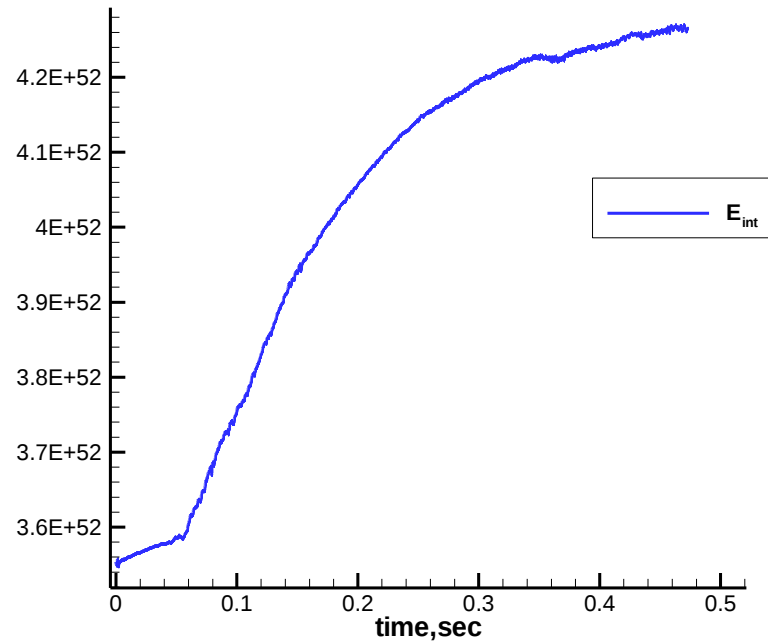
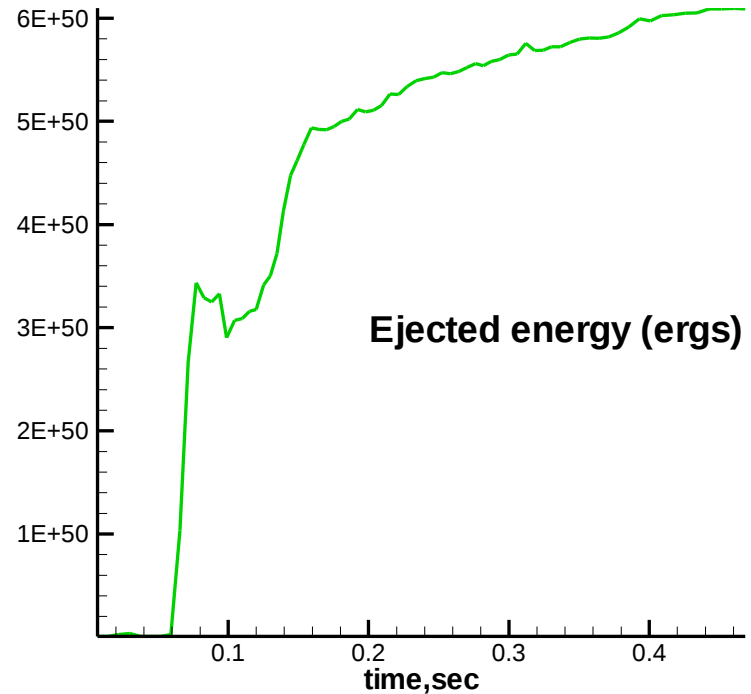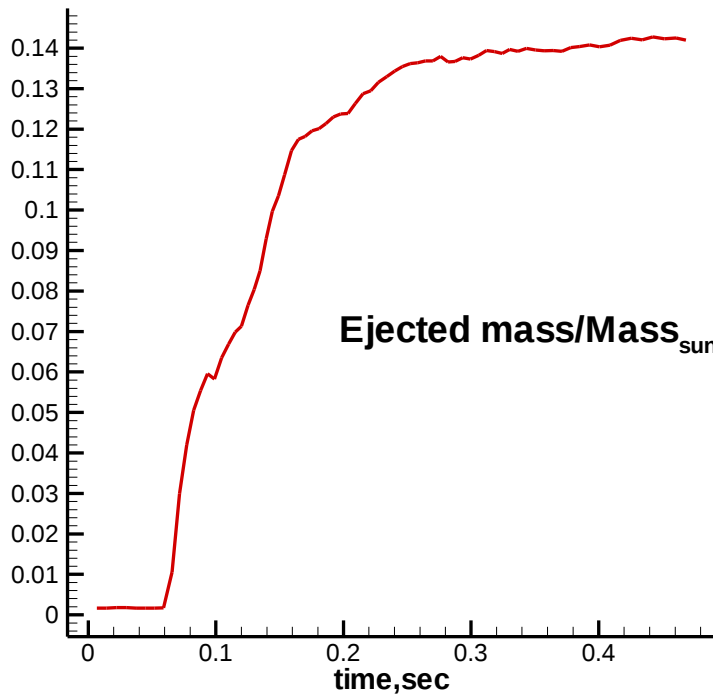# Time evolution of the energies

Neutrino losses (ergs)
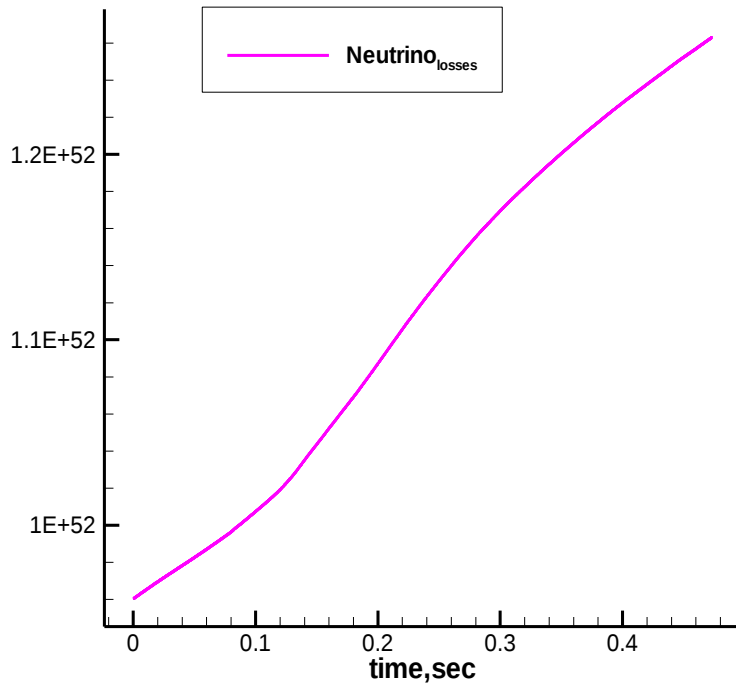
Neutrino luminosity (ergs/sec)
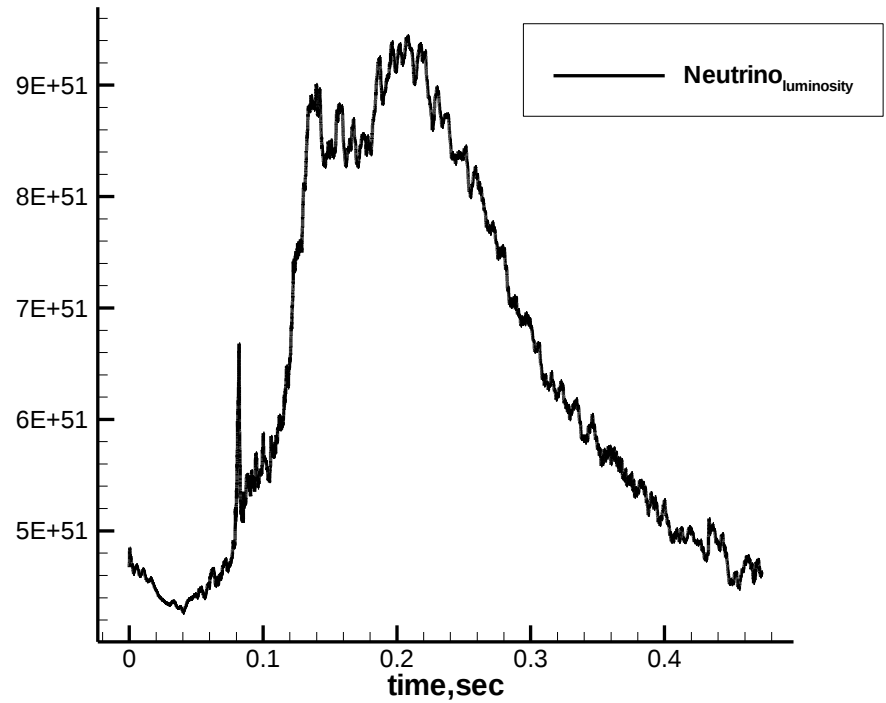
Ejected energy and mass

Ejected energy $\quad 0.6 \times 10^{51} erg \quad$ Ejected mass 0.14M$_\bullet$

Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy

Ejected mass/Mass$_{sun}$

Ejected energy (ergs)

Magnetorotational supernova in 1D

Bisnovaty-Kogan et al. 1976, Ardeljan et al. 1979

$$t_{\text{explosion}} : \frac{1}{\sqrt{\alpha}}, \qquad \left(\alpha = \frac{E_{\text{mag0}}}{E_{\text{grav0}}} \right)$$

Example: $\alpha = 10^{-2} \Rightarrow t_{\text{explosion}} = 10,$

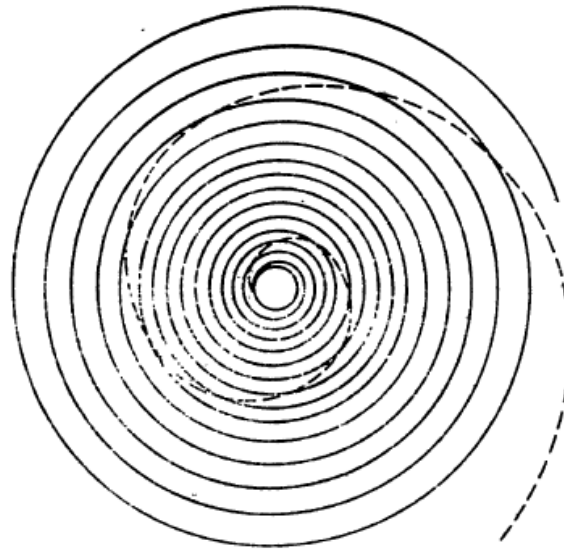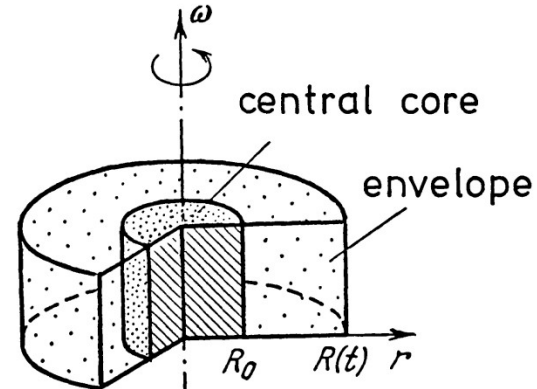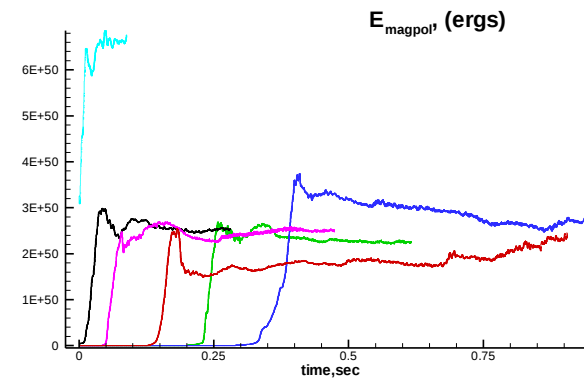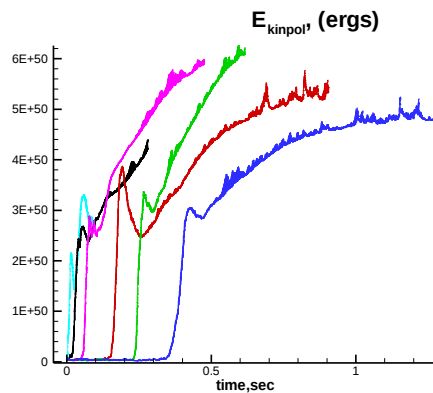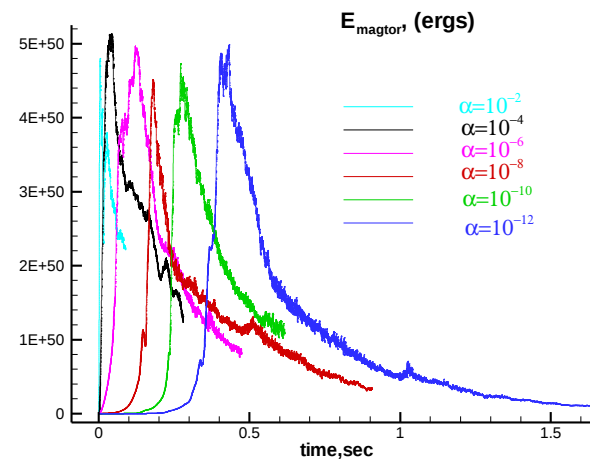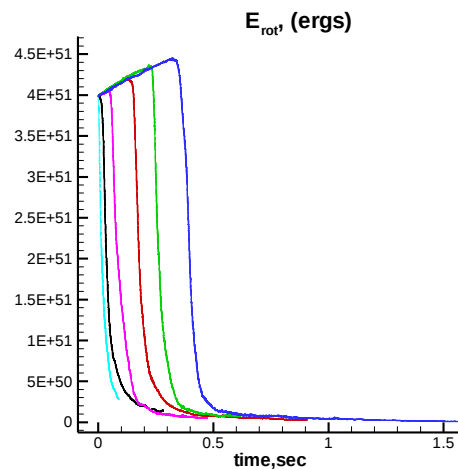$\alpha = 10^{-12} \Rightarrow t_{\text{explosion}} = 10^6$!!!

FIG. 3. Shape of a field line in the region near the core at the time $t_\alpha = 7$ for $\alpha = 10^{-2}$ (dashed line) and $\alpha = 10^{-4}$ (solid line).

Magnetorotational explosion for the different $\alpha = \dfrac{E_{mag0}}{E_{grav0}} = 10^{-2} - 10^{-12}$

Magnetorotational instability⬜ mag. field grows exponentially
(Dungey 1958,Velikhov 1959, Chandrasekhar, Balbus & Hawley 1991,
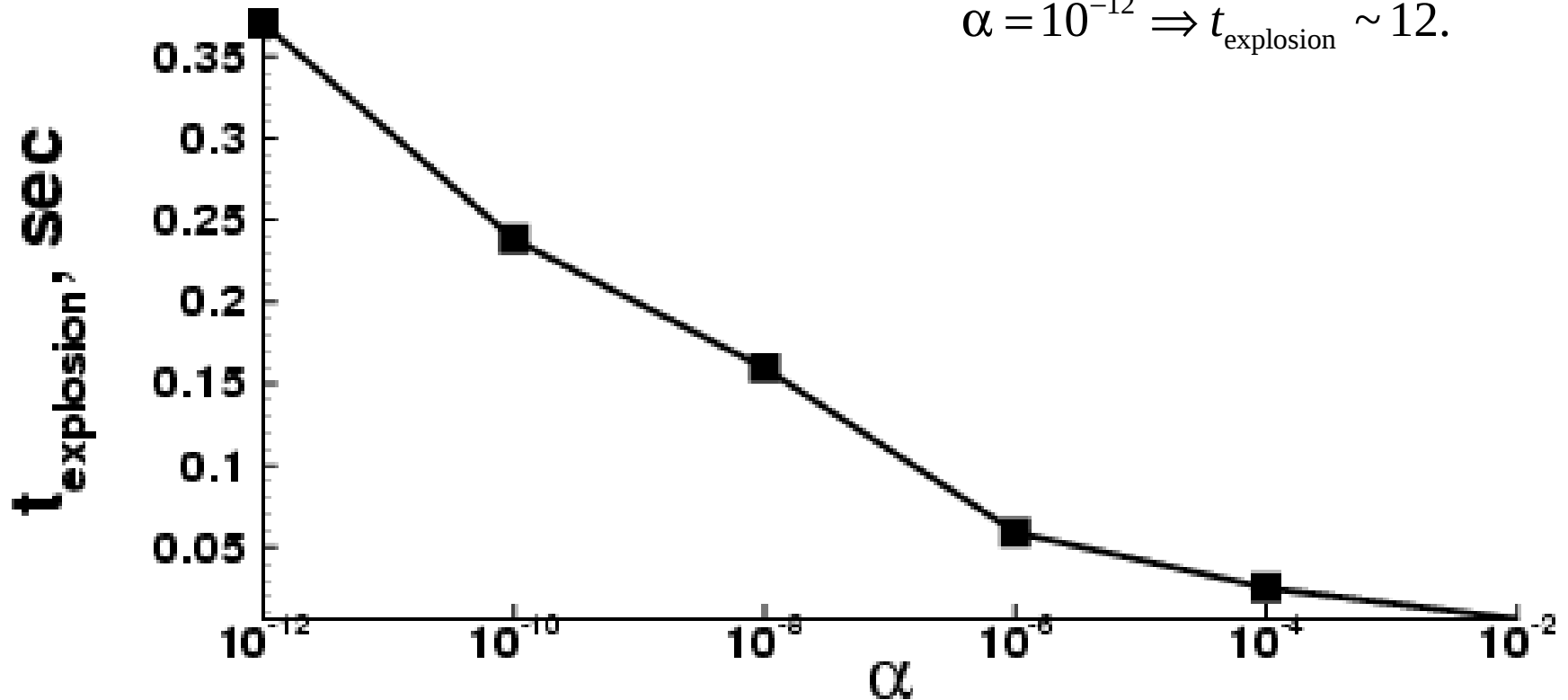Spruit 2002, Akiyama et al. 2003…)

Dependence of the explosion time from $\alpha = \dfrac{E_{mag0}}{E_{grav0}}$

$t_{explosion} \sim -\log(\alpha)$ (for small α)

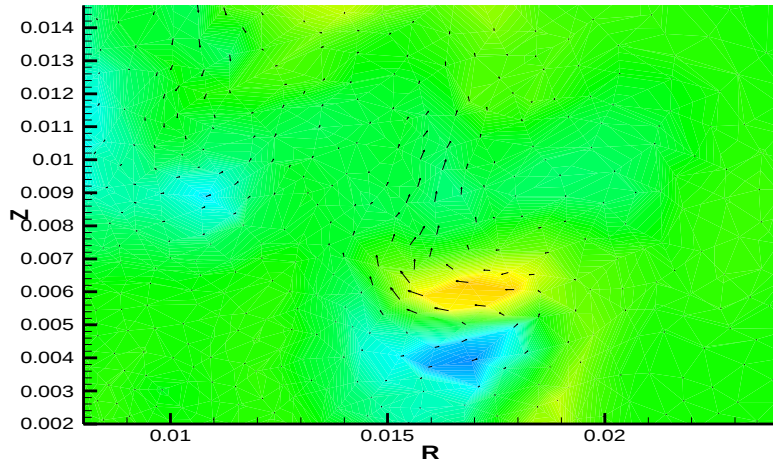$$\alpha = 10^{-6} \Rightarrow t_{explosion} \sim 6,$$

Example:

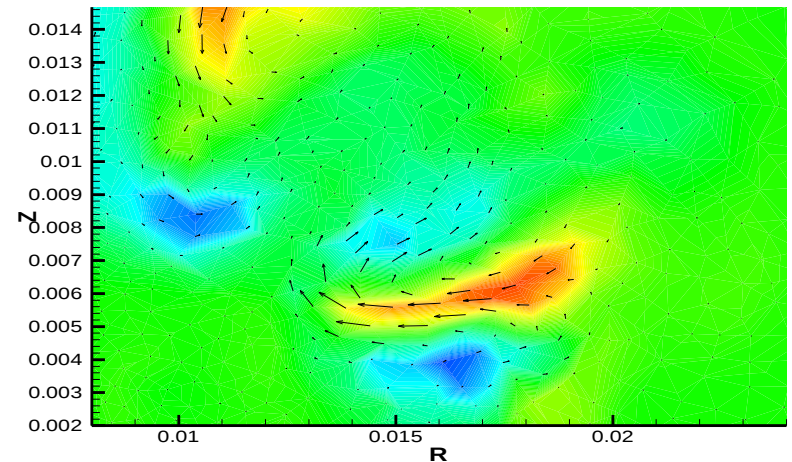$$\alpha = 10^{-12} \Rightarrow t_{explosion} \sim 12.$$

Magnetorotational instability

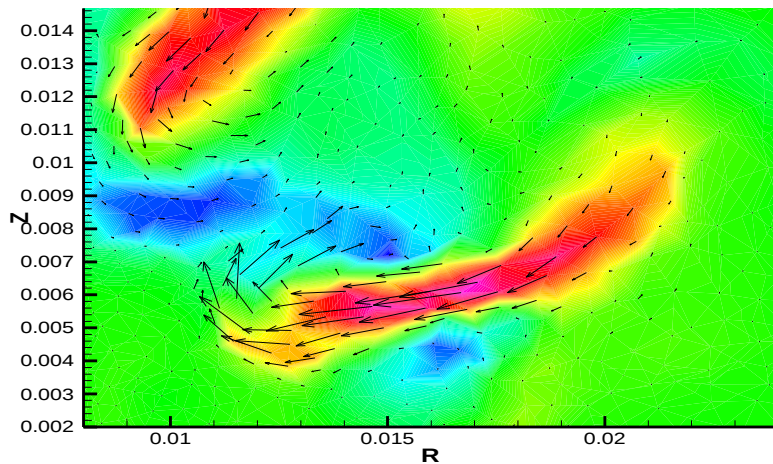**Central part of the computational domain . Formation of the MRI.**
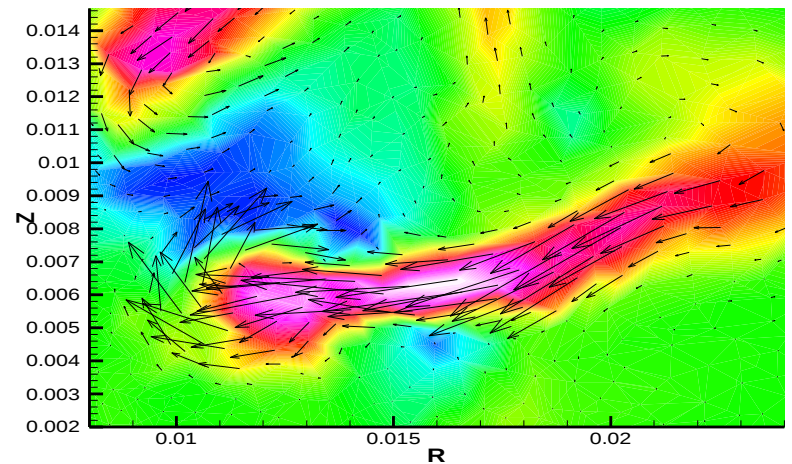

TIME= 34.83616590 (1.20326837sec)

TIME= 35.08302173 (1.21179496sec)

TIME= 35.26651529 (1.21813298sec)

TIME= 35.38772425 (1.22231963sec)

Toy model for MRI in the magnetorotational supernova

$$\frac{dH_\varphi}{dt} = H_r \left(r \frac{d\Omega}{dr} \right);$$ at the initial stage of the process $H_\varphi < H_\varphi^*$: $H_r \left(r \frac{d\Omega}{dr} \right) \approx \text{const},$

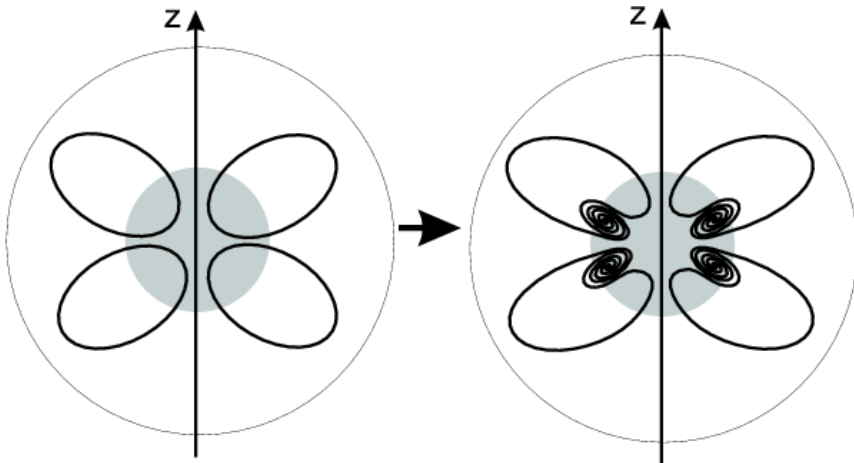beginning of the MRI => formation of multiple *poloidal* differentially rotating

vortexes $\frac{dH_r}{dt} = H_{r0} \left(\frac{d\omega_v}{dl} l \right),$ in general we may approximate: $\left(\frac{d\omega_v}{dl} l \right) \approx \alpha(H_\varphi - H_\varphi^*).$

Assuming for the simplicity that $\left(r \frac{d\Omega}{dr} \right) = A$ is a constant during the first stages of MRI, and taking H_φ^* as a constant we come to the following equation:

$$\frac{d^2}{dt^2} \left(H_\varphi - H_\varphi^* \right) = A H_{r0} \alpha (H_\varphi - H_\varphi^*)$$
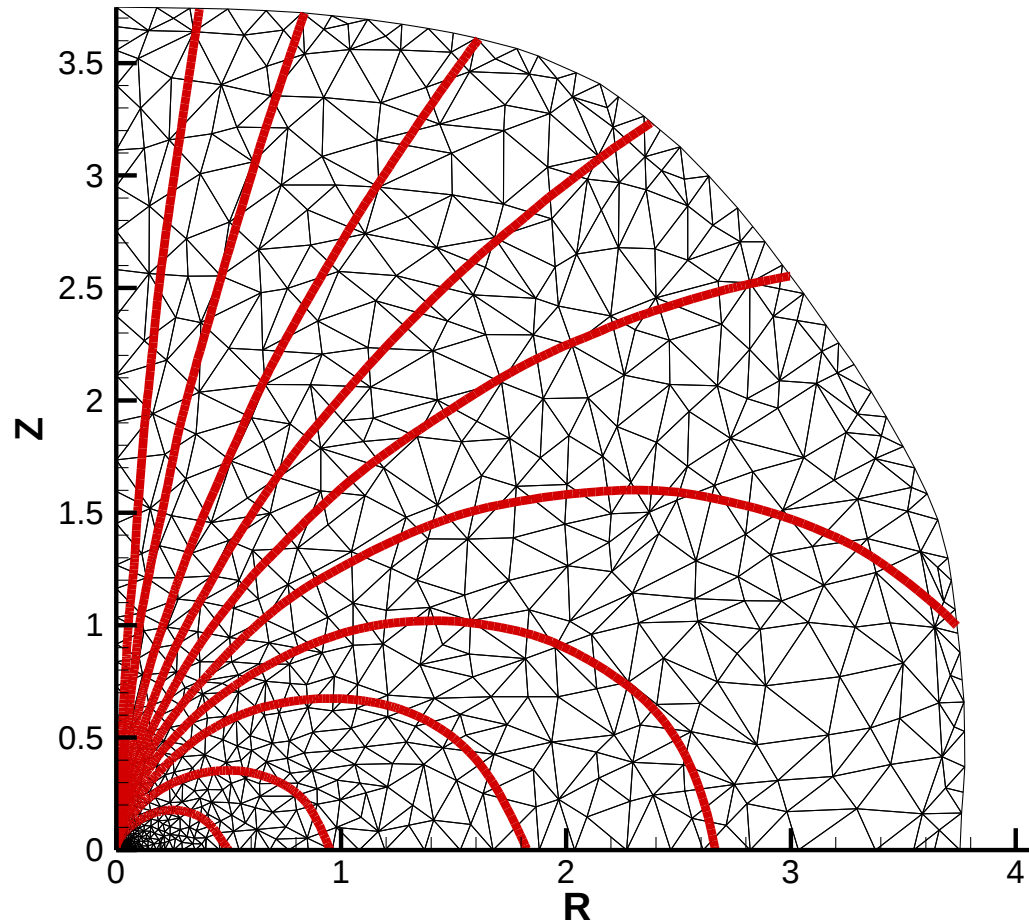
$$\Downarrow$$

$$\begin{cases} H_\varphi = H_\varphi^* + H_{r0} e^{\sqrt{A\alpha H_{r0}}\,(t-t^*)}, \\ \\ H_r = H_{r0} + \dfrac{H_{r0}^{3/2}\alpha^{1/2}}{\sqrt{A}} \left(e^{\sqrt{A\alpha H_{r0}}\,(t-t^*)} - 1 \right). \end{cases}$$

Initial magnetic field – dipole-like symmetry

Magnetorotational explosion for the dipole-like magnetic field

TIME= 31.60366286 (1.09161519sec)
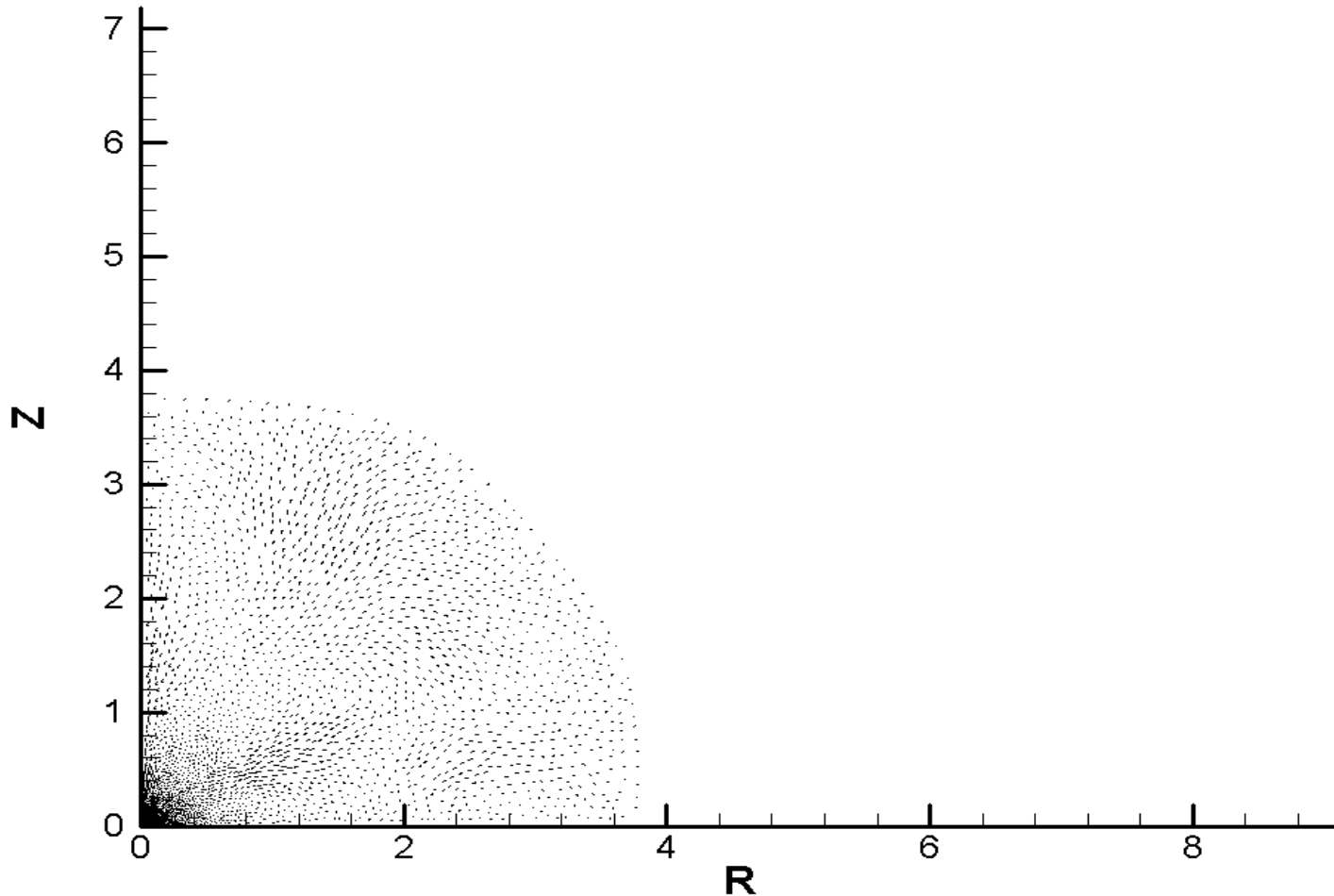
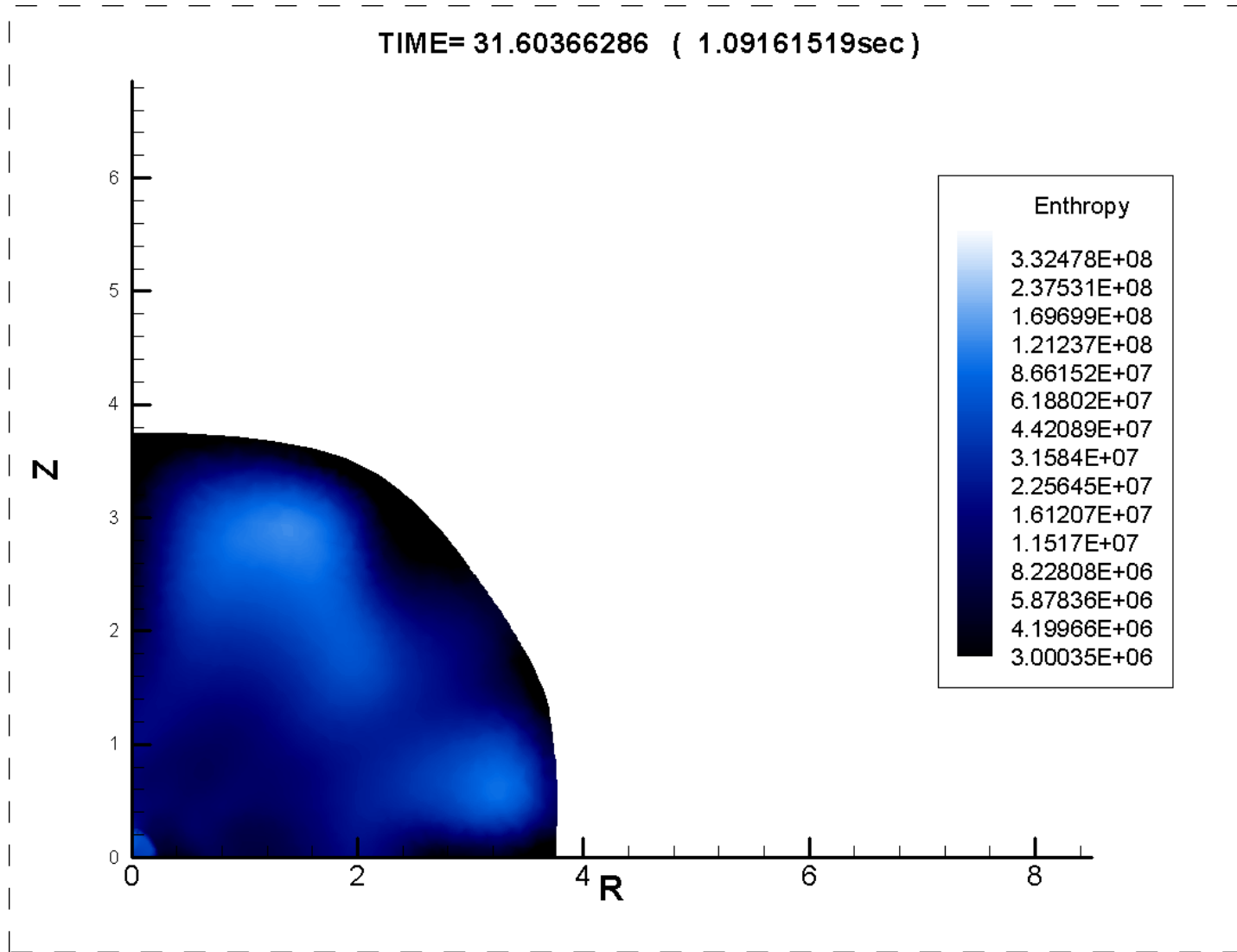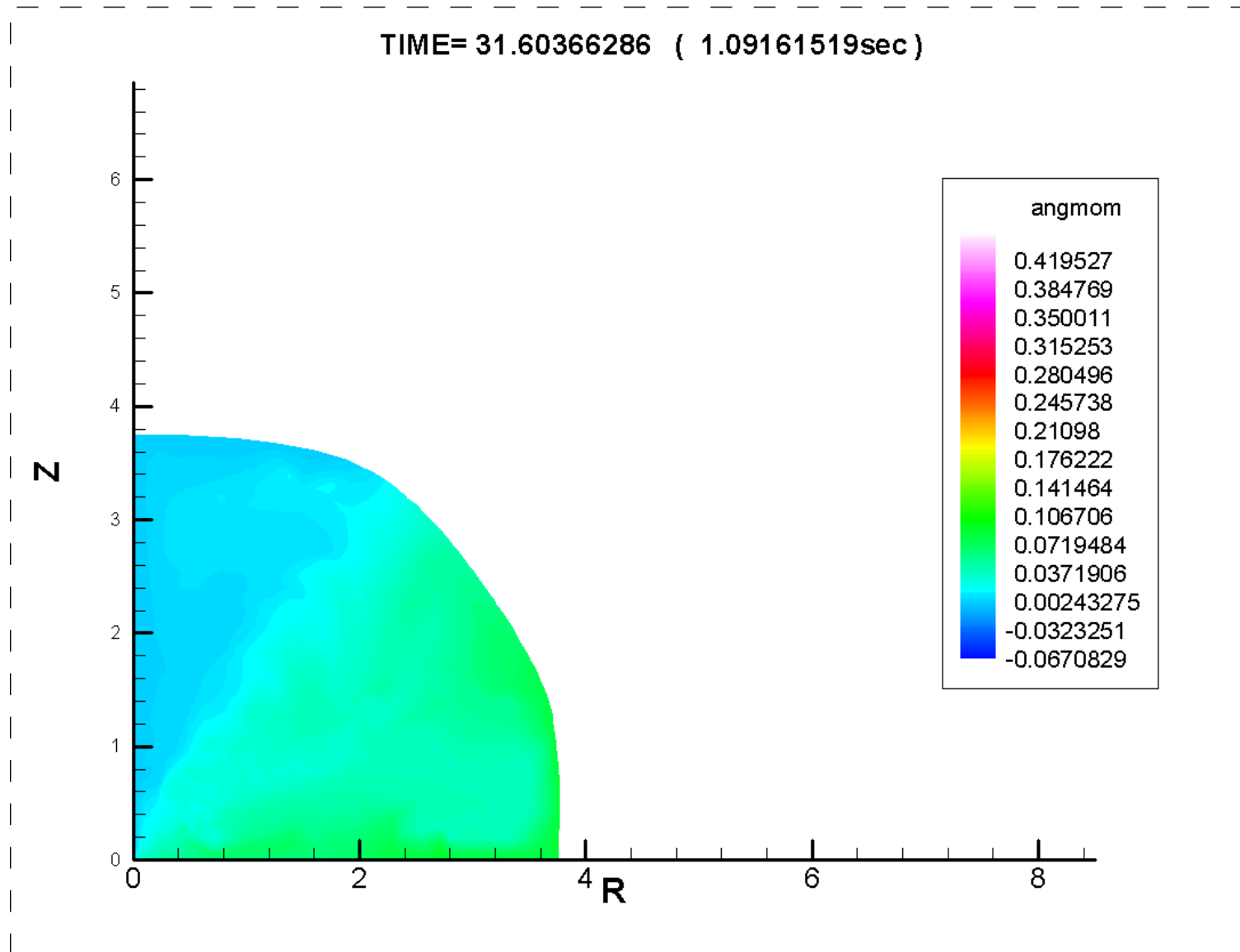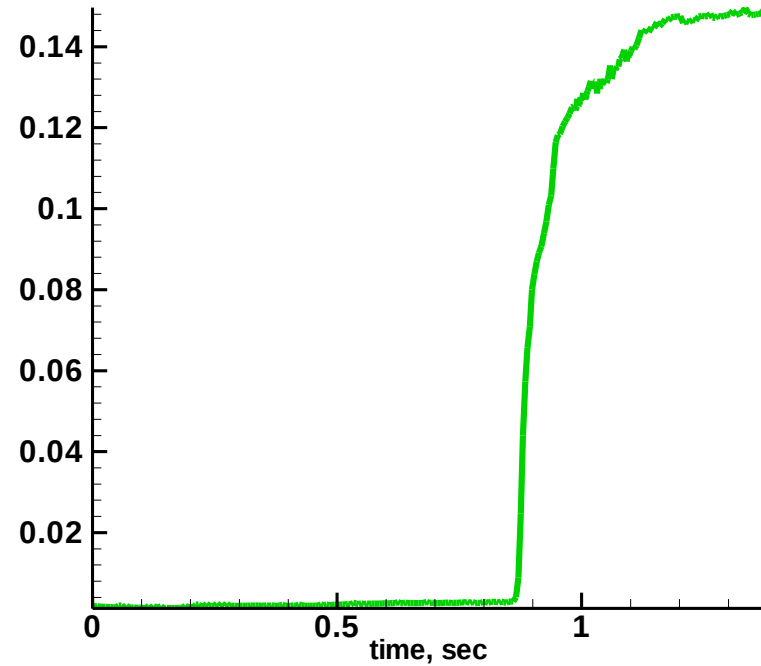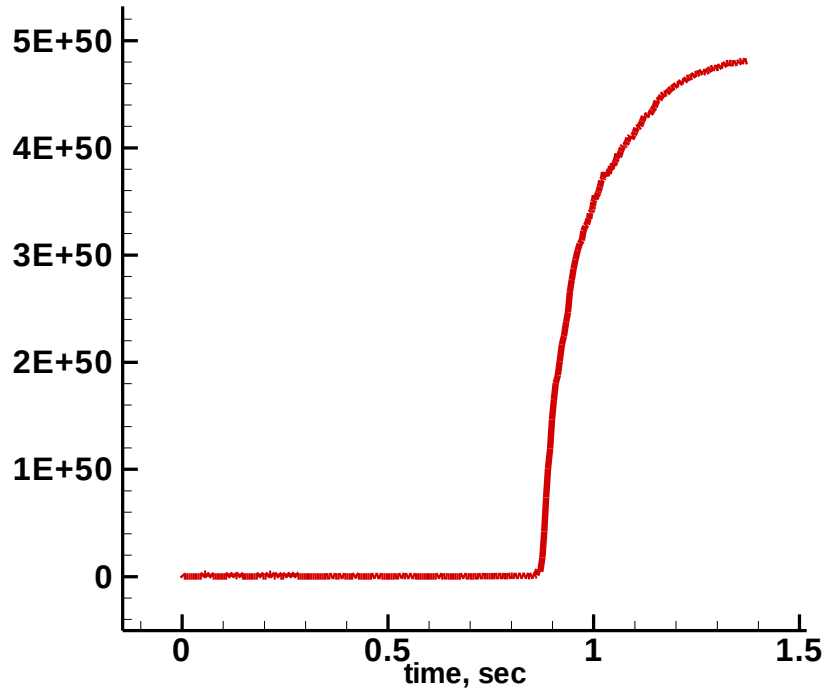# Magnetorotational explosion for the dipole-like magnetic field

Magnetorotational explosion for the dipole-like magnetic field

Ejected energy and mass (dipole)

Ejected energy $\approx 0.5 \times 10^{51} erg$ Ejected mass $\approx 0.14 M_\bullet$

**Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy**

Characteristic time of the magnetic field reconnection (rough estimation)

Petcheck mechanism – characteristic reconnection time

$$\tau_{reconn} = \frac{4(\ln(\mathrm{Re_m}) + 0.74)}{\pi v_A l^{-1}}$$

Our estimations show:
conductivity ~ $8 \; 10^{20} c^{-1}$
Magnetic Reynolds number ~ 10^{15}

Characteristic time of the magnetic field reconnection
For the magnetorotational supernova is:

$$\tau_{reconn} \approx 5 \; c$$

(approximately 10 times larger than
characteristic time of magnetorotational
supernova explosion). .

Reconnection of the magnetic field does not influence
significantly on the supernova explosion.

MR supernova – different core masses

Bisnovatyi-Kogan, SM, 2008 (in preparation)

Dependence of the MR supernova explosion energy on the core mass and initial angular
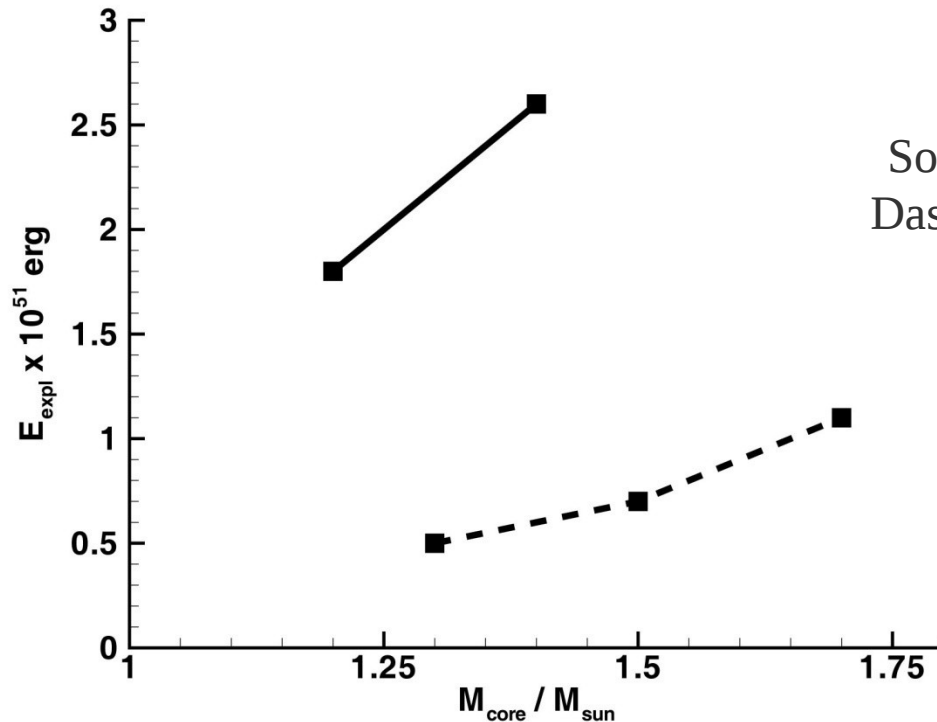
Solid line – initial angular velocity = 3.53s^{-1}
Dashed line - initial angular velocity = 2.52s^{-1}

The main difference of bounce shock and neutrino driven mechanisms from magnetorotational supernovae ⟹ . magnetic field works like a piston. This MHD piston supports the supernova MHD shock wave for some time.

3D features of MR supernova simulations

•Lagrangian grid (tetrahedrons) method requires frequent grid reconstruction near protoneutron star surface (due to differential rotation) => violation of the solution.
•Unstructured grid (Dirichlet sells) – construction of 3D grid of Diriclet cells is expensive.
•SPH (Smooth Particle Hydrodynamics) – poor spatial accuracy, concentration of particles near gravitational center.
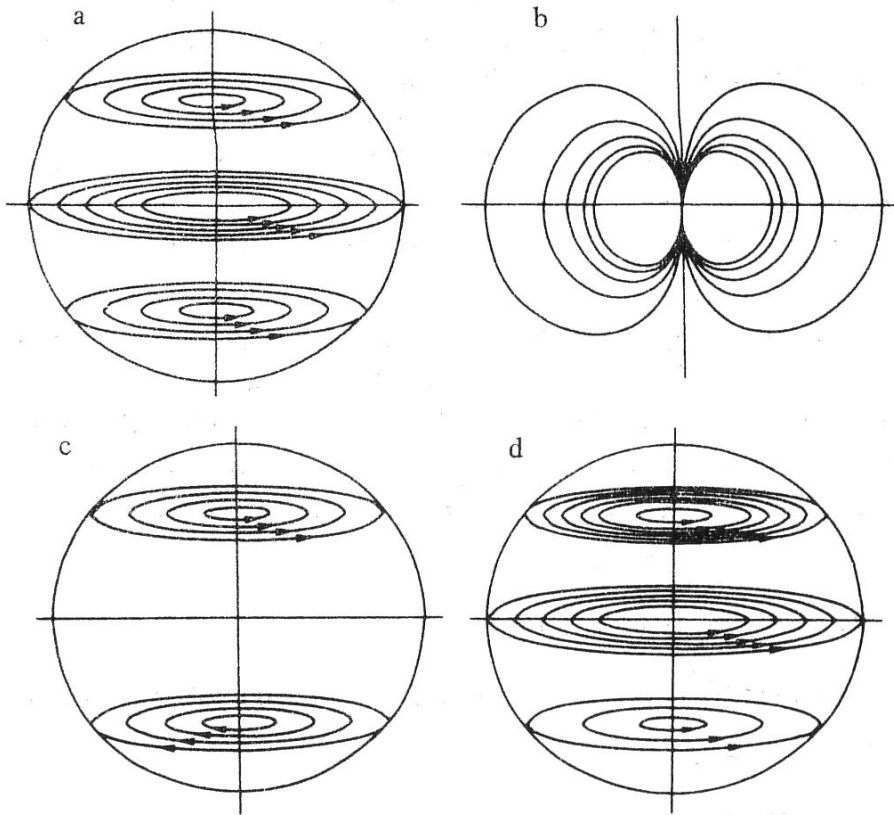
Optimal :
Eulerian scheme with Automatic Mesh Refinement : Approximate MHD Riemann solvers (problems with degeneration of eigenvectors and eigenvalues)
Finite-difference schemes (e.g. central numerical schemes *Kurganov&Tadmor, Ziegler*).
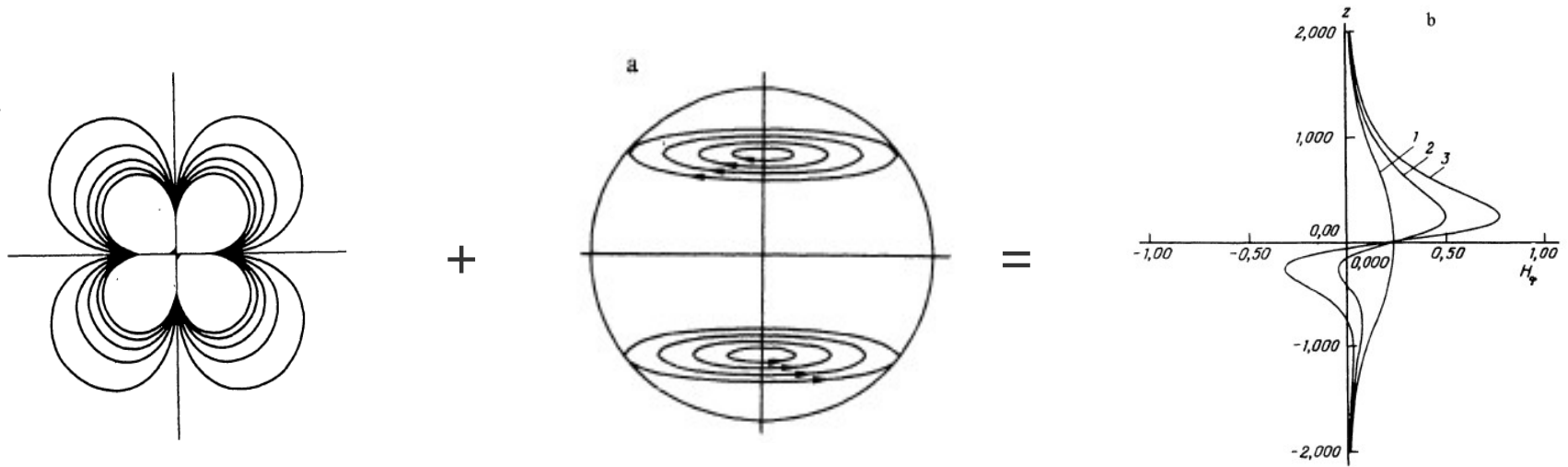
Mirror symmetry violation of the magnetic field in rotating stars

Bisnovatyi-Kogan, S.M. Sov. Astr. 1992, 36, 285

- a. Initial toroidal field

- b. Initial dipole field

- c. Generated toroidal field

- d. Resulting toroidal field

Mirror symmetry violation of the magnetic field in rotating stars

Resulting toroidal filed is larger in the upper hemisphere.

Violation of mirror symmetry of the magnetic field in magnetorotational explosion leads to: One sided ejections **along the rotational axis.**

Rapidly moving radiopulsarss (up to 300 km/s).

In reality we have dipole + quadrupole + other multipoles…

(Lovelace et al. 1992)

Dipole ~ $\dfrac{1}{r^3}$

Quadrupole ~ $\dfrac{1}{r^4}$

The magnetorotational supernova explosion is always asymmetrical.

For high magnetic fields neutrino cross-section depends on the magnetic field values.

(Bisnovatyi-Kogan, Astron. & Astroph. Trans., 1993, 3, 287, See also Lai, 1998)

The pulsar kick velocity can be up to 1000 km/s along rotational axis

Jet, kick and axis of rotation are **aligned** in MR supernovae.

Evidence for alignment of the rotation and velocity vectors in pulsars

"We present strong observational evidence for a relationship between the direction of a pulsar's motion and its rotation axis. We show carefully calibrated polarization data for 25pulsars, 20 of which display linearly polarized emission from the pulse longitude at closest approach to the magnetic pole…

**we conclude that the velocity vector and the rotation axis are aligned at birth".**

Rapidly moving pulsar VLBI observations

"Determination of pulsar parallaxes and proper motions addresses fundamental astrophysical questions. We have recently finished a VLBI astrometry project to determine the proper motions and parallaxes of 27 pulsars, thereby doubling the total number of pulsar parallaxes. Here we summarize our astrometric technique and present the *discovery of a pulsar moving in excess of 1000 kms*, PSR B1508+55".

Cassiopea A- supernova with jets-an example of the magnetorotational supernova

(Hwang et al. ApJL, 2004, 615, L117)
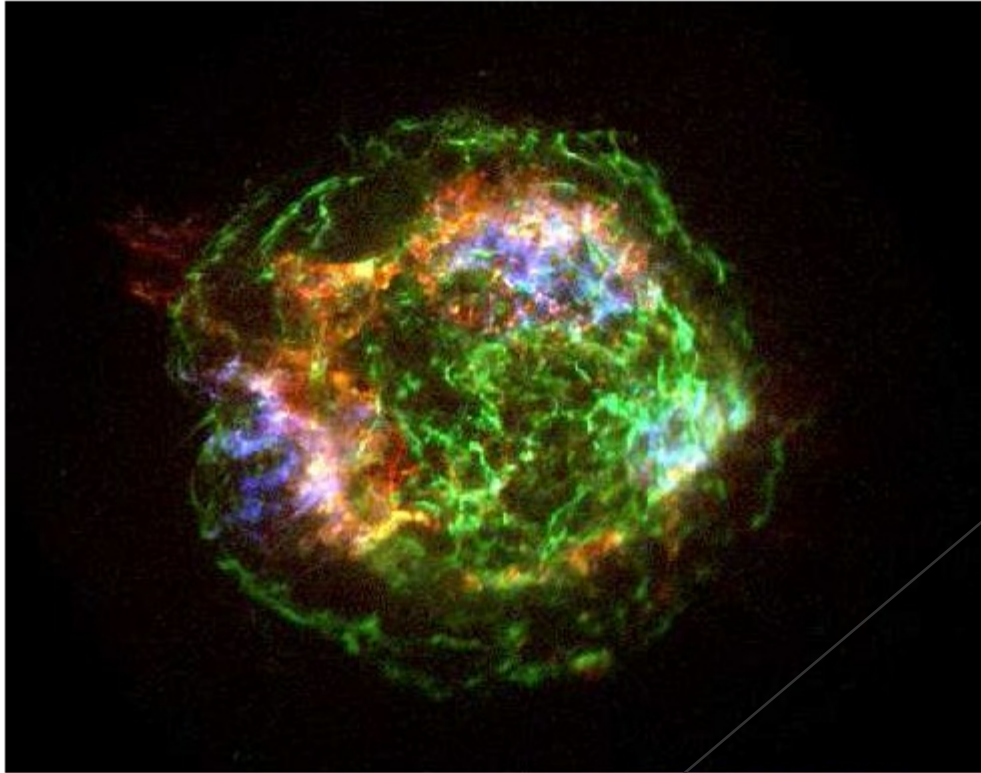
1 million Chandra servey of Cas A. Second jet was found.
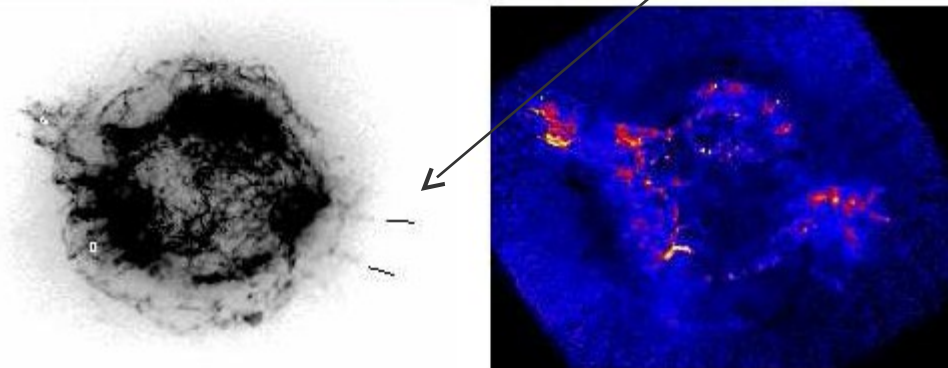
Conclusions

- Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.

- The MRM is weakly sensitive to the neutrino cooling mechanism.

- MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.

- MRI developes in MR supernova explosion.

- One sided jets and rapidly moving pulsars can appear due to MR supernovae.

- 3D simulations of MR supernova are necessary.

Thank you!