# From Newton to Einstein Relativistic Dynamics of Black Holes in Galactic Nuclei

Rainer Spurzem, Astronomisches Rechen-Institut
Zentrum für Astronomie Univ.Heidelberg, Germany

spurzem@ari.uni-heidelberg.de
http://www.ari.uni-heidelberg.de/mitarbeiter/spurzem/

ASTRONOMISCHES RECHEN-INSTITUT
HEIDELBERG

VolkswagenStiftung

GRACE
GRACE = GRAPE + RACE

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**Team:** ***@ ARI / ITA (ZAH):***

**Peter Berczik, Ingo Berentzen, Miguel Preto**
**Jonathan Downing, Andreas Ernst,**
**Jose Fiestas, Oliver Porth, Kristina Wäcken**

*Previously @ARI:*

**Gabor Kupi, Christoph Eichhorn**
**Pau Amaro-Seoane, Marc Freitag**
**Andrea Borch**

***Further important collaborators:***

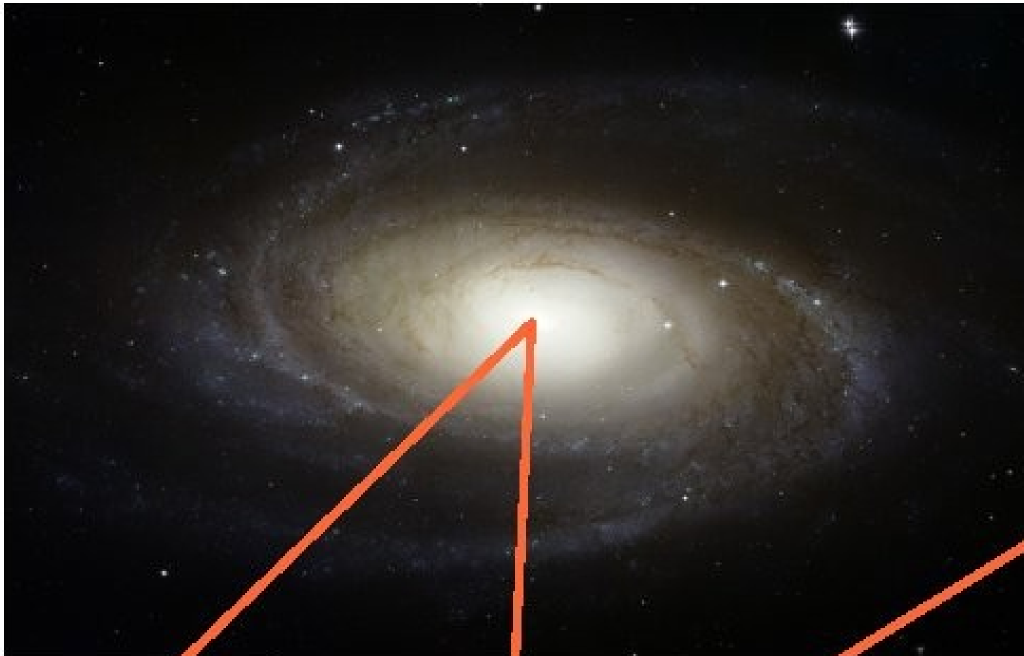
**David Merritt (RIT), Sverre Aarseth (IoA)**
**Doug Lin (KIAA / UCOLick),**
**G. Schäfer, A. Gopakumar (Univ. Jena, D)**
**M. Benacquista (UT Brownsville, USA)**
**M. Giersz (CAMK Warsaw), H.M. Lee (Korea)**

## Impressions of JENAM 2007 in Yerevan

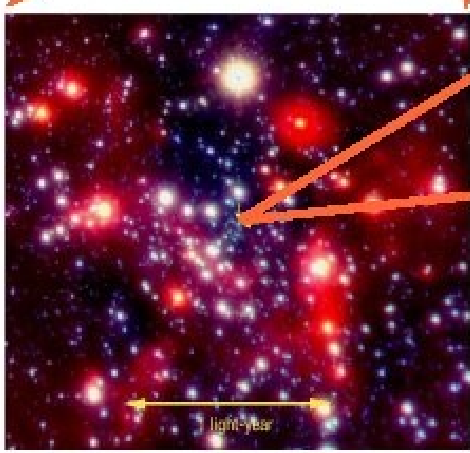
## from Yerevan (below) and Byurakan (right)

# Contents

# Setting the stage: the galactic nucleus dense star cluster



Size ~ 10 Kpc
Density ~ 0.05 $M_{sun}$ $pc^{-3}$
Vel. Disp. ~ 40 Km/s
Relaxation time ~ $10^{15}$ yrs.




Size ~ 1-10 pc
Density ~ $10^{6-8}$ $M_{sun}$ $pc^{-3}$
Vel. Disp. ~ $10^{2-3}$ Km/s
Relaxation time ~ $10^{8-9}$ yrs.

Size ~ $10^{-7}$-$10^{-4}$ pc
$R_s = 2G\,M_{BH}/c^2$
$R_t \sim (\alpha\, M_{BH}/m_*)^{1/3} R_*$
Loss cone aperture: $\theta$

*Slide: Miguel Preto*

# Astrophysical Introduction

- Dynamical Time Scale
- Relaxation Time Scale
- Age of Universe

$$t_{\rm cr} = \frac{r_h}{\sigma_h} \, ,$$

$$t_{\rm rx} = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho \ln(\gamma N)} \, .$$

$10^6$ yrs

$10^8$ yrs

$10^{10}$ yrs

Laboratories for gravothermal N-Body Systems!

Note: Cosmological and Galactic N-Body Simulations need few crossing times, and less than a relaxation time, while gravothermal systems need multiples of N crossing times, several relaxation times! Complexity goes as $N^3$ !

$$t_{\rm cr} \approx \sqrt{\frac{r_h^3}{G M_h}} \, .$$

← Virial Equilibrium →

$$\frac{t_{\rm rx}}{t_{\rm dyn}} \propto \frac{N}{\log(\gamma N)} \, .$$

# Dense Star Clusters

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[ \frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

- $N = \infty$

negative specific Heat

gravothermal Collapse

gravothermal Oscillations

- $N = 3 \ (N = 2, \ldots, \approx 100)$

History

Exponential Instability

Chaos and Resonance

Regularisation

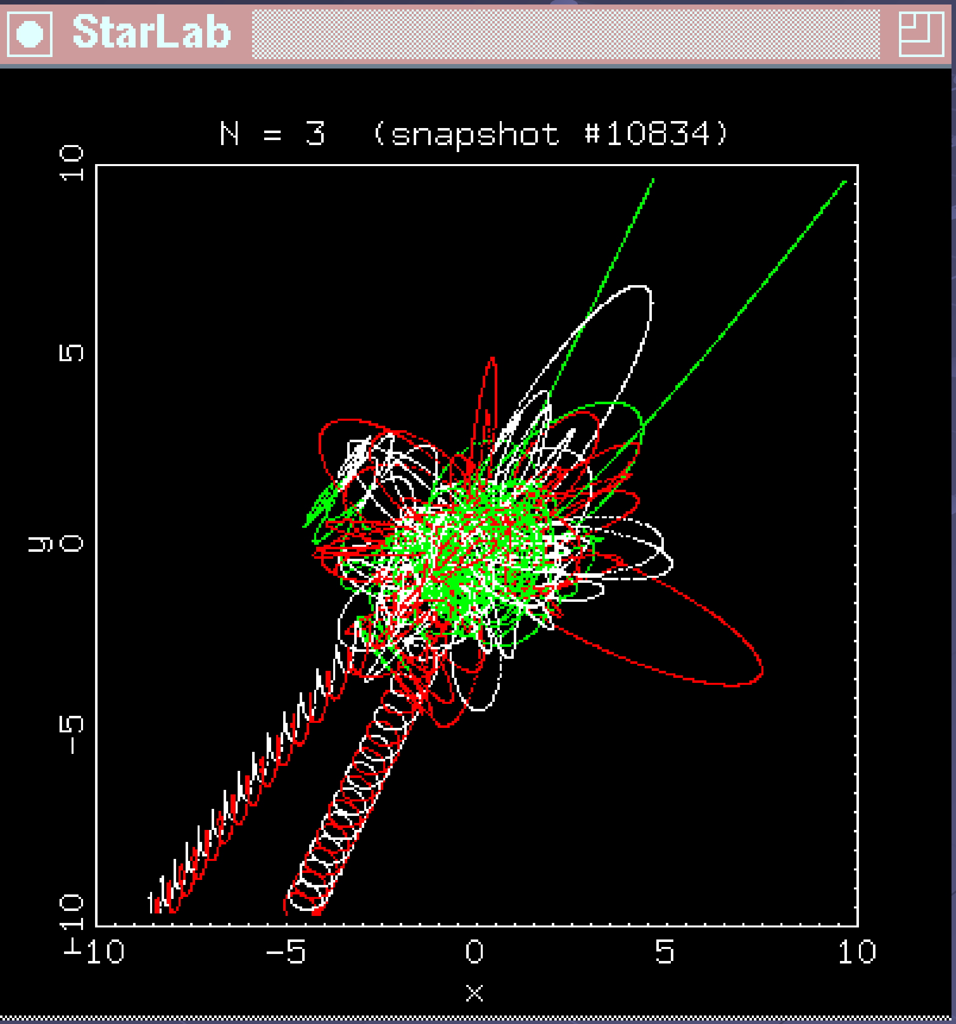
- $N = 10^6 \ (N = 10^4, 10^5)$

Post-Kollaps-Evolution

Binaries

Globular Clusters

# Dense Star Clusters




- Unstable 3-body Encounters Starlab Simulation (S.L.W. McMillan)

http://www.physics.drexel.edu/~steve/
-> Three-Body-Problem

***Here: some classical 3-body movies...***

# Dense Star Clusters

- Chaos in the 3-Body Problem
  (by S.L.W. McMillan)

1 pixel in image =
1 simulated 3-body encounter

X-axis: initial phase of binary

Y-axis: impact parameter

Colour: angle by which escaping star leaves the system.

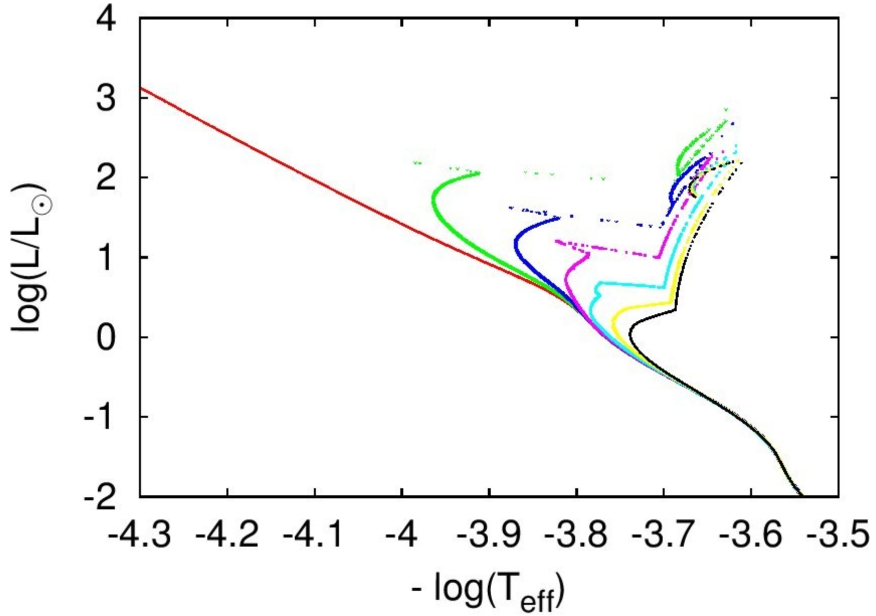
Fortunately there exist statistical averages for cross sections

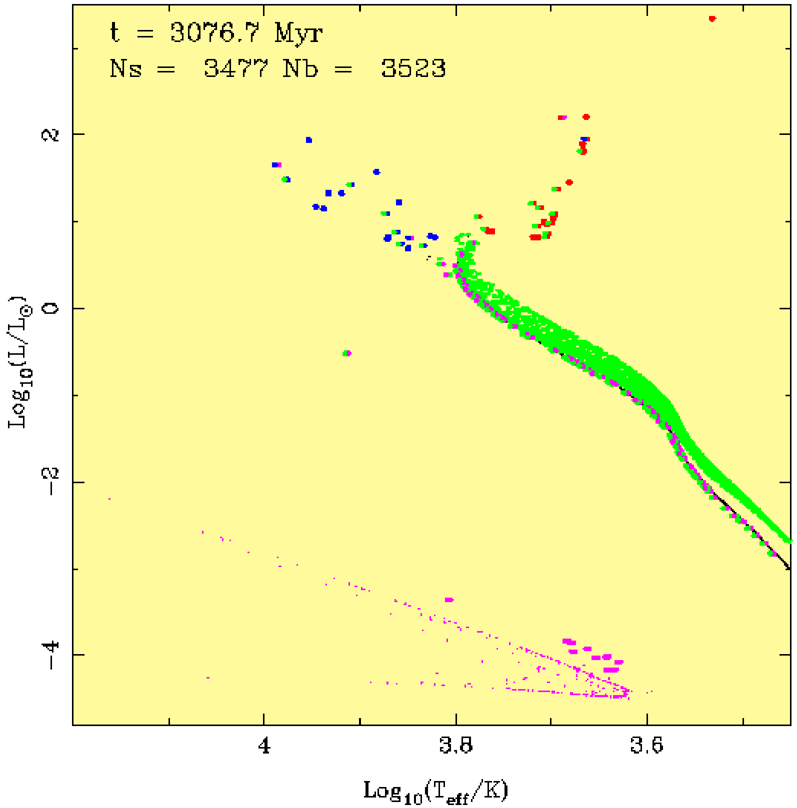
# Three-Body – Million Body

Cambridge Stellar Evolution Tracks in NBODY6++
Kyoto Millennium Project
Mardling, Spurzem, Borch, Giersz, Hurley, Downing

Andrea Borch et al.: NBODY meets stellar

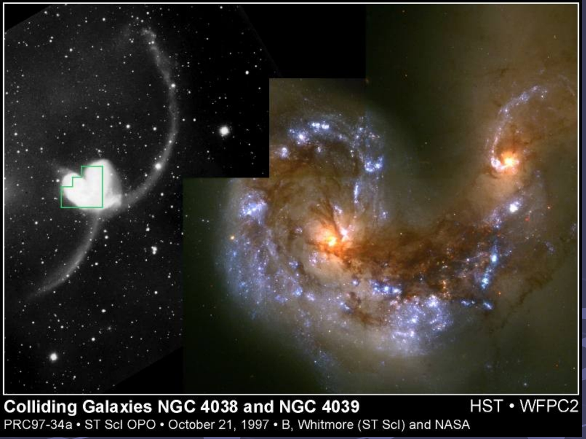
**population: the HYDE-PARC project**



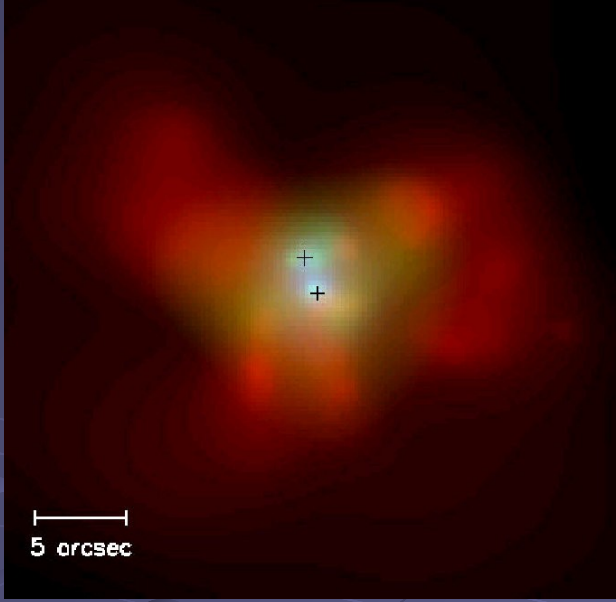




J. Hurley et al. 2001:
A star cluster like M67
Synthetic Hertzsprung-Russell Diagram from Simulation
A load of blue stragglers!
About 10 so-called DD's found – "double degenerates"
"loaded guns"

N : 50.000




# Contents




**A) Black Holes in Galactic Nuclei**

*1. Introduction*

**2. Classical Black Hole Dynamics – Stalling or not Stalling?**

**B. Gravitational Waves – Post-Newtonian Relativistic Dynamics**

*3. From Newton to Einstein - Grav. Wave Prediction from Simulations*

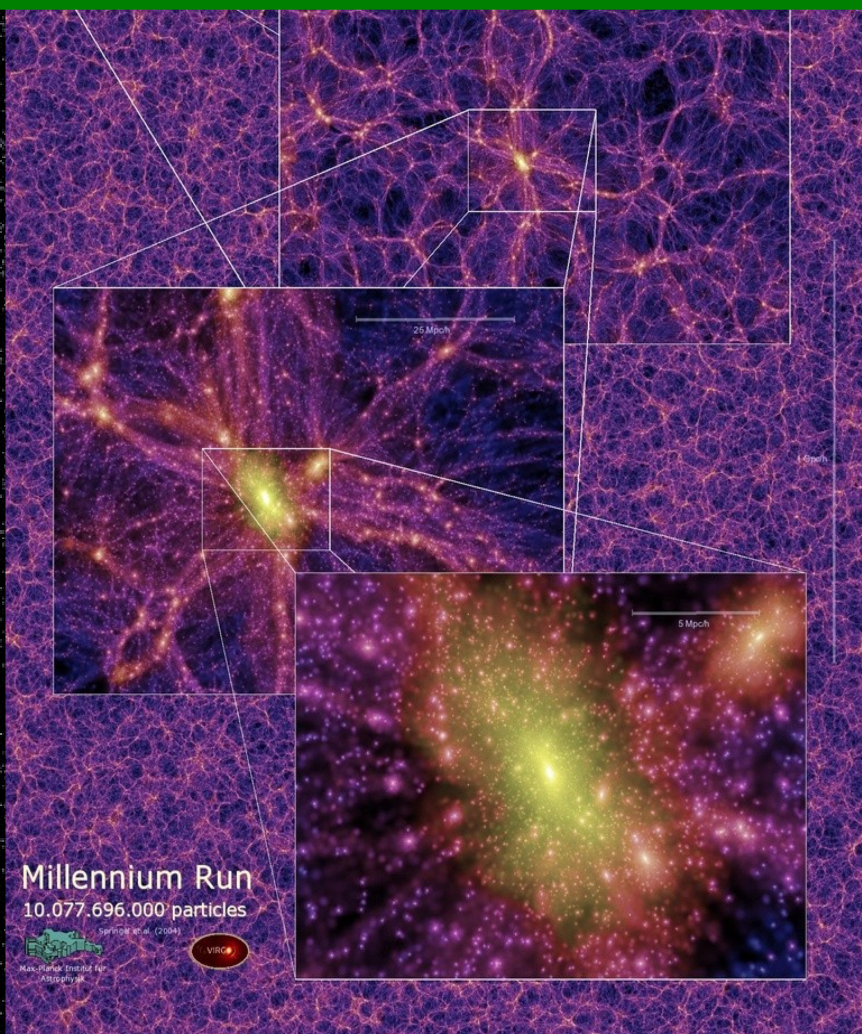
*4. Gravitational Wave Detection*

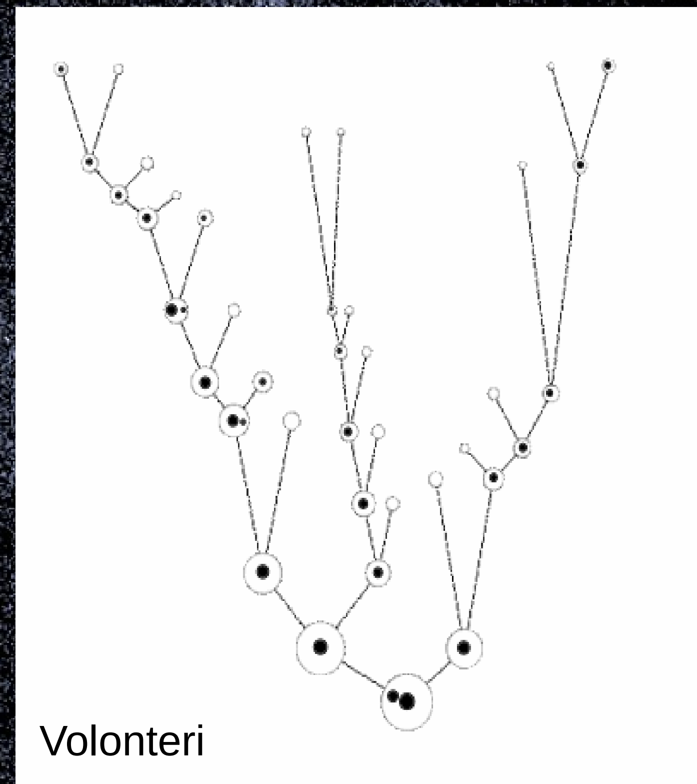
**C. Computational Astrophysics**

*5. Special Software, Parallel N-Body Simulations*

*6. Special Hardware, Computer Games*

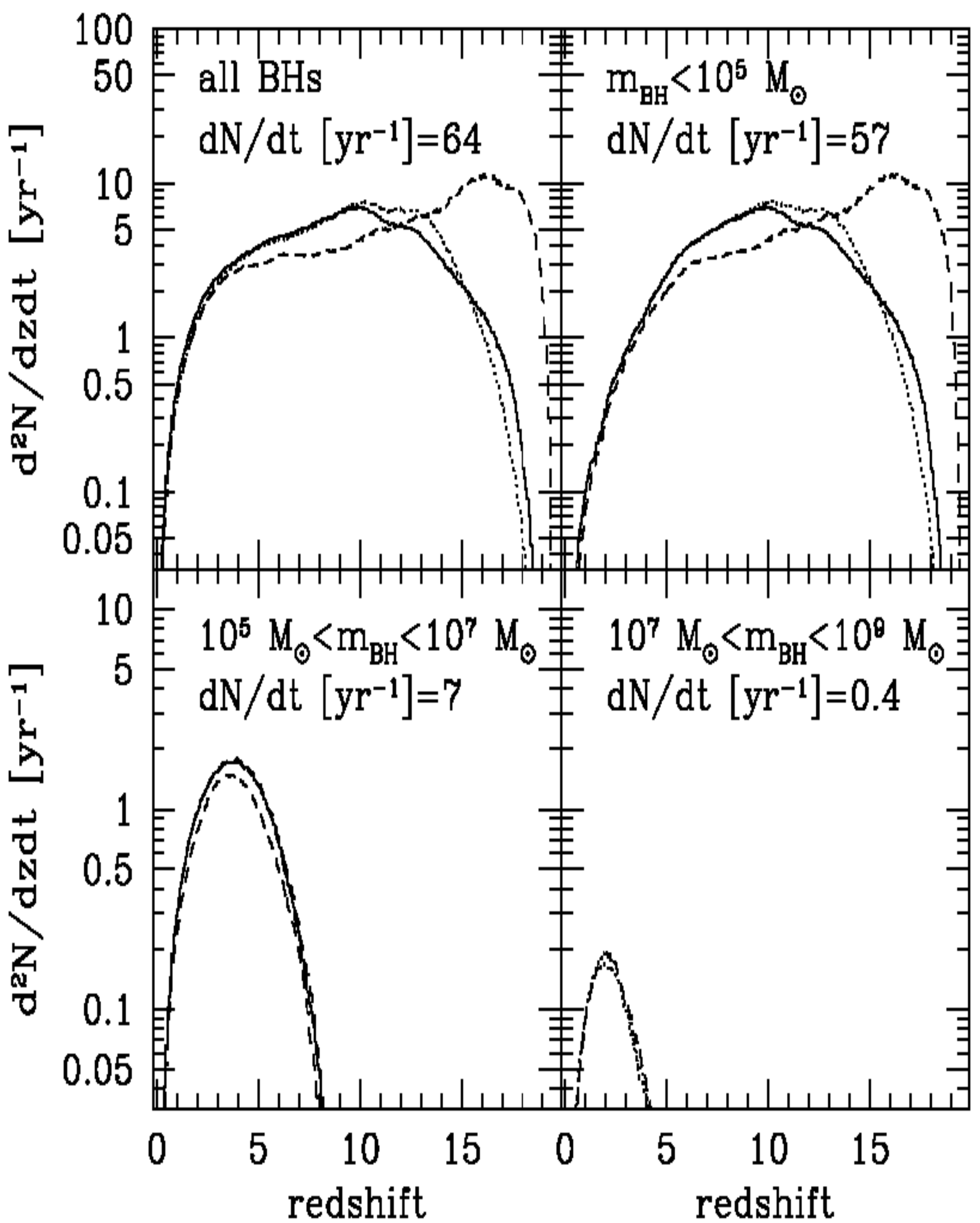
# Galaxies merge, hierarchical Structure formation, their centres? Black Holes?




*Slide: Ingo Berentzen*

# Classical Black Holes

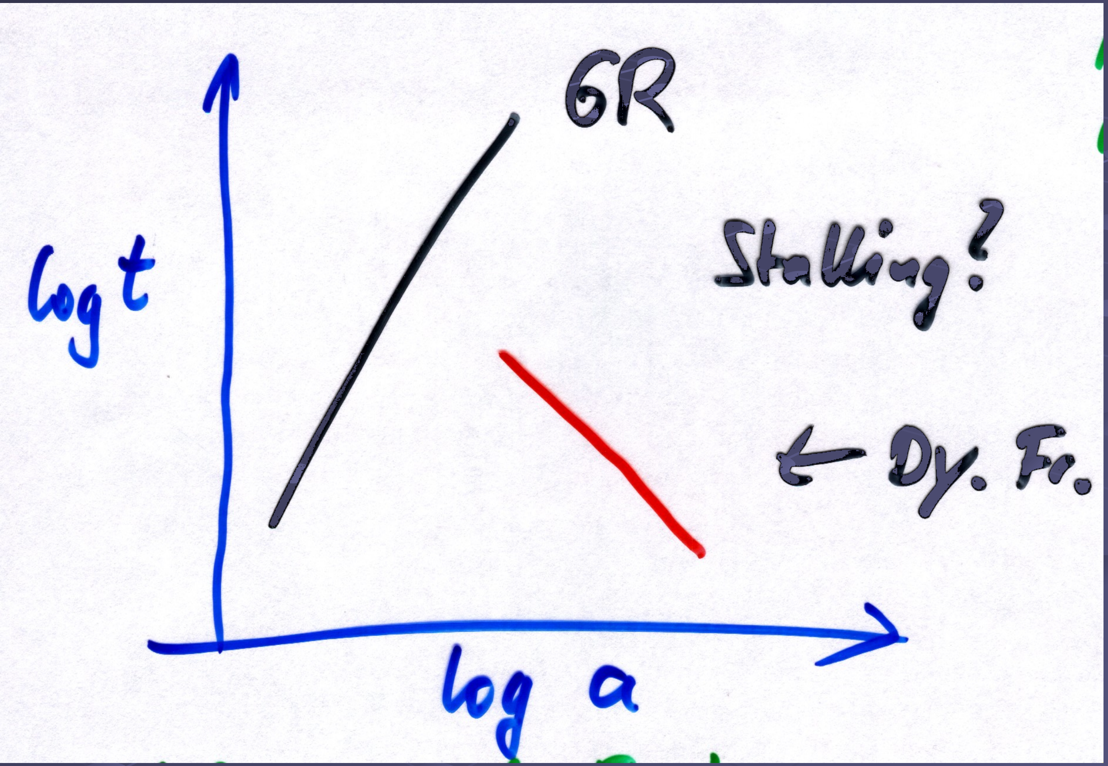



Sesana et al. 2004

**Extending study of Volonteri et al. 03: Rate of expected Black Hole Mergers in Galaxies**

Number of MBH binary coalescences observed per year at $z = 0$, per unit redshift, in different $m_{BH} = m_1 + m_2$ mass intervals. Each panel also lists the integrated event rate, *dN/dt*, predicted by our model. The rates (*solid lines*) are compared to a case in which triple black hole interactions are switched off (*dotted lines*). Triple black hole interactions increase the coalescence rate at very high redshifts, while for $10 < z < 15$, the rate is decreased because of the reduced number of surviving binaries. *Dashed lines*: Rates computed assuming binary hardening is instantaneous, i.e., MBHs coalesce after a dynamical friction timescale.

# Classical Black Holes




d ~10*R_BH

**BH coalescence?**

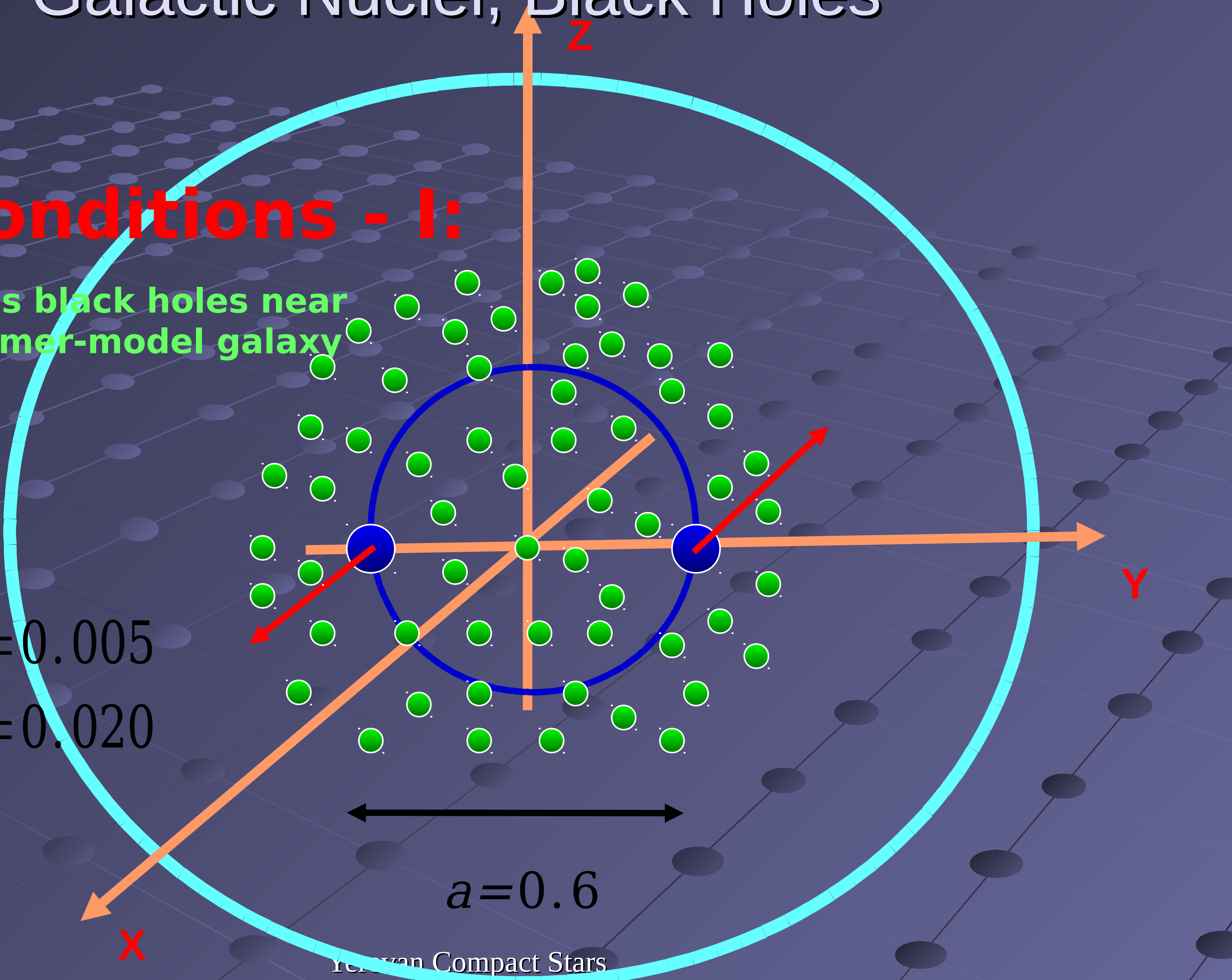
Begelman,
Blandford & Rees
1980

$$R_{BH} = \frac{2 \cdot G \cdot M_{BH}}{c^2} \approx 10^{-7} \cdot \left( \frac{M_{BH}}{10^6 \cdot M_o} \right) [\mathrm{pc}]$$

# Galactic Nuclei, Black Holes

## Initial Conditions - I:

**Two equal-mass black holes near center of Plummer-model galaxy**



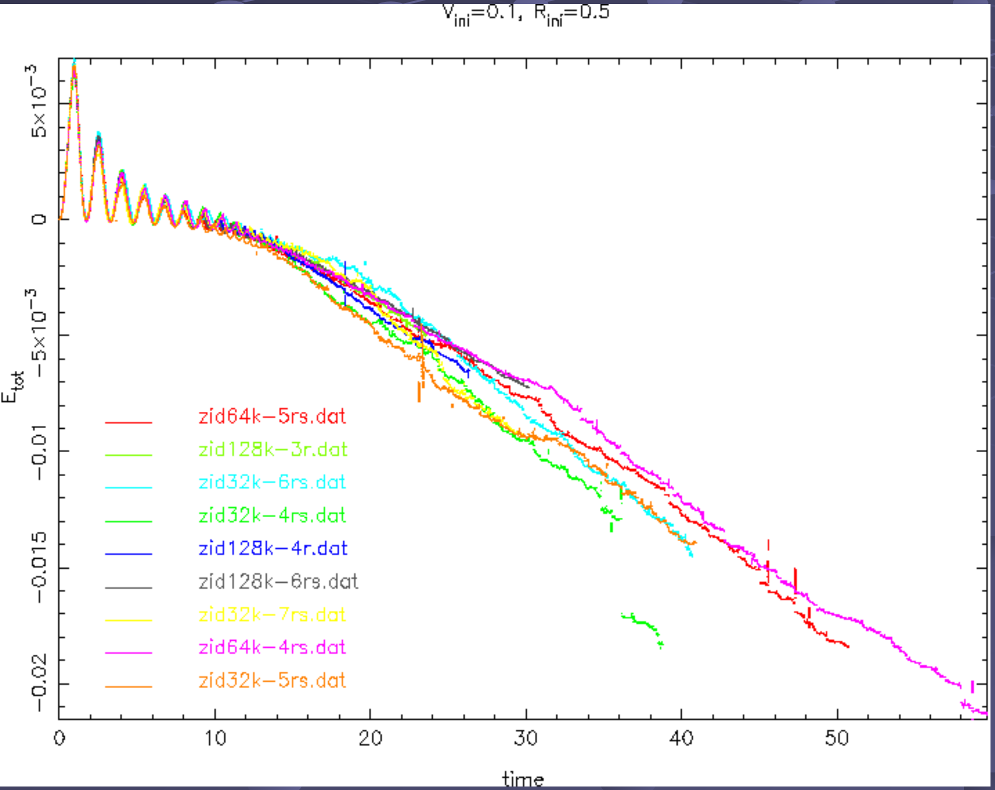

$$m_{BH1} = m_{BH2} = 0.005$$

$$m_{BH1} = m_{BH2} = 0.020$$

$$G = M = 1$$

$$E_{TOT} = -\frac{1}{4}$$

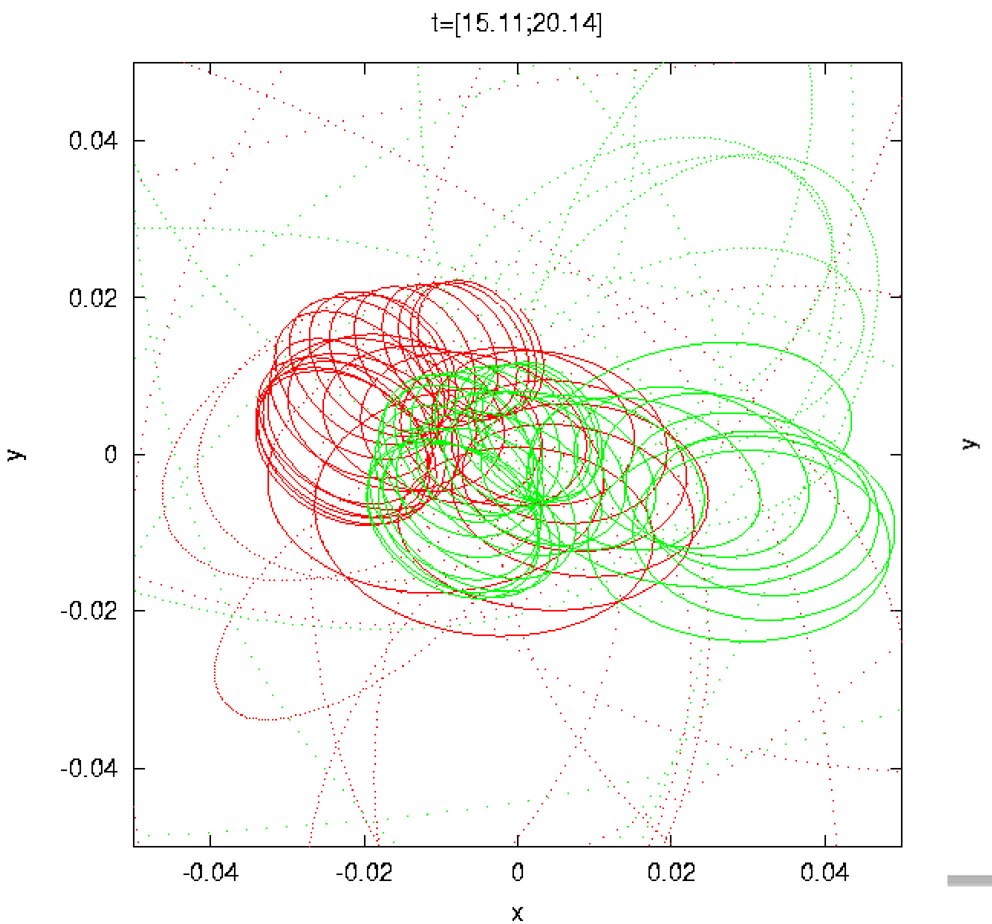
# Galactic Nuclei, Black Holes

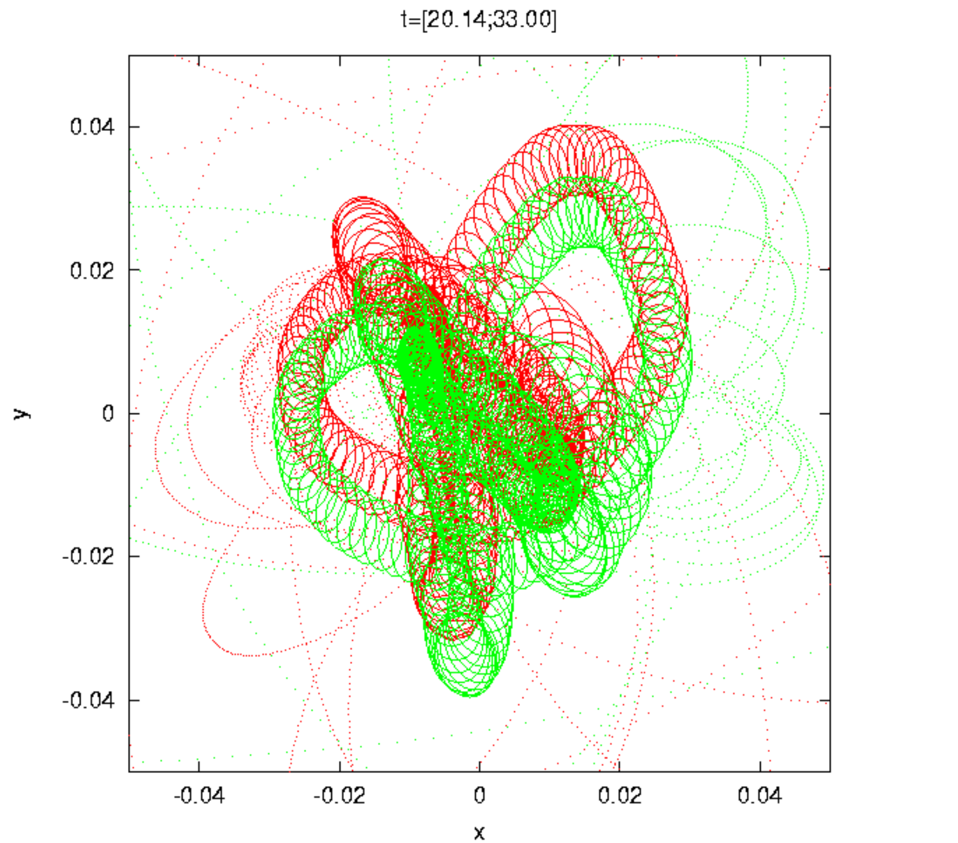



- Stochastic Orbit variations of binary black holes due to superelastic scatterings with stars

Hemsendorf, Sigurdsson, Spurzem, 2002, Astroph. Jl

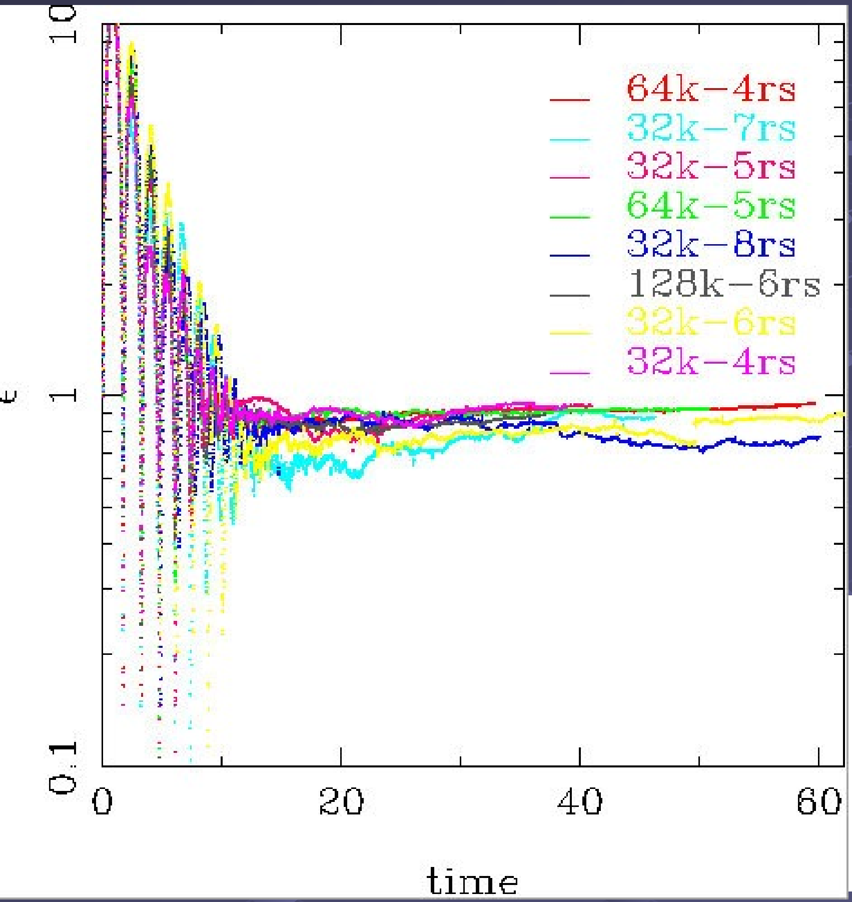
# Galactic Nuclei, Black Holes




Christoph Eichhorn (Diploma Thesis Univ. Heidelberg), Eichhorn, Amaro-Seoane & Spurzem 2008 (in prep)

# Galactic Nuclei, Black Holes




- Hemsendorf, Sigurdsson, Spurzem, 2002 , Astroph. Jl.
- Cf. also Aarseth & Mikkola (2003), Funato & Makino (2005), Makino et al. 1993

Time Scale for GR merger: $t_{GR} \propto M^{-3}(1-e^2)^{7/2}$

Important for Background of Ultra-Low GR frequencies (LISA!) 0.01-1 $\mu$Hz

But: other authors (Milosavljevic & Merritt 2001, Berczik, Merritt & Spurzem 2005) find: eccentricity remains small if N is large enough!

# Galactic Nuclei, Black Holes

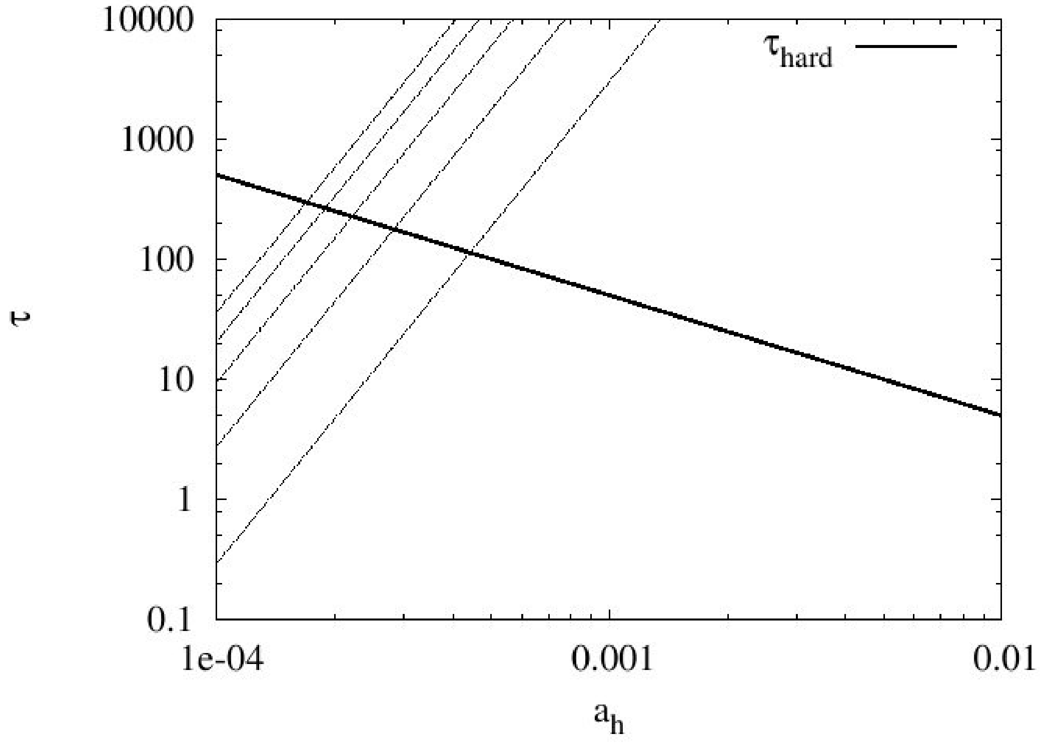
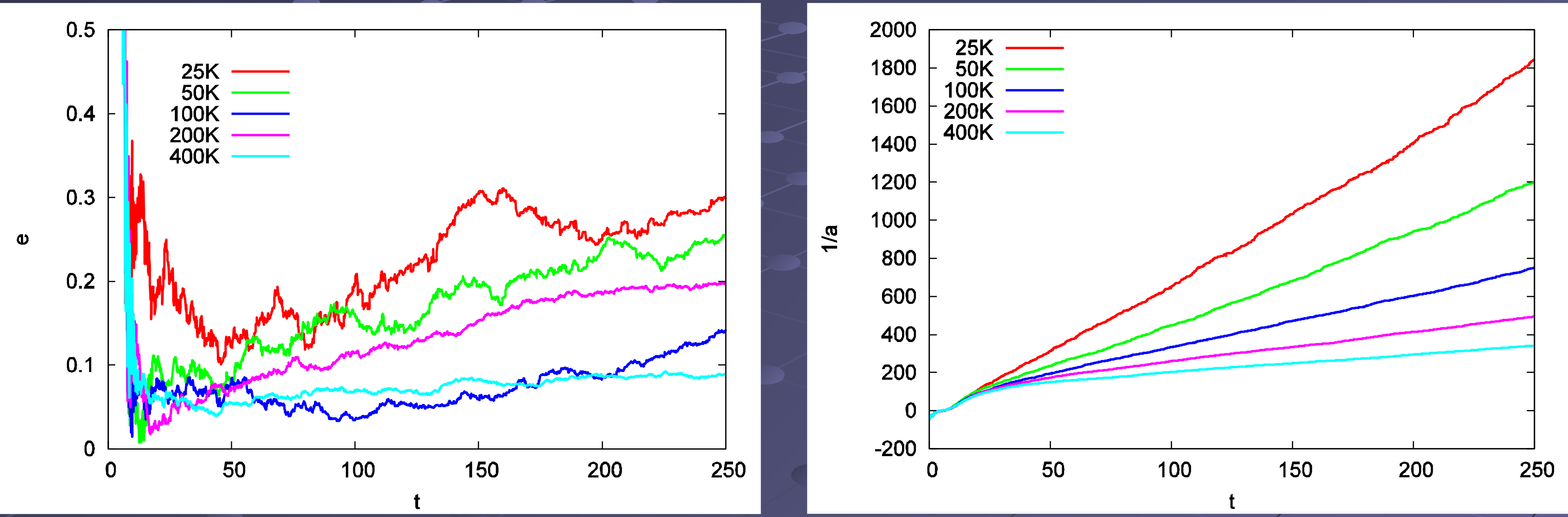



FIG. 5.— Characteristic timescales as a function of the semi-major axis $a_h$ for a) the Newtonian hardening (thick full line) and b) Peters&Mathews (dotted lines) for eccentricities ranging from 0.5 to 0.9 from top to bottom in steps of 0.1. The speed of light has been chosen as $c \approx 500$ for the latter timescale.

Berentzen, Berczik, Merritt, Spurzem, Preto 2008, to be subm. ApJ

Binary
Black Hole
with PN treatment

# Galactic Nuclei, Black Holes



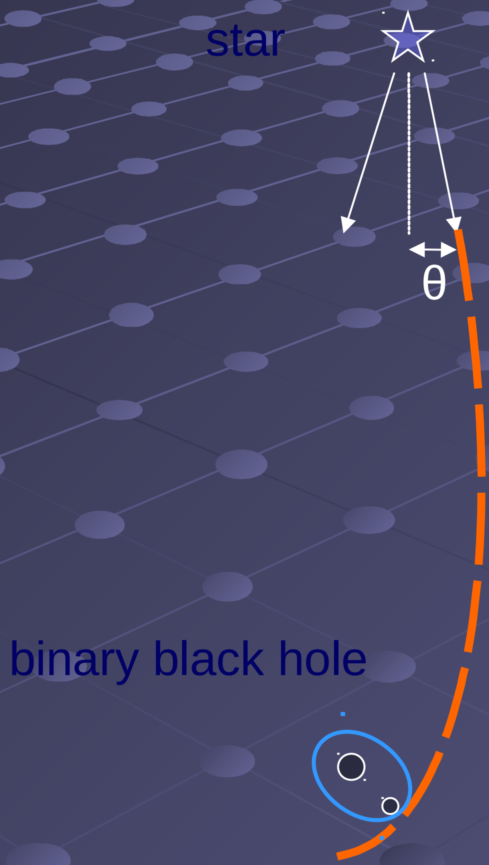

$m_{BH1} = m_{BH2} = 0.020$

**Results – I (Plummer): going to large N**
**low ecc., stalling occurs!**

Berczik, Merritt & Spurzem, 2005, ApJ

# Galactic Nuclei, Black Holes

**N-body Integration of Binary Black Hole Dynamical Evolut**




**Example: loss-cone around a binary black ho**

**Stars scattered into the binary are ejected via the gravitational slingshot. The binary responds by shrinking.**

**Full loss-cone**

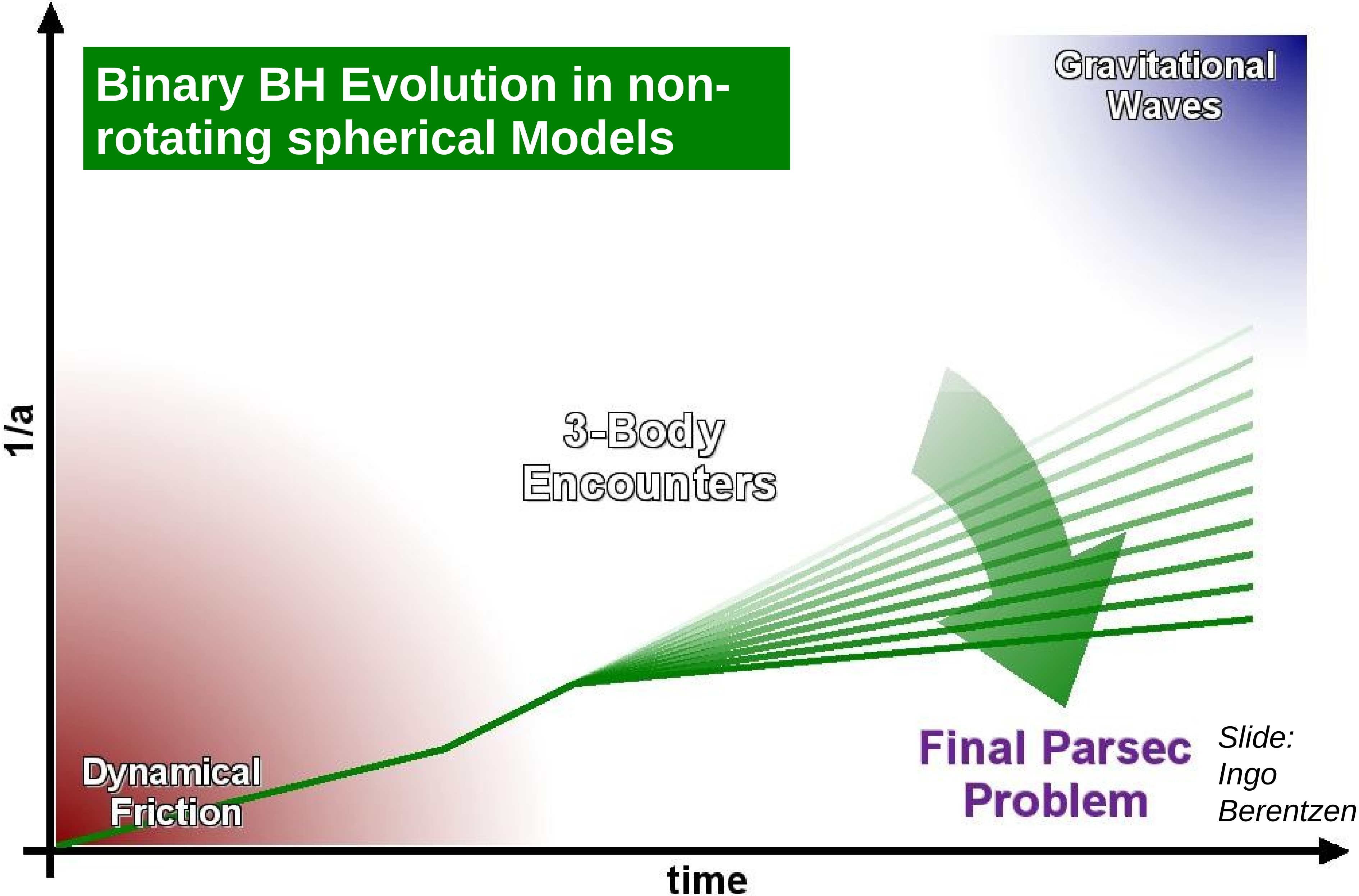
$$\frac{1}{a} \propto t$$

**Diffusive regime**

$$\frac{1}{a} \propto \frac{t}{N}$$

**In a real galaxy, the shrinking rate *(d/dt)(1/a)* would be limited by the rate of diffusion of stars into the loss cone.**

# Binary BH Evolution in non-rotating spherical Models




Slide: *Ingo Berentzen*

# Galactic Nuclei, Black Holes

## Initial Conditions - II:

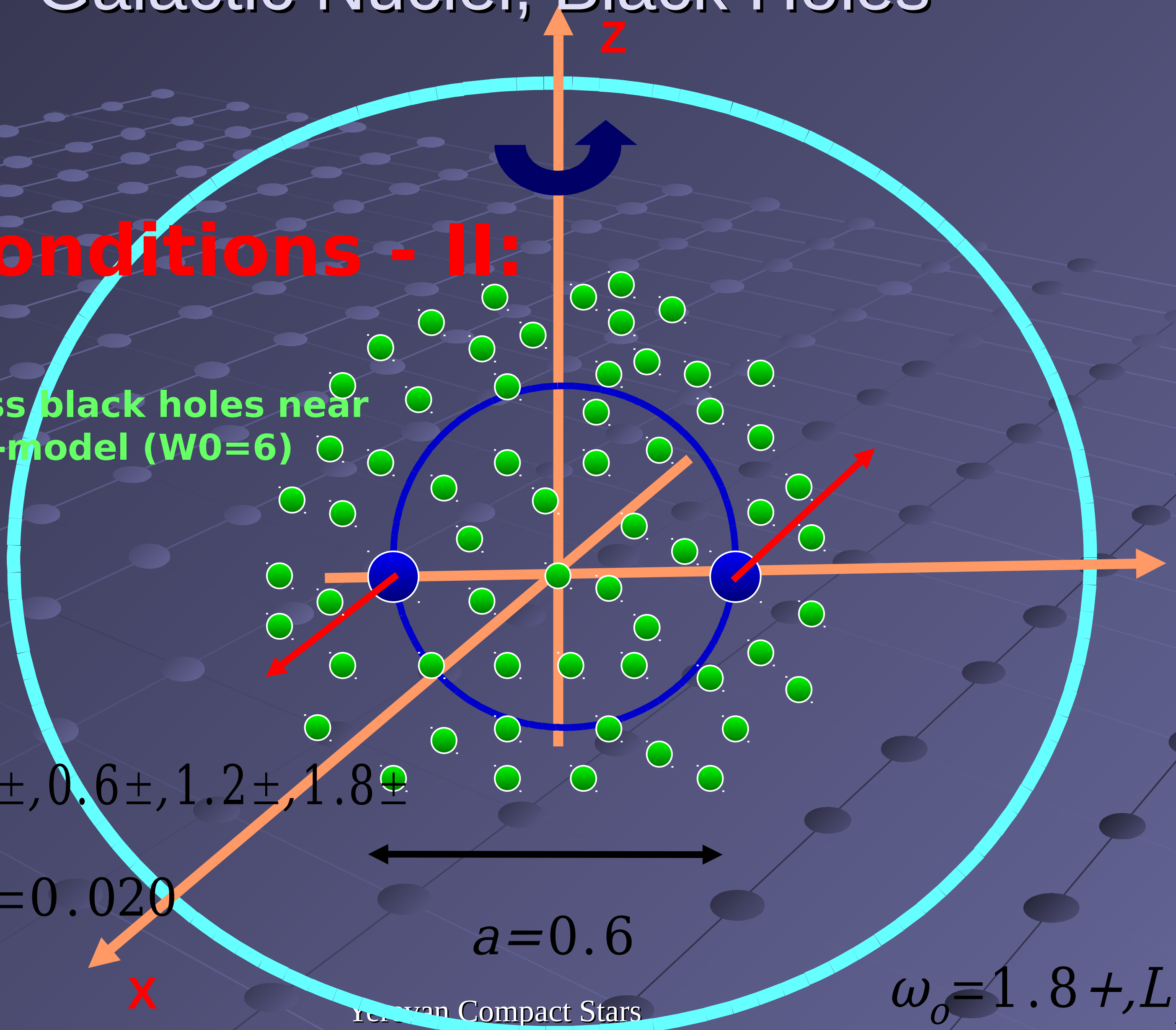
**Two equal-mass black holes near center of King-model (W0=6) galaxy**

$$G = M = 1$$

$$E_{TOT} = -\frac{1}{4}$$

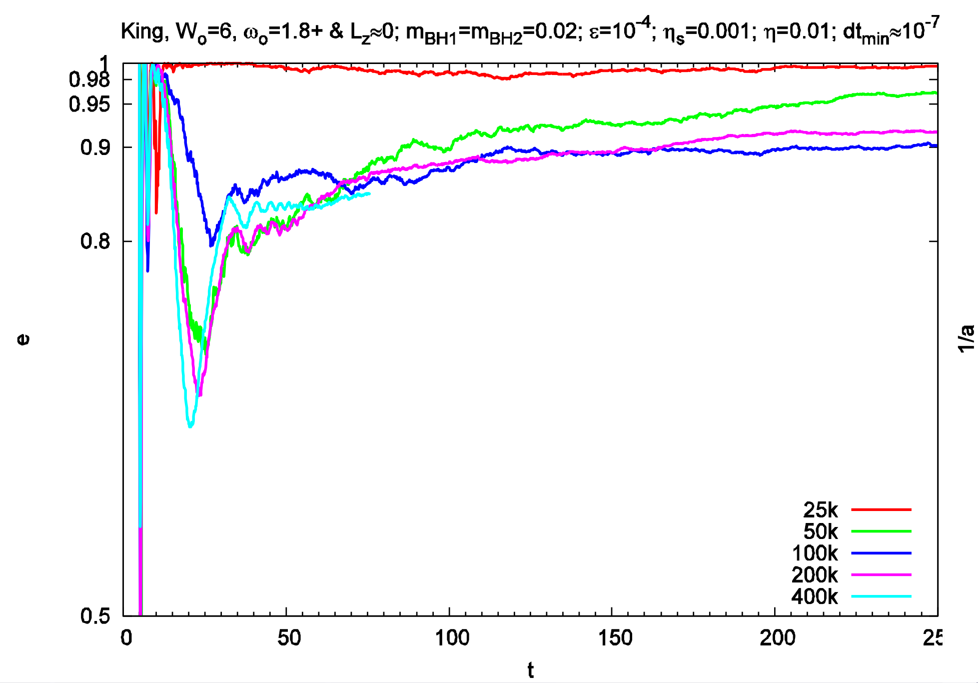
$$\omega_o = 0.0+, 0.3\pm, 0.6\pm, 1.2\pm, 1.8\pm$$

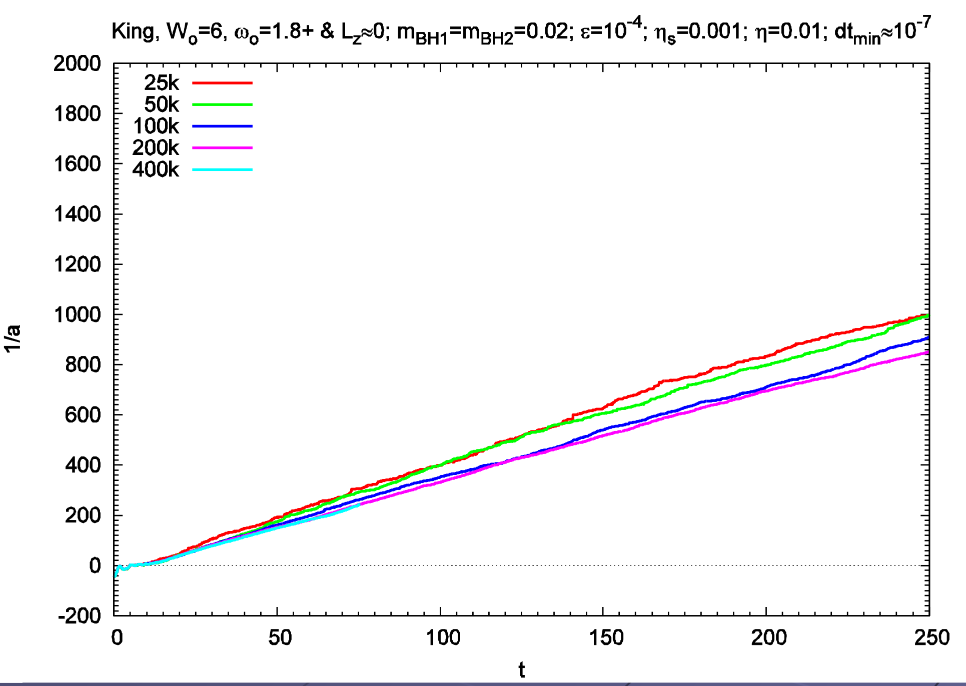
$$m_{BH1} = m_{BH2} = 0.020$$




$$\omega_o = 1.8+, L_z \approx 0$$

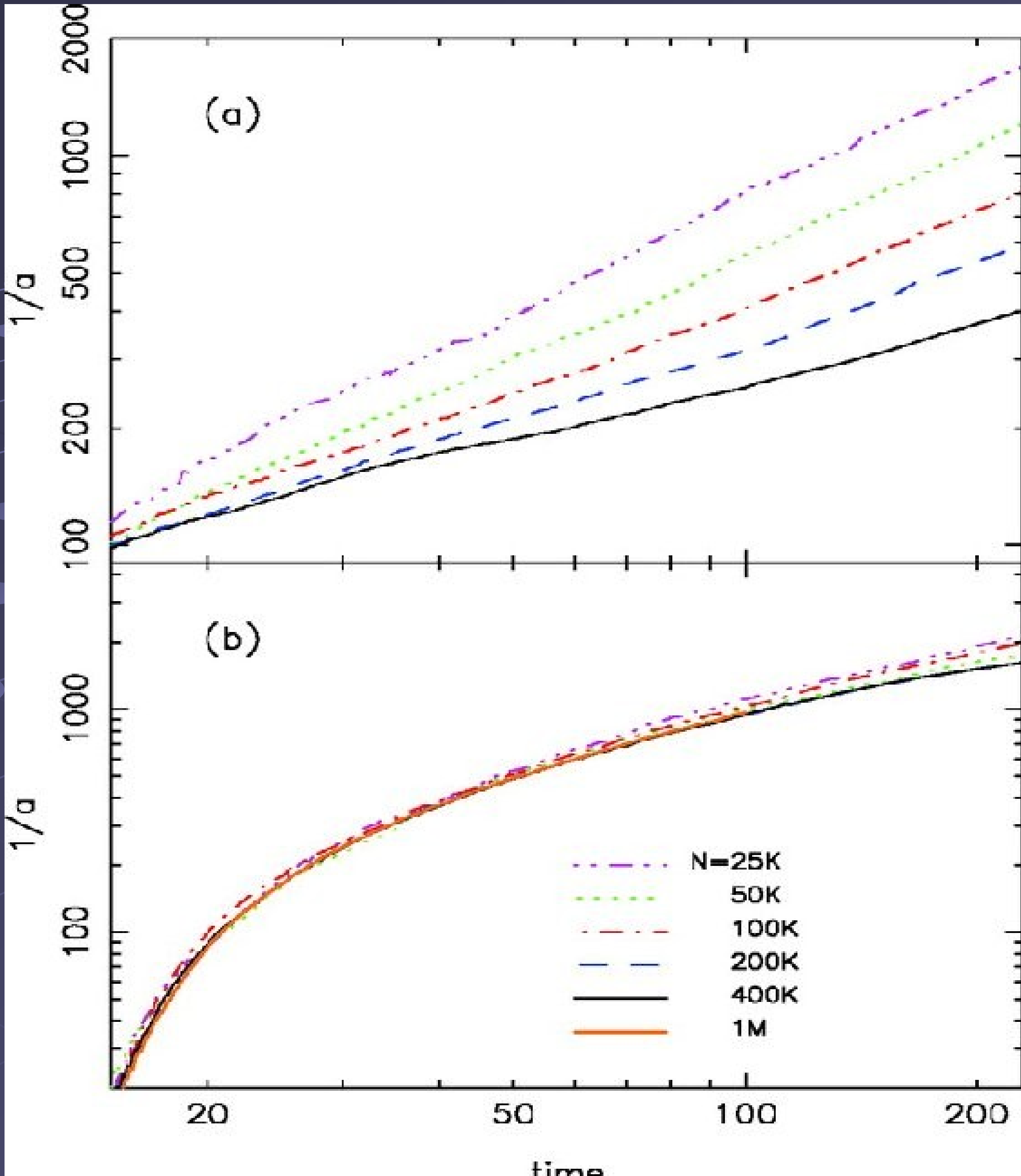
# Galactic Nuclei, Black Holes




**Results – II (King): rotating -> high ecc**
**no N dependence, full loss c**

Berczik, Merritt, Spurzem, Bischof, 2006, ApJL



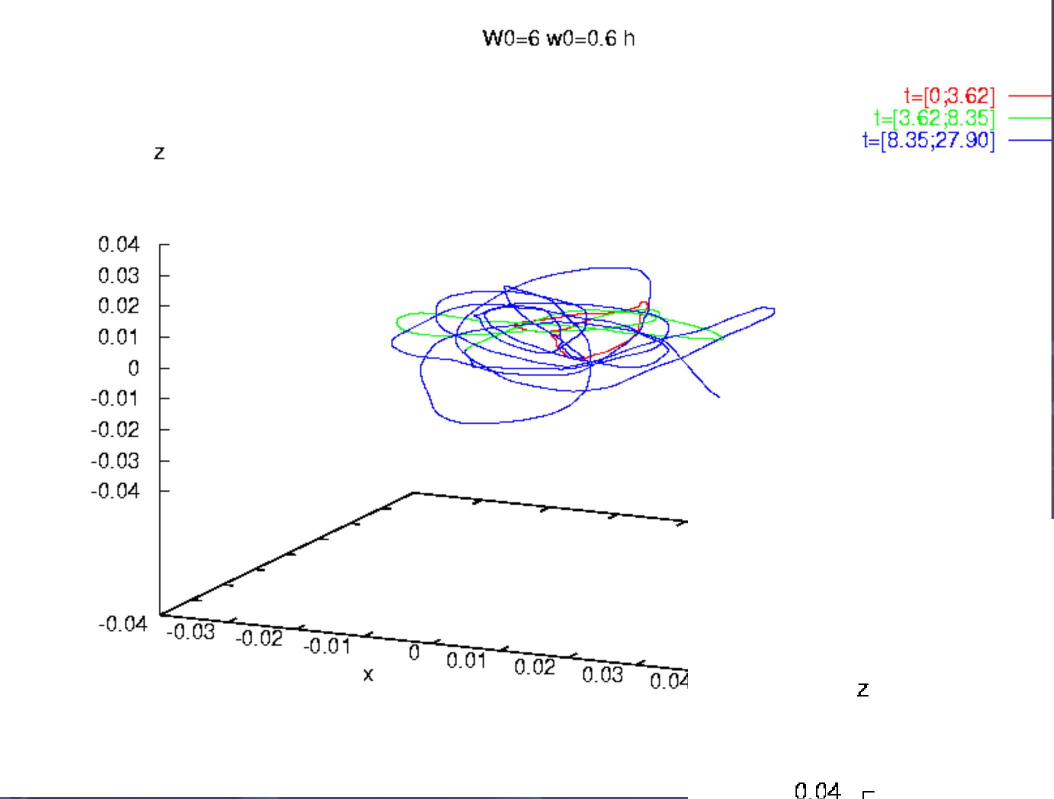

## Classical Dynamics:

*Spherical Systems:*
*BH binary stalls with N*

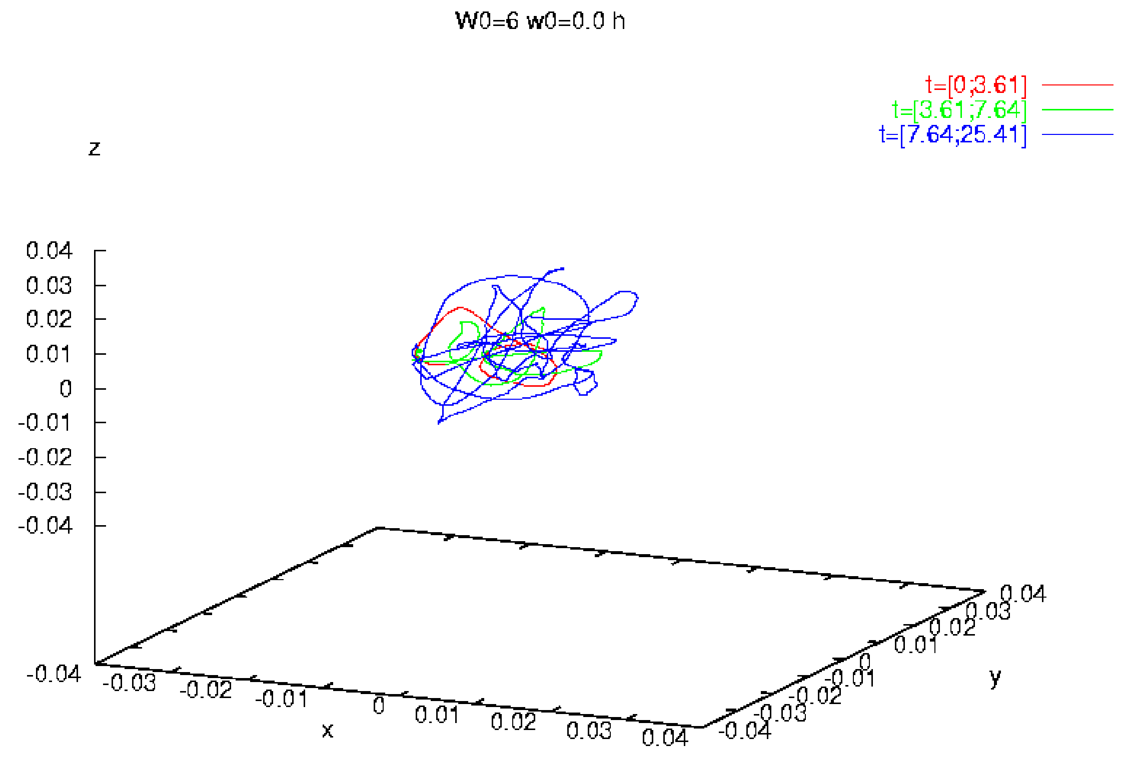
*Axisymmetric,*
*Rotating systems:*
*No stalling observed*

Berczik, Merritt, Spurzem, 2005, ApJ
Berczik, Merritt, Spurzem, Bischof, 2006, ApJ

*Movie!*



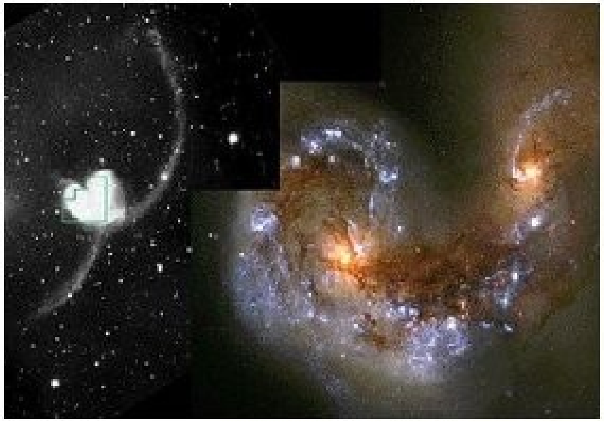

# Galactic Nuclei, Black Holes

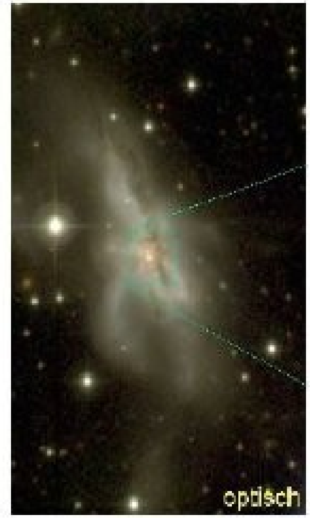


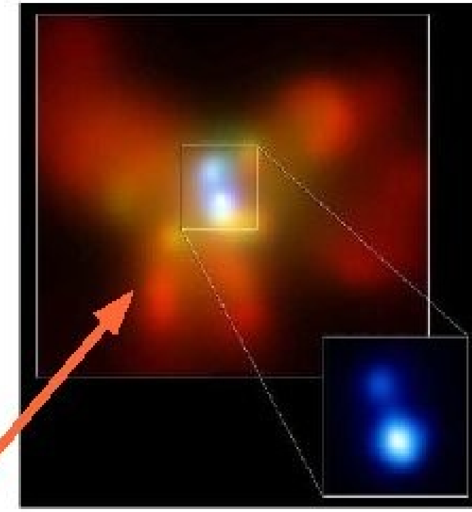

Christoph Eichhorn
(Diploma Thesis Univ. Heidelberg),
Eichhorn, Amaro-Seoane &
Spurzem 2008 (in prep)

Sep. 2008

# Galaxy Mergers and Massive Black Hole Binaries

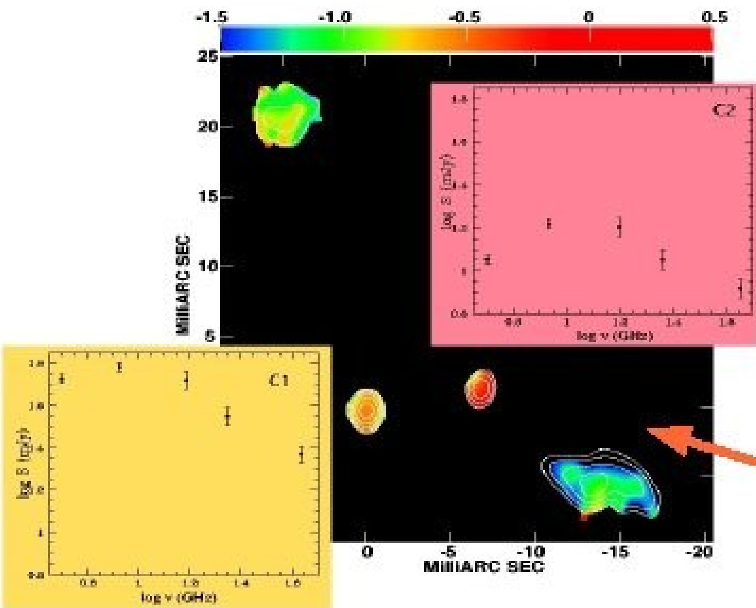




Komossa et al. 2003

NGC 6240: Merger in action
AGN in both galactic nuclei
Separation ~ 1 Kpc



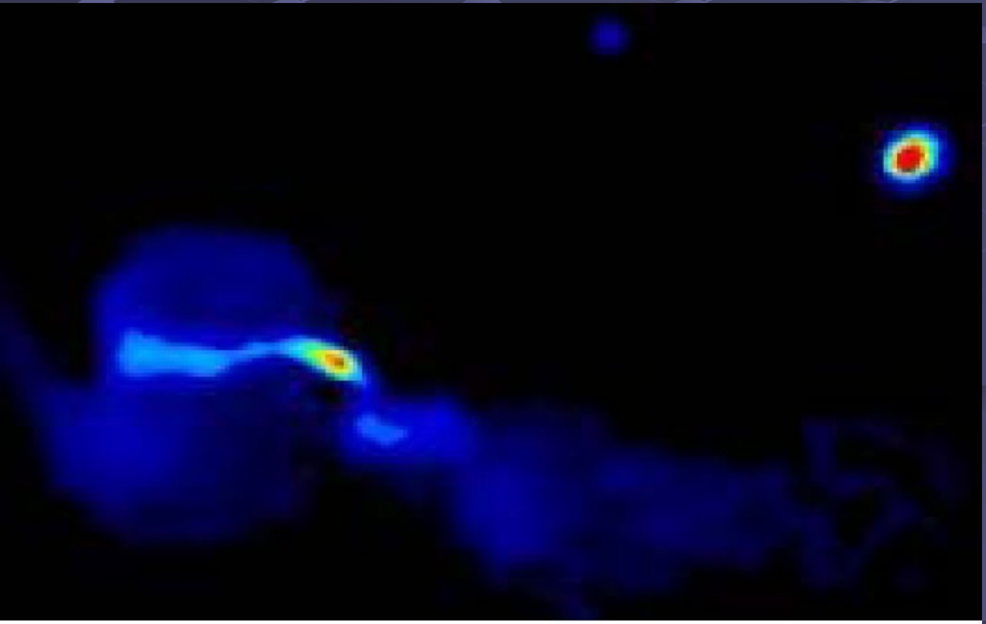

Separation ~ 7.3 pc
Total mass ~ $10^8$ $M_S$

Rodriguez et al. 2006

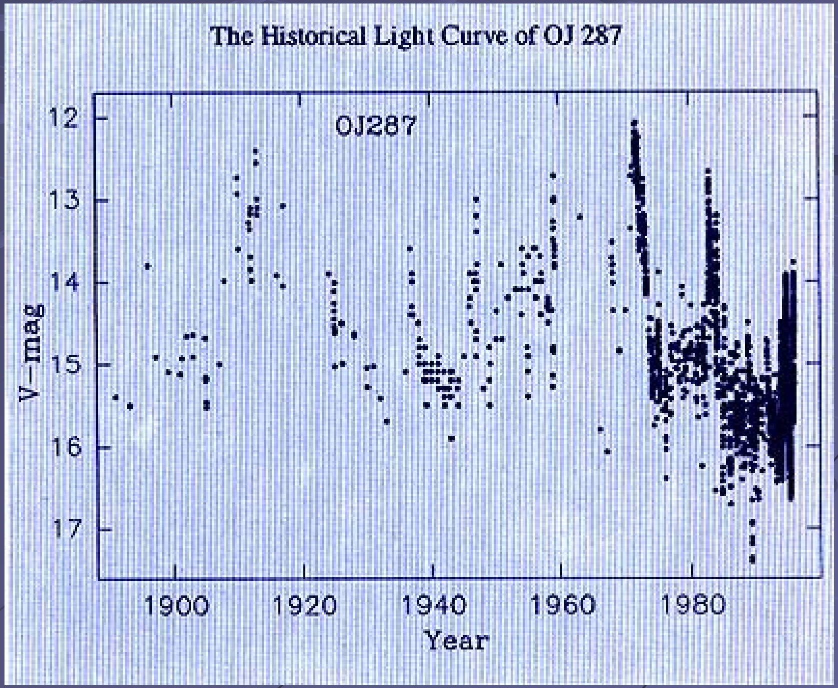
*Slide: Miguel Preto*

# Observations of Binary Black Holes?

*Quasi-Periodic Variations, optical or Radio, sub-pc separation (10 yrs)*



Hardcastle, M. J., et al., 1996, MNRAS **278**, 273

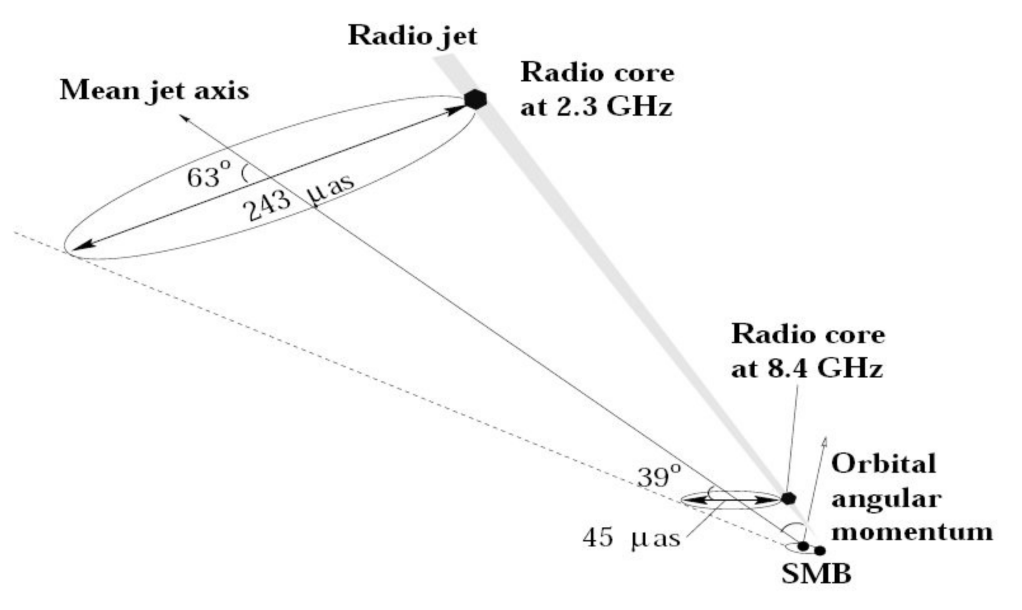


# Observations of Binary Black Holes?

*Orbital Motion of Radfio Core, with VLBI, < 0.01 pc, 1 yr*

Table 1: Parameters of the fitted orbital motion of the radio core in 3C 66B. The errors in the parameters correspond to a change of 1 in chi-square from the value at the best fitted parameter. The offset angle is the angle between the major axis and the point where the orbital motion started. R.A. center is the relative position in right ascension. Dec. center is the relative position in declination.

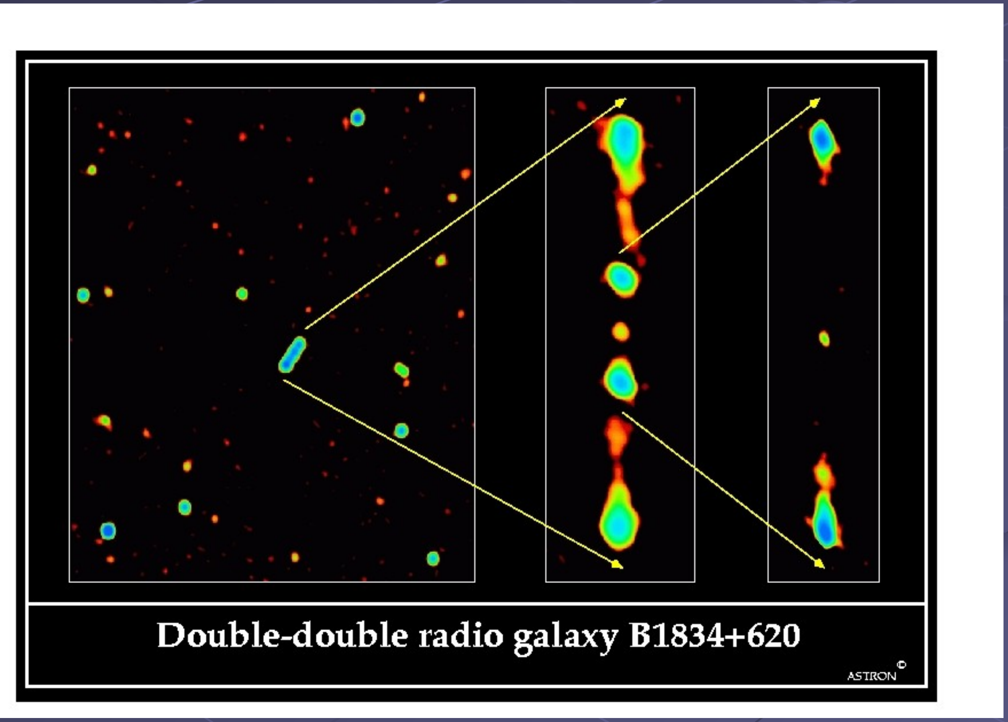
| | | 2.3 GHz | 8.4 GHz |
|---|---|---|---|
| Major axis | ($\mu$as) | 243±30 | 45±4 |
| Axial ratio | | 0.31±0.17 | 0.24±0.14 |
| Orbital period | (yr) | 1.10±0.06 | 1.02±0.04 |
| Position angle | (°) | 113±9 | 89±6 |
| Offset Angle | (°) | 60±7 | 101±5 |
| R.A. center | (mas) | 1.441±0.020 | 0.970±0.002 |
| Dec. center | (mas) | −0.888±0.033 | −1.861±0.004 |

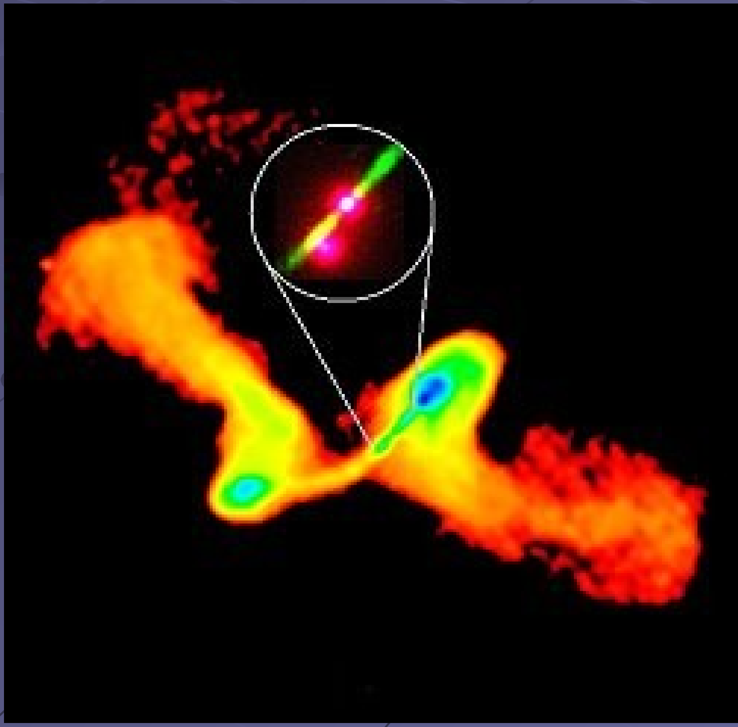



*Schoenemakers, 2000*

# Observations of Binary Black Holes?

*Left: double-double radio galaxies, clear out of inner disk, milliparsec, small mass ratio*
*Right: Jet Flipping, large mass ratio, depend on viscous time scales*
*(Theory: Liu 2004, Liu et al. 2006)*



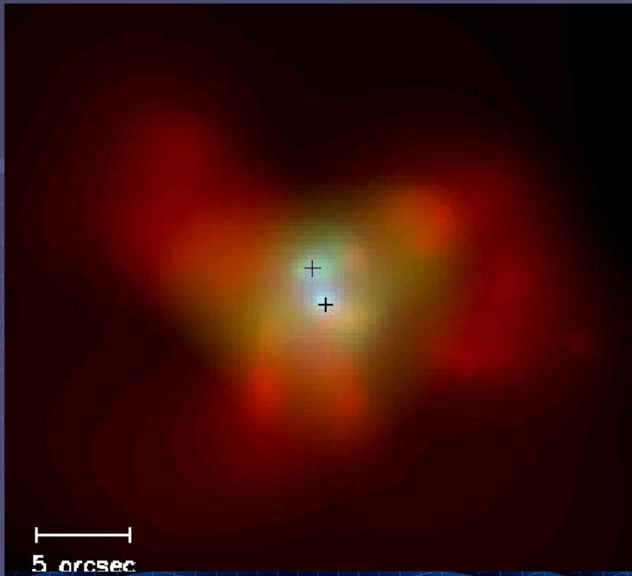



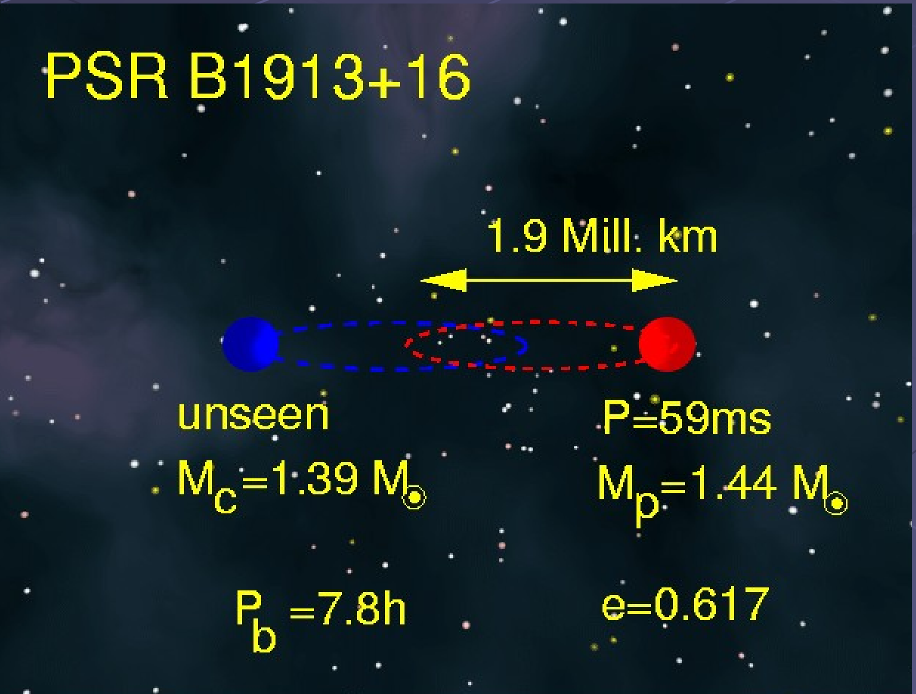
# Contents

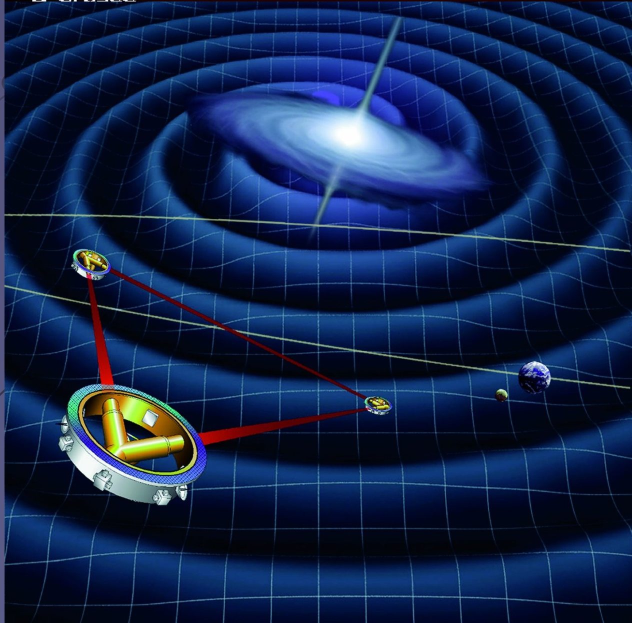
Colliding Galaxies NGC 4038 and NGC 4039 HST • WFPC2
PRC97-34a • ST ScI OPO • October 21, 1997 • B. Whitmore (ST ScI) and NASA

# Detection of Gravitational Waves?

Was Einstein right?

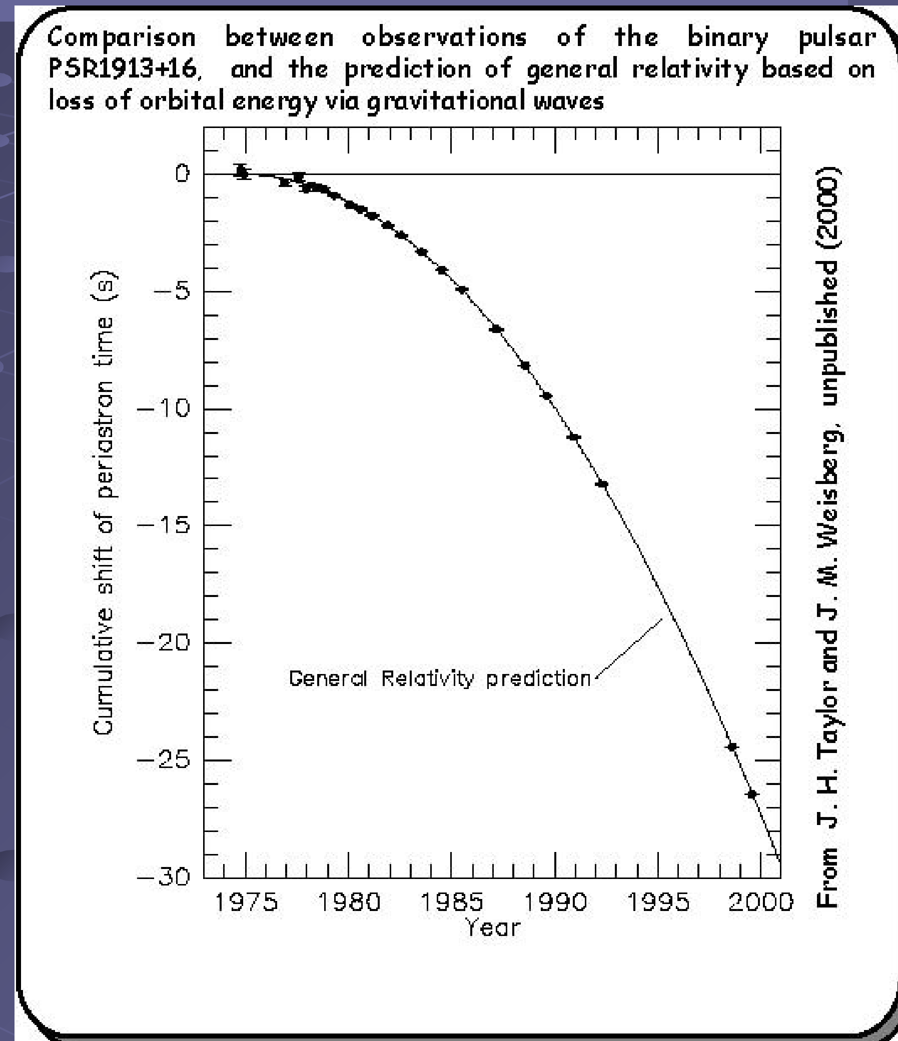


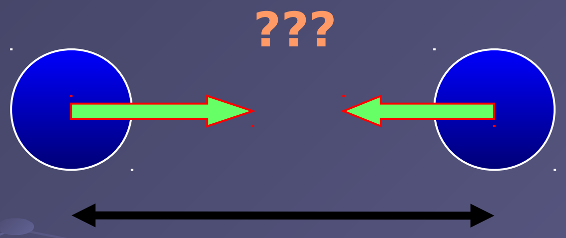



# Gravity Waves

Indirect Proof by Hulse and Taylor, binary pulsar (Nobel prize 1993)

# Post-Newtonian Dynamics




**BH mergers – conservative view:?**

**d ~10*R_BH**

$$R_{BH} = \frac{2 \cdot G \cdot M_{BH}}{c^2} \approx 10^{-7} \cdot \left( \frac{M_{BH}}{10^6 \cdot M_o} \right) [\mathrm{pc}]$$

**If we scaled up our numerical results, for the typical galaxy bulge (~10^9 Mo & ~3 kpc: 10 Gyr = 130) we see that the BH's separation never come closer ~1 – 0.1 pc…**

**For the typical BH's mass (10^6 Mo) the "gravitational merging" regime start with ~10^-6 pc!!!**

# Post-Newtonian Dynamics

Two approaches: 1. Quasi-Newtonian Source
compute far-field grav. radiation,
Einstein quadrupole formula, Landau-Lifschitz.
from energy loss compute orbital change

$$h_{ij}^{\rm TT} = \frac{2G}{c^4 R}\mathcal{P}_{ijab}(\mathbf{N})\left\{\frac{d^2{\rm Q}_{ab}}{dT^2}(T - R/c) + \mathcal{O}\left(\frac{1}{c}\right)\right\} + \mathcal{O}\left(\frac{1}{R^2}\right), \qquad (2)$$

where $R = |\mathbf{X}|$ is the distance to the source, $\mathbf{N} = \mathbf{X}/R$ is the unit direction from the source to the observer, and $\mathcal{P}_{ijab} = \mathcal{P}_{ia}\mathcal{P}_{jb} - \frac{1}{2}\delta_{ij}\mathcal{P}_{ij}\mathcal{P}_{ab}$ is the TT projection operator, with $\mathcal{P}_{ij} = \delta_{ij} - N_i N_j$ being the projector onto the plane orthogonal to $\mathbf{N}$. The source's quadrupole moment takes the familiar Newtonian form

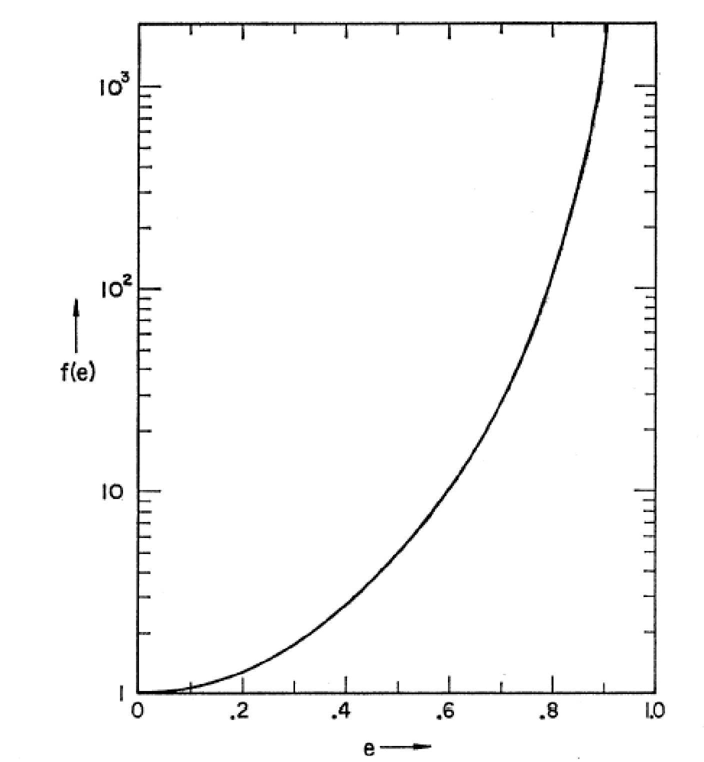
$${\rm Q}_{ij}(t) = \int_{\rm source} d^3\mathbf{x}\,\rho(\mathbf{x},t)\left(x_i x_j - \frac{1}{3}\delta_{ij}\mathbf{x}^2\right), \qquad (3)$$

where $\rho$ is the Newtonian mass density. The total gravitational power emitted by the source in all directions is given by the Einstein quadrupole formula

$$\mathcal{L} = \frac{G}{5c^5}\left\{\frac{d^3{\rm Q}_{ab}}{dT^3}\frac{d^3{\rm Q}_{ab}}{dT^3} + \mathcal{O}\left(\frac{1}{c^2}\right)\right\}. \qquad (4)$$

Pioneers: Peters & Mathews 1963, Peters 1964 Tyson & Giffard 1978

# Post-Newtonian Dynamics




initial orbital ecc. determines ecc. at final merger;
waveforms and emitted frequency of G.W. changes!

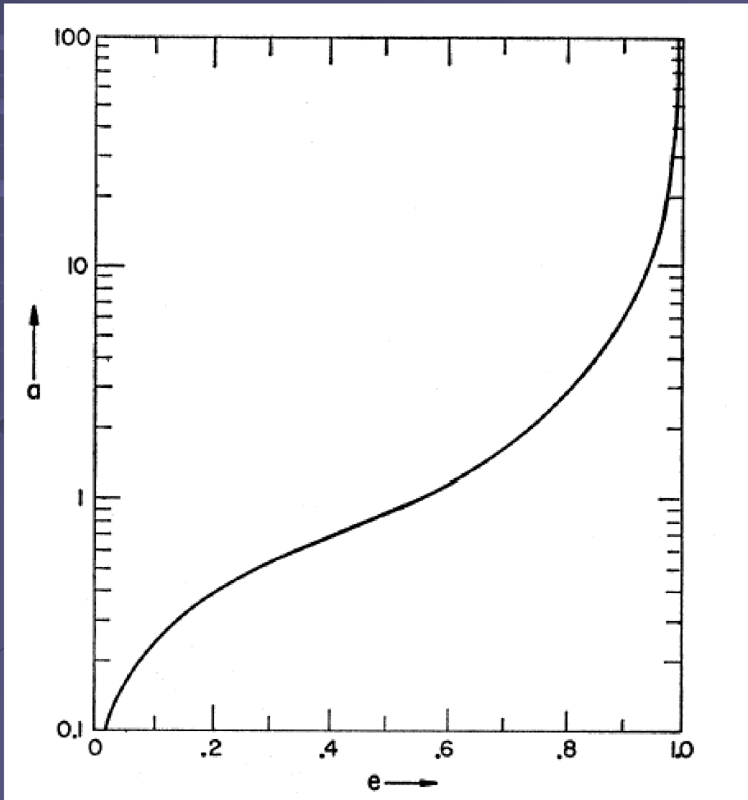



Fig. 1. The semimajor axis $a$ as a function of the eccentricity $e$ in the decay of a two-point mass system. Here, $c_0$ is chosen to be 1.

Peters, P.C., Phys. Rev. 1964, 136, 1224
Peters, P.C., Mathews, J., 1963, Phys. Rev. 131, 435

$$\frac{da}{dt} = -\frac{64G^3 m_1 m_2(m_1 + m_2)}{5c^5 a^3(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

$$\frac{de}{dt} = -\frac{304G^3 m_1 m_2(m_1 + m_2)}{15c^5 a^4(1-e^2)^{5/2}}\, e\left(1 + \frac{121}{304}e^2\right),$$

# Post-Newtonian Dynamics

Method A: use geodetic equations, harmonic gauge, directly obtain eqs. of motion (Blanchet et al., Living Reviews)
Method B: Hamiltonian approach using ADM gauge (Schaefer et al.)

A and B equivalent till PN2.5 (1/c**5), higher order gauge functions appear.

$$\frac{dv^i}{dt} = -\frac{Gm}{r^2}\left[(1+\mathcal{A})\, n^i + \mathcal{B}\, v^i\right] + \mathcal{O}\left(\frac{1}{c^8}\right), \qquad (181)$$

and find [43] that the coefficients $\mathcal{A}$ and $\mathcal{B}$ are

$$\begin{aligned}\mathcal{A} = {} & \frac{1}{c^2}\left\{-\frac{3\dot{r}^2\nu}{2} + v^2 + 3\nu v^2 - \frac{Gm}{r}(4+2\nu)\right\} \\ & + \frac{1}{c^4}\left\{\frac{15\dot{r}^4\nu}{8} - \frac{45\dot{r}^4\nu^2}{8} - \frac{9\dot{r}^2\nu v^2}{2} + 6\dot{r}^2\nu^2 v^2 + 3\nu v^4 - 4\nu^2 v^4 \right. \\ & \qquad \left. + \frac{Gm}{r}\left(-2\dot{r}^2 - 25\dot{r}^2\nu - 2\dot{r}^2\nu^2 - \frac{13\nu v^2}{2} + 2\nu^2 v^2\right) + \frac{G^2m^2}{r^2}\left(9 + \frac{87\nu}{4}\right)\right\} \\ & + \frac{1}{c^5}\left\{-\frac{24\dot{r}\nu v^2}{5}\frac{Gm}{r} - \frac{136\dot{r}\nu}{15}\frac{G^2m^2}{r^2}\right\}\end{aligned}$$

# Post-Newtonian Dynamics

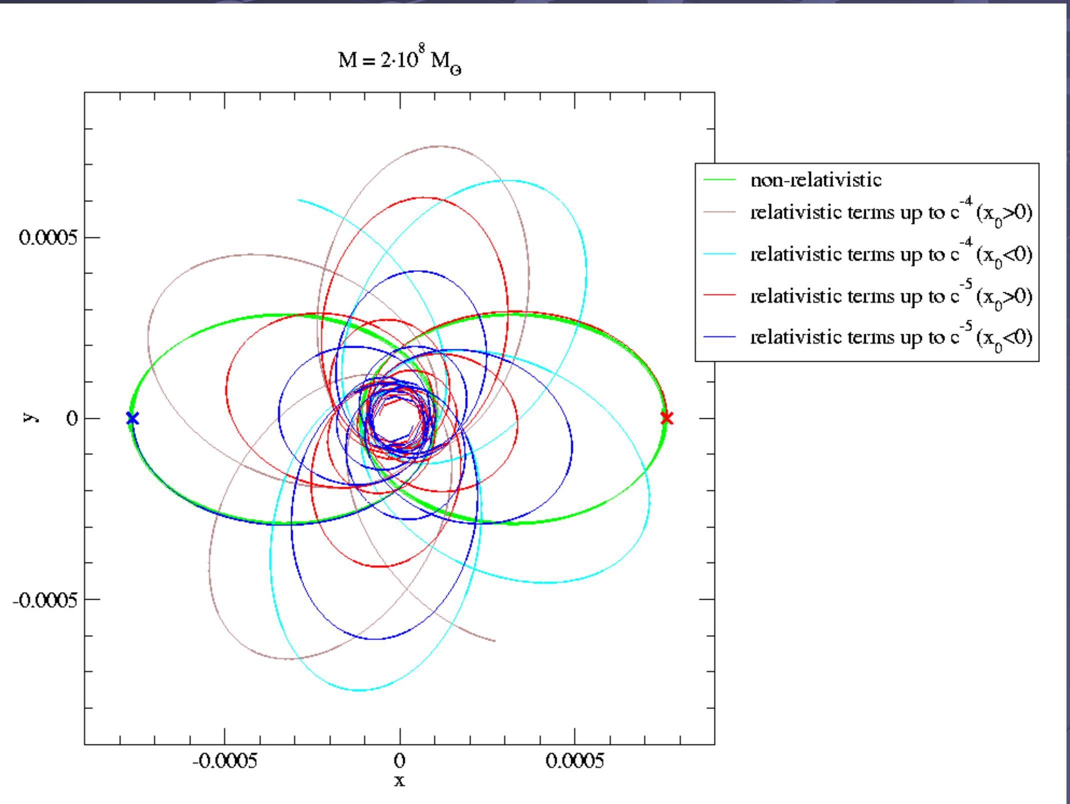
$$
\begin{aligned}
&+\frac{1}{c^6}\Bigg\{-\frac{35\dot{r}^6\nu}{16}+\frac{175\dot{r}^6\nu^2}{16}-\frac{175\dot{r}^6\nu^3}{16}+\frac{15\dot{r}^4\nu v^2}{2}-\frac{135\dot{r}^4\nu^2 v^2}{4}+\frac{255\dot{r}^4\nu^3 v^2}{8}\\
&\quad-\frac{15\dot{r}^2\nu v^4}{2}+\frac{237\dot{r}^2\nu^2 v^4}{8}-\frac{45\dot{r}^2\nu^3 v^4}{2}+\frac{11\nu v^6}{4}-\frac{49\nu^2 v^6}{4}+13\nu^3 v^6\\
&\quad+\frac{Gm}{r}\Bigg(79\dot{r}^4\nu-\frac{69\dot{r}^4\nu^2}{2}-30\dot{r}^4\nu^3-121\dot{r}^2\nu v^2+16\dot{r}^2\nu^2 v^2+20\dot{r}^2\nu^3 v^2+\frac{75\nu v^4}{4}\\
&\qquad+8\nu^2 v^4-10\nu^3 v^4\Bigg)\\
&\quad+\frac{G^2m^2}{r^2}\Bigg(\dot{r}^2+\frac{32573\dot{r}^2\nu}{168}+\frac{11\dot{r}^2\nu^2}{8}-7\dot{r}^2\nu^3+\frac{615\dot{r}^2\nu\pi^2}{64}-\frac{26987\nu v^2}{840}+\nu^3 v^2\\
&\qquad-\frac{123\nu\pi^2 v^2}{64}-110\dot{r}^2\nu\ln\left(\frac{r}{r_0'}\right)+22\nu v^2\ln\left(\frac{r}{r_0'}\right)\Bigg)\\
&\quad+\frac{G^3m^3}{r^3}\left(-16-\frac{437\nu}{4}-\frac{71\nu^2}{2}+\frac{41\nu\pi^2}{16}\right)\Bigg\}\\
&+\frac{1}{c^7}\Bigg\{\frac{Gm}{r}\left(\frac{366}{35}\nu v^4+12\nu^2 v^4-114v^2\nu\dot{r}^2-12\nu^2 v^2\dot{r}^2+112\nu\dot{r}^4\right)\\
&\quad+\frac{G^2m^2}{r^2}\left(\frac{692}{35}\nu v^2-\frac{724}{15}v^2\nu^2+\frac{294}{5}\nu\dot{r}^2+\frac{376}{5}\nu^2\dot{r}^2\right)\\
&\quad+\frac{G^3m^3}{r^3}\left(\frac{3956}{35}\nu+\frac{184}{5}\nu^2\right)\Bigg\},
\end{aligned}
\qquad (182)
$$

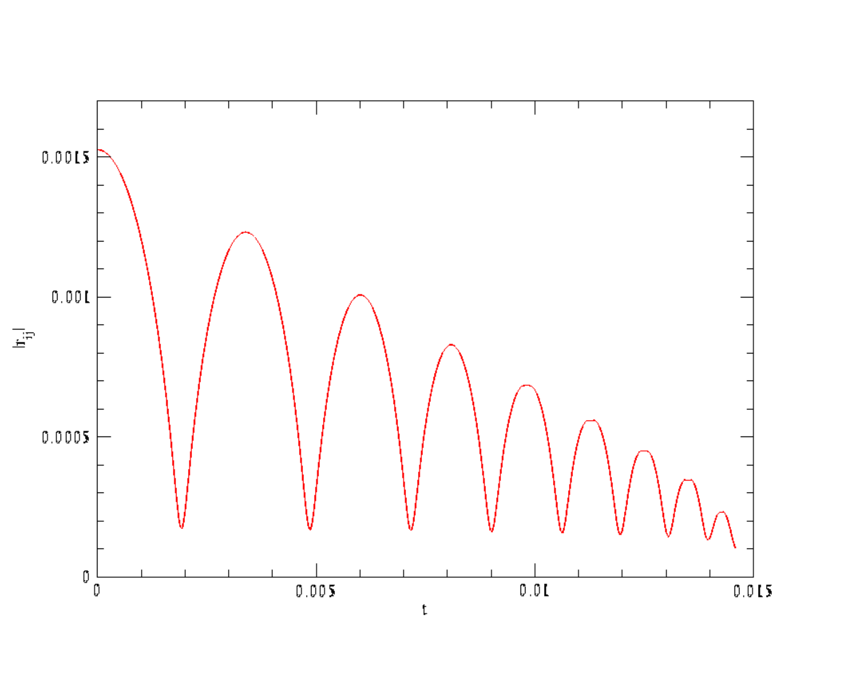
# Post-Newtonian Dynamics

$$
\begin{aligned}
\mathcal{B} = {} & \frac{1}{c^2}\left\{-4\dot{r} + 2\dot{r}\nu\right\} \\
& + \frac{1}{c^4}\left\{\frac{9\dot{r}^3\nu}{2} + 3\dot{r}^3\nu^2 - \frac{15\dot{r}\nu v^2}{2} - 2\dot{r}\nu^2 v^2 + \frac{Gm}{r}\left(2\dot{r} + \frac{41\dot{r}\nu}{2} + 4\dot{r}\nu^2\right)\right\} \\
& + \frac{1}{c^5}\left\{\frac{8\nu v^2}{5}\frac{Gm}{r} + \frac{24\nu}{5}\frac{G^2m^2}{r^2}\right\} \\
& + \frac{1}{c^6}\left\{-\frac{45\dot{r}^5\nu}{8} + 15\dot{r}^5\nu^2 + \frac{15\dot{r}^5\nu^3}{4} + 12\dot{r}^3\nu v^2 - \frac{111\dot{r}^3\nu^2 v^2}{4} - 12\dot{r}^3\nu^3 v^2 - \frac{65\dot{r}\nu v^4}{8} \right. \\
& \qquad + 19\dot{r}\nu^2 v^4 + 6\dot{r}\nu^3 v^4 \\
& \qquad + \frac{Gm}{r}\left(\frac{329\dot{r}^3\nu}{6} + \frac{59\dot{r}^3\nu^2}{2} + 18\dot{r}^3\nu^3 - 15\dot{r}\nu v^2 - 27\dot{r}\nu^2 v^2 - 10\dot{r}\nu^3 v^2\right) \\
& \qquad \left. + \frac{G^2m^2}{r^2}\left(-4\dot{r} - \frac{18169\dot{r}\nu}{840} + 25\dot{r}\nu^2 + 8\dot{r}\nu^3 - \frac{123\dot{r}\nu\pi^2}{32} + 44\dot{r}\nu\ln\left(\frac{r}{r'_0}\right)\right)\right\} \\
& + \frac{1}{c^7}\left\{\frac{Gm}{r}\left(-\frac{626}{35}\nu v^4 - \frac{12}{5}\nu^2 v^4 + \frac{678}{5}\nu v^2\dot{r}^2 + \frac{12}{5}\nu^2 v^2\dot{r}^2 - 120\nu\dot{r}^4\right) \right. \\
& \qquad + \frac{G^2m^2}{r^2}\left(\frac{164}{21}\nu v^2 + \frac{148}{5}\nu^2 v^2 - \frac{82}{3}\nu\dot{r}^2 - \frac{848}{15}\nu^2\dot{r}^2\right) \\
& \qquad \left. + \frac{G^3m^3}{r^3}\left(-\frac{1060}{21}\nu - \frac{104}{5}\nu^2\right)\right\}. \qquad (183)
\end{aligned}
$$

# Post-Newtonian Dynamics

What happens afterwards? Post-Newton Order „2.5"...



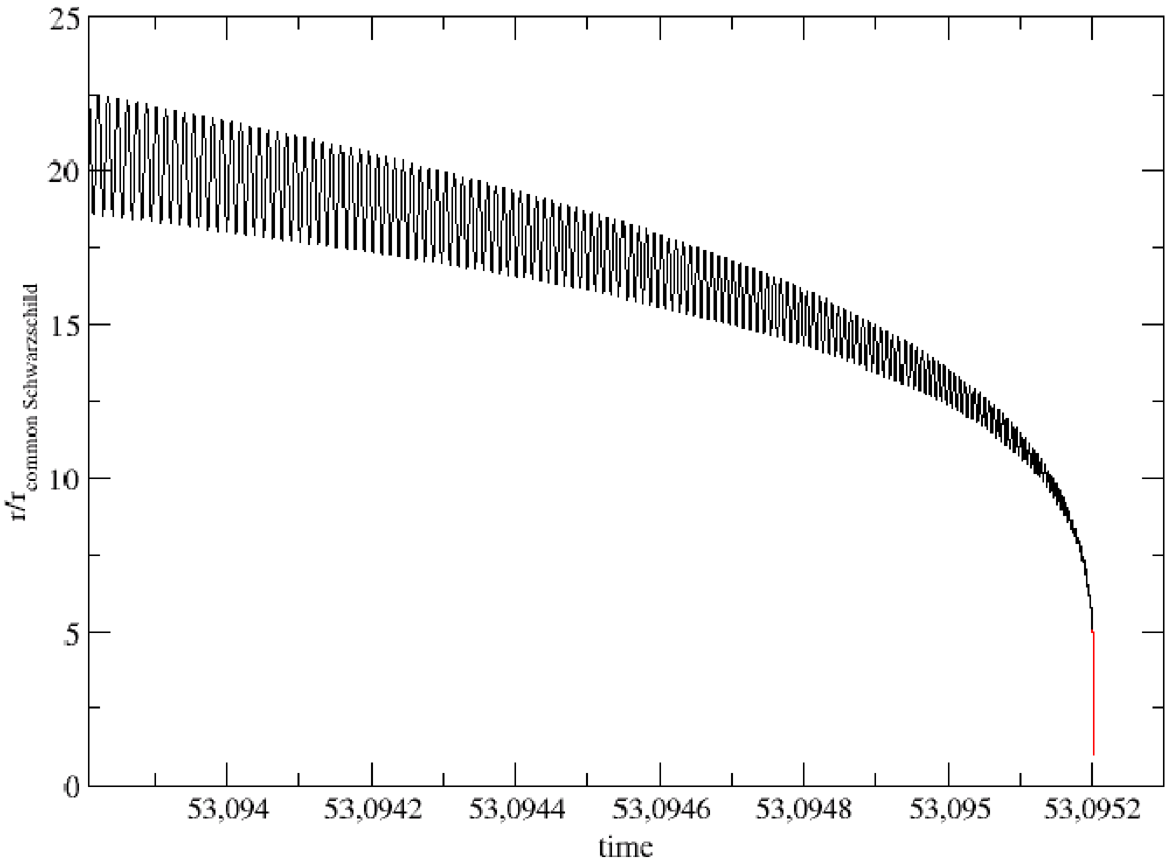

Kupi, Amaro-Seoane & Spurzem 2006

<u>Final Ring-Down and Merger ⇒ grav. wave signal!</u>

$t_{GW} = 1.1 \cdot 10^6$ a $M_8^{-5/3}$ $(P/\mathrm{a})^{8/3}$ $(1+q)^2/q$ (Jaffe & Backer 2003)

# Post-Newtonian Dynamics

*A complete inspiral in NBODY6++…*

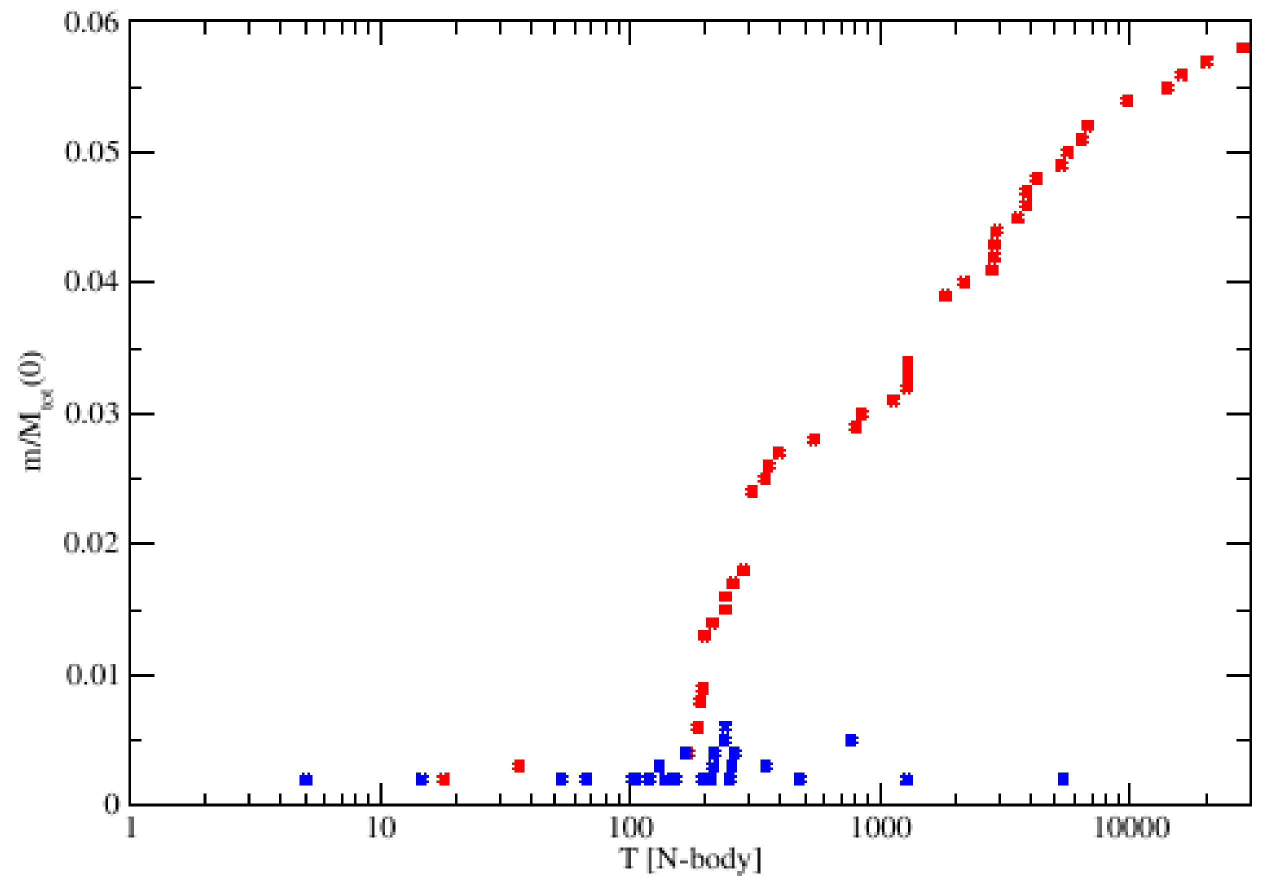



*Kupi, Amaro-Seoane, Spurzem, MNRAS 2006*

# Post-Newtonian Dynamics

Runaway Growth due to GR induced BH merging…
…. high ecc. mergers… we do all in Post-Newtonian…

Kupi & Amaro-Seoane & Spurzem 2006



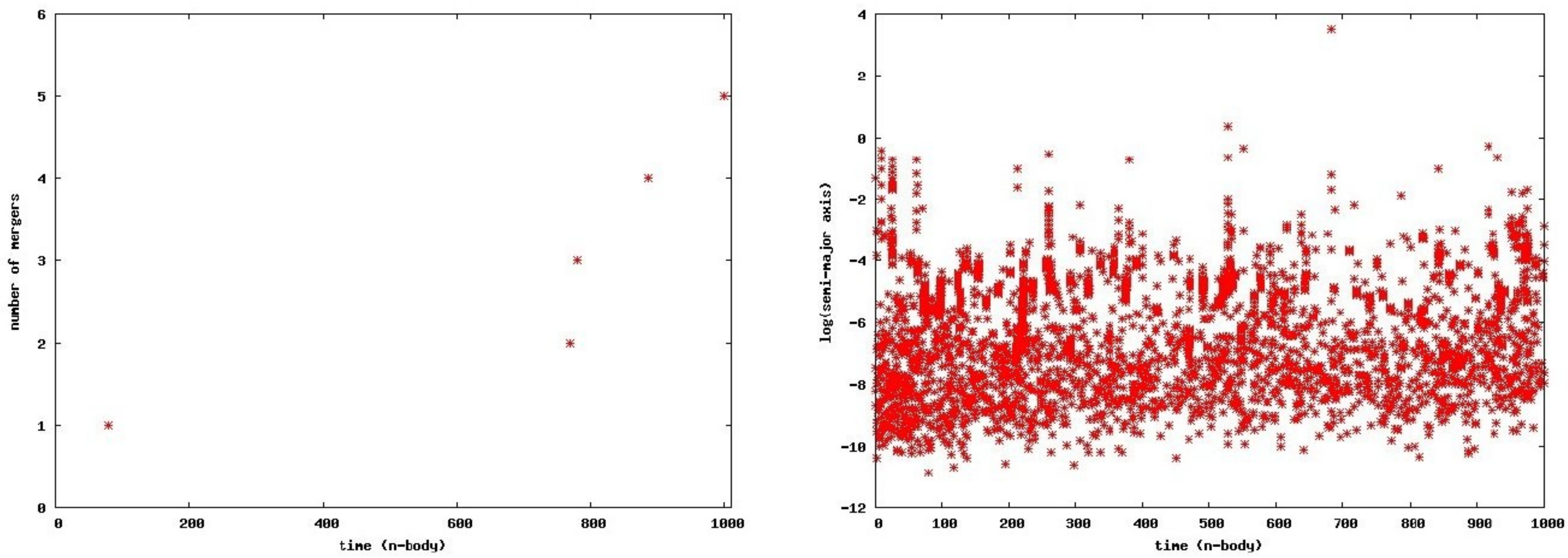

Cluster of 1000 very massive black Holes.

Gravitational Radiation Feedback in Dynamics of N-Body System

# Direct N-Body

Relativistic Compact Objects in Dense Star Clusters:

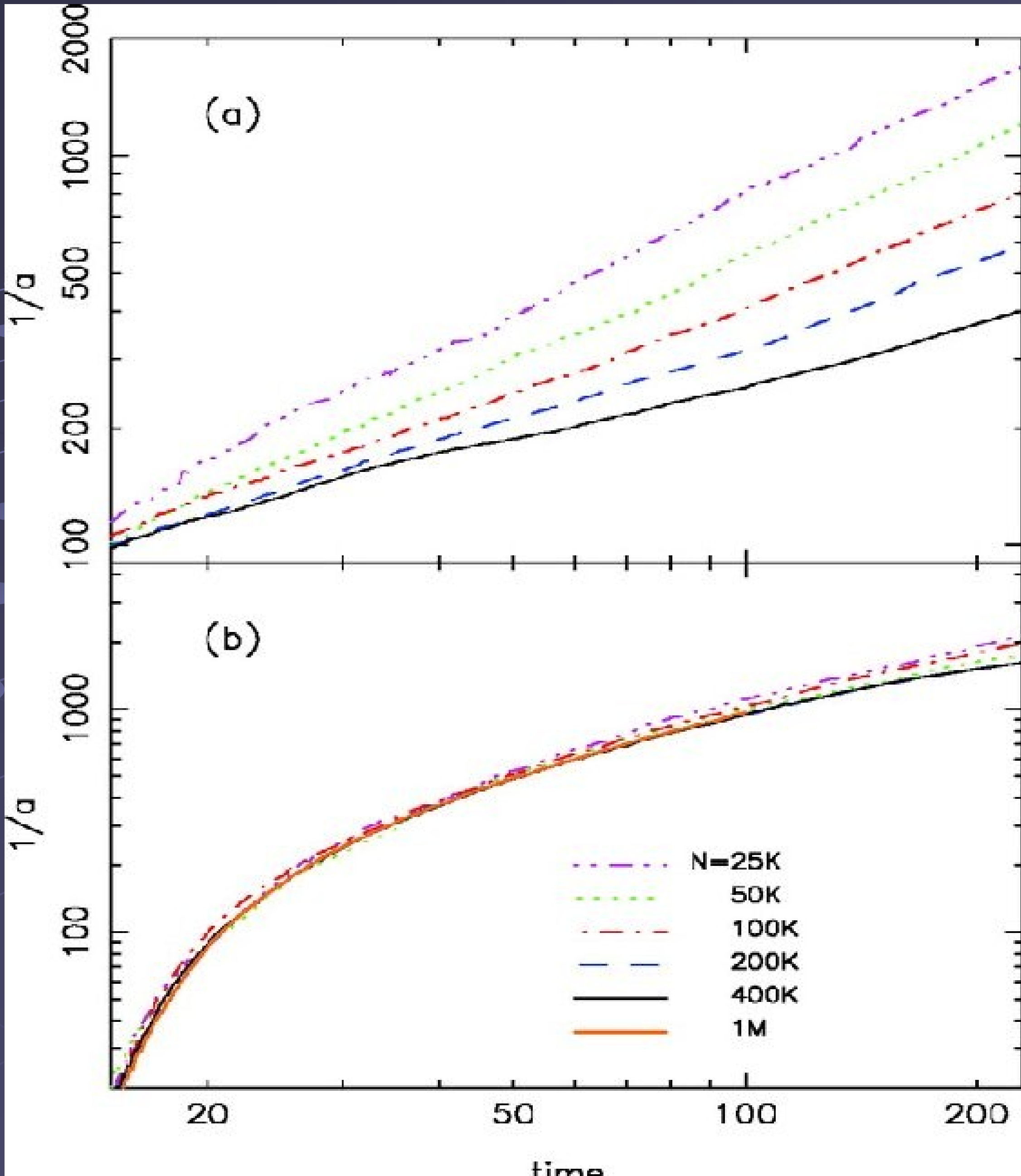
***by Jonathan Downing***




On the left are the total number of relativistic mergers as a function of time. This data us useful for providing event rates. On the left are the semi-major axes of individual relativistic binaries as a function of time. By producing orbital parameters we are able to provide initial conditions for gravitational wave templates and for full numerical simulations of relativistic mergers.

right

Returning to
Classical Dynamics:



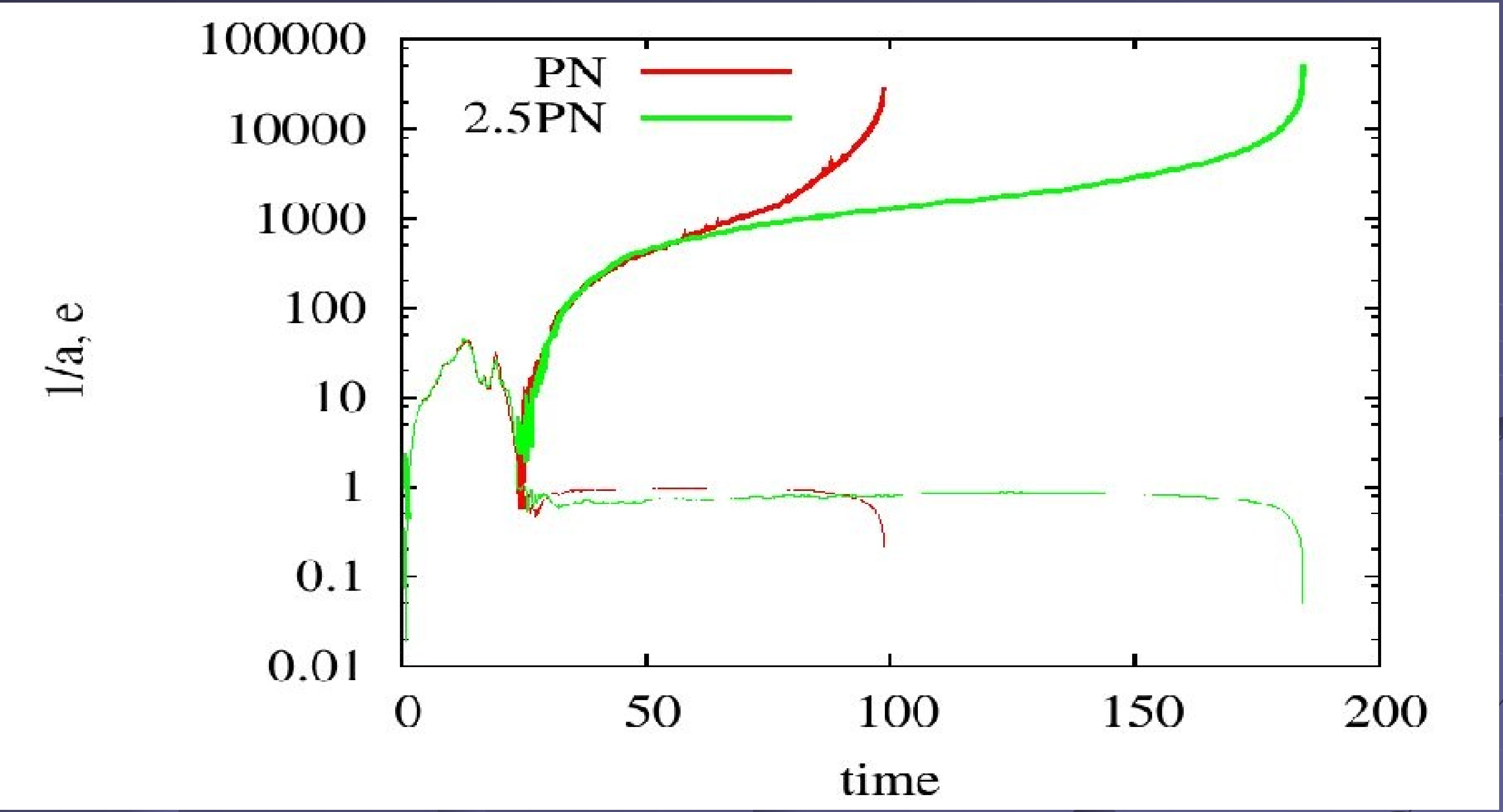

*Spherical Systems:*
*BH binary stalls with N*

*Axisymmetric,*
*Rotating systems:*
*No stalling observed*

Berczik, Merritt, Spurzem, 2005, ApJ
Berczik, Merritt, Spurzem, Bischof,
2006, ApJ

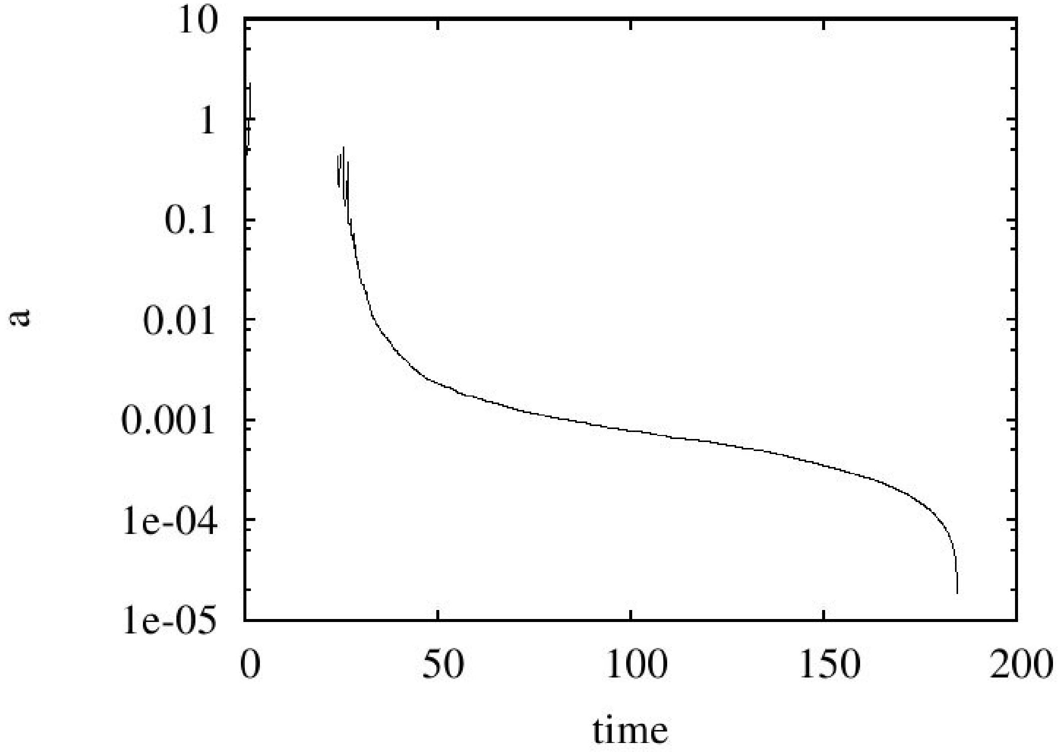
*Movie!*

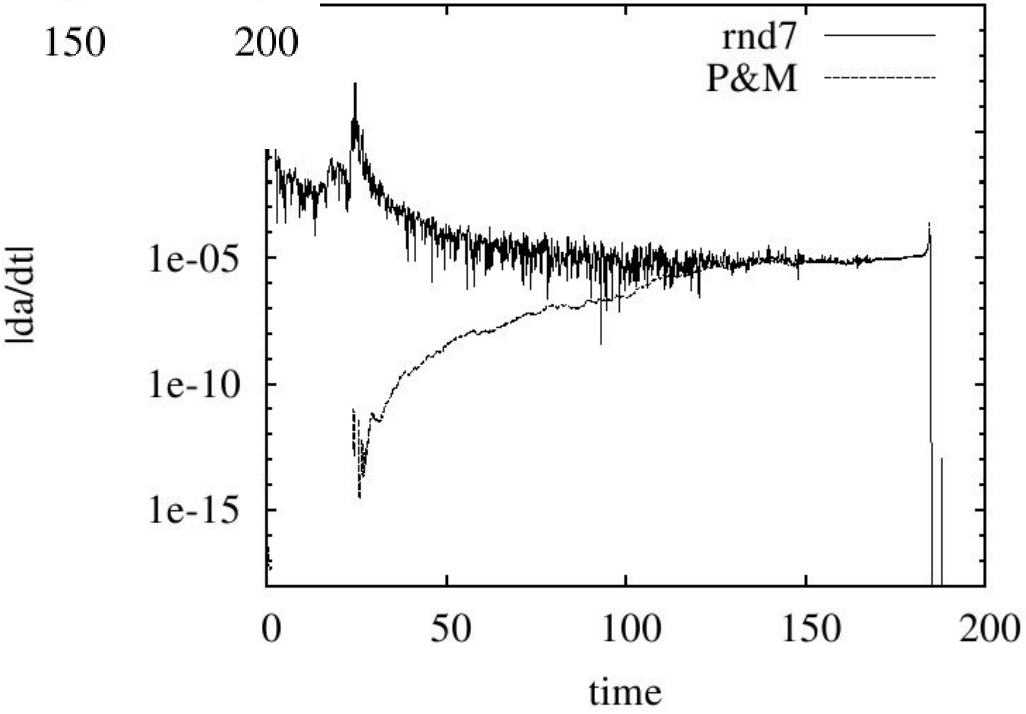
# Post-Newtonian Dynamics




Berentzen, Preto, Berczik, Merritt, Spurzem 2008

# Post-Newtonian Dynamics

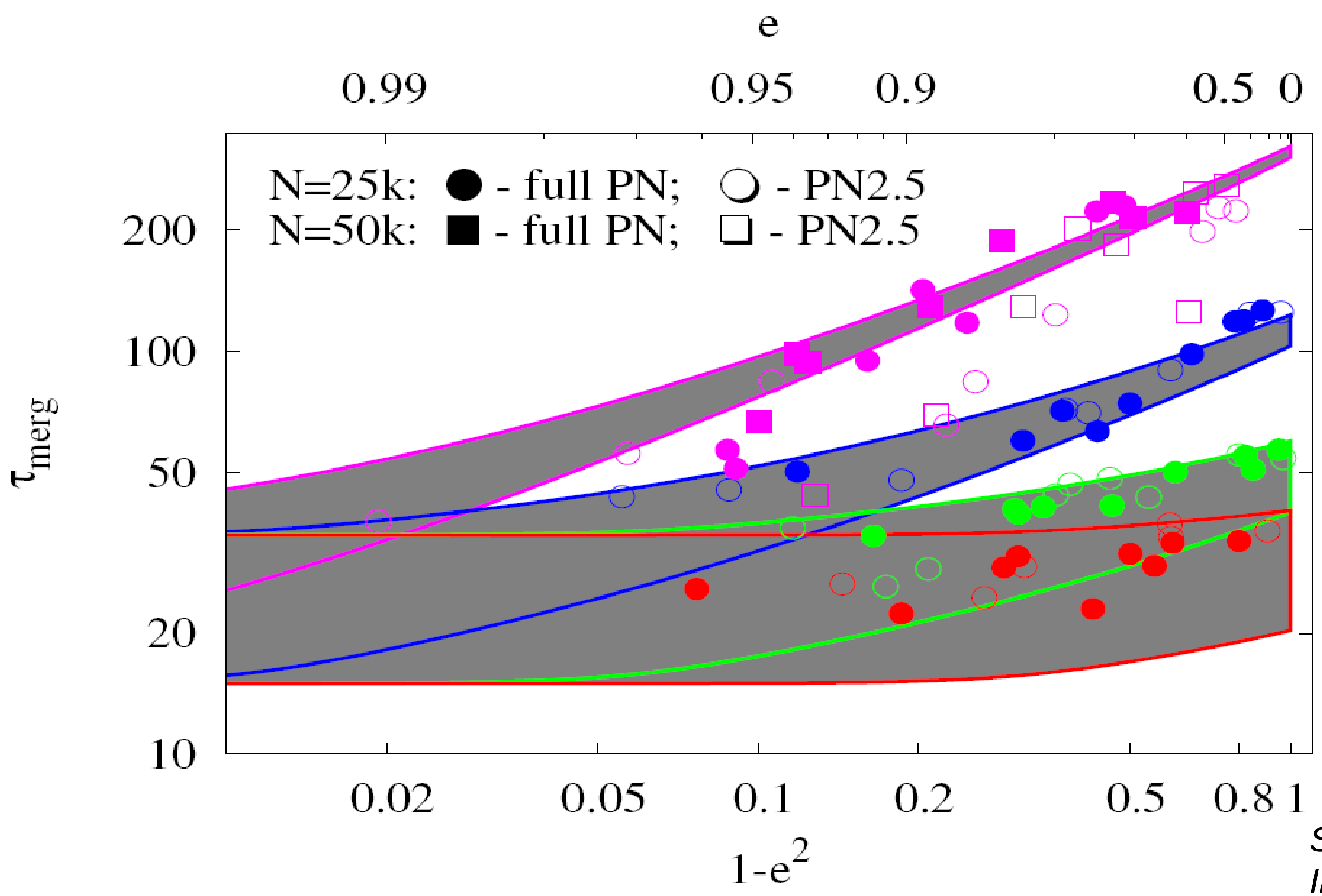



Berentzen, Berczik, Merritt, Spurzem, Preto
2008, to be subm. ApJ

Binary Black Hole
with full PN treatment

# Post-Newtonian Dynamics




*Slide: Ingo Berentzen*

# Post-Newtonian Dynamics

## Spin-Orbit Interaction S / Spin-Spin SS

Note: $S^{true} = m\, a\, v_{spin}$

$S = c\, S^{true}$

Maximally rotating body: $a \sim Gm/c^2$ ; $v_{sprin} = c$

$S^{true} \sim Gm^2/c$ ; S contains no c-Power

If not maximally rotating, PN order's change.

$$\frac{d\mathbf{v}_1}{dt} = \mathbf{A}_{\mathrm{N}} + \frac{1}{c^2}\mathbf{A}_{1\mathrm{PN}} + \frac{1}{c^3}\underset{\mathrm{S}}{\mathbf{A}}_{1.5\mathrm{PN}} + \frac{1}{c^4}\left[\mathbf{A}_{2\mathrm{PN}} + \underset{\mathrm{SS}}{\mathbf{A}}_{2\mathrm{PN}}\right] + \frac{1}{c^5}\left[\mathbf{A}_{2.5\mathrm{PN}} + \underset{\mathrm{S}}{\mathbf{A}}_{2.5\mathrm{PN}}\right] + \mathcal{O}\left(\frac{1}{c^6}\right). \qquad (5.1)$$

Faye, Blanchet, Buonanno 2006

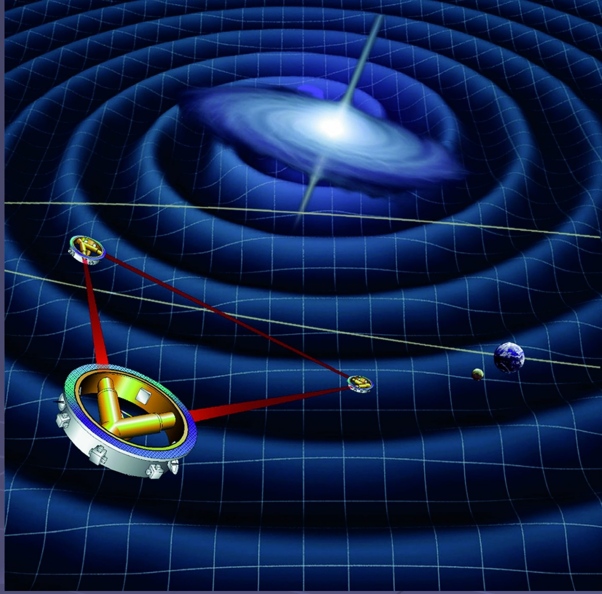
$$\underset{\mathrm{S}}{\mathbf{A}}_{1.5\mathrm{PN}} = \frac{Gm_2}{r_{12}^3}\left\{\left[6\frac{(S_1, n_{12}, v_{12})}{m_1} + 6\frac{(S_2, n_{12}, v_{12})}{m_2}\right]\mathbf{n}_{12} + 3(n_{12}v_{12})\frac{\mathbf{n}_{12}\times\mathbf{S}_1}{m_1} + 6(n_{12}v_{12})\frac{\mathbf{n}_{12}\times\mathbf{S}_2}{m_2} - 3\frac{\mathbf{v}_{12}\times\mathbf{S}_1}{m_1} - 4\frac{\mathbf{v}_{12}\times\mathbf{S}_2}{m_2}\right\}. \qquad (5.3a)$$

# Spin-Orbit Interaction S / Spin-Spin SS



$$\mathbf{A}^{2.5\mathrm{PN}}_{\mathrm{S}} = \frac{Gm_2}{r_{12}^3}\Big\{\mathbf{n}_{12}\Big[-6(n_{12},v_1,v_2)\Big(\frac{(v_1S_1)}{m_1}+\frac{(v_2S_2)}{m_2}\Big) - \frac{(S_1,n_{12},v_{12})}{m_1}\Big(15(n_{12}v_2)^2+6(v_{12}v_2)+26\frac{Gm_1}{r_{12}}+18\frac{Gm_2}{r_{12}}\Big)$$
$$-\frac{(S_2,n_{12},v_{12})}{m_2}\Big(15(n_{12}v_2)^2+6(v_{12}v_2)+\frac{49}{2}\frac{Gm_1}{r_{12}}+20\frac{Gm_2}{r_{12}}\Big)\Big]+\mathbf{v}_1\Big[-3\frac{(S_1,n_{12},v_1)}{m_1}((n_{12}v_1)+(n_{12}v_2))$$
$$+6(n_{12}v_1)\frac{(S_1,n_{12},v_2)}{m_1}-3\frac{(S_1,v_1,v_2)}{m_1}-6(n_{12}v_1)\frac{(S_2,n_{12},v_1)}{m_2}+\frac{(S_2,n_{12},v_2)}{m_2}(12(n_{12}v_1)-6(n_{12}v_2))$$
$$-4\frac{(S_2,v_1,v_2)}{m_2}\Big]+\mathbf{v}_2\Big[6(n_{12}v_1)\frac{(S_1,n_{12},v_{12})}{m_1}+6(n_{12}v_1)\frac{(S_2,n_{12},v_{12})}{m_2}\Big]-\mathbf{n}_{12}\times\mathbf{v}_1\Big[3(n_{12}v_{12})\frac{(v_1S_1)}{m_1}$$
$$+4\frac{Gm_1}{r_{12}}\frac{(n_{12}S_2)}{m_2}\Big]-\mathbf{n}_{12}\times\mathbf{v}_2\Big[6(n_{12}v_{12})\frac{(v_2S_2)}{m_2}-4\frac{Gm_1}{r_{12}}\frac{(n_{12}S_2)}{m_2}\Big]+\mathbf{v}_1\times\mathbf{v}_2\Big[3\frac{(v_1S_1)}{m_1}+4\frac{(v_2S_2)}{m_2}\Big]$$
$$+\frac{\mathbf{n}_{12}\times\mathbf{S}_1}{m_1}\Big[-\frac{15}{2}(n_{12}v_{12})(n_{12}v_2)^2+3(n_{12}v_2)(v_{12}v_2)-14\frac{Gm_1}{r_{12}}(n_{12}v_{12})-9\frac{Gm_2}{r_{12}}(n_{12}v_{12})\Big]$$
$$+\frac{\mathbf{n}_{12}\times\mathbf{S}_2}{m_2}\Big[-15(n_{12}v_{12})(n_{12}v_2)^2-6(n_{12}v_1)(v_{12}v_2)+12(n_{12}v_2)(v_{12}v_2)+\frac{Gm_1}{r_{12}}\Big(-\frac{35}{2}(n_{12}v_1)+\frac{39}{2}(n_{12}v_2)\Big)$$
$$-16\frac{Gm_2}{r_{12}}(n_{12}v_{12})\Big]+\frac{\mathbf{v}_{12}\times\mathbf{S}_1}{m_1}\Big[-3(n_{12}v_1)(n_{12}v_2)+\frac{15}{2}(n_{12}v_2)^2+\frac{G}{r_{12}}(14m_1+9m_2)+3(v_{12}v_2)\Big]$$
$$+\frac{\mathbf{v}_{12}\times\mathbf{S}_2}{m_2}\Big[6(n_{12}v_2)^2+4(v_{12}v_2)+\frac{23}{2}\frac{Gm_1}{r_{12}}+12\frac{Gm_2}{r_{12}}\Big]\Big\}. \qquad (5.3b)$$

# Contents



# Post-Newtonian Dynamics

## Observable Properties of Gravity Waves

- Luminosity ***L*** = (Asymm.) ***v***[10]

$$L=\frac{32}{5}\frac{G^4 M^5}{c^5 a^5 (1-e^2)^{7/2}}$$

- Frequency ***f = 2π/ T***
  - Orbital Frequency of System
  - Gravitaty Wave Frequency:
  - from ***f*** to about *10f*
  - circular orbit: *2f*

- Amplitude

*h*

- Polarisation
  - Two Polarisations in Einstein's Theory: + und x

*B.S. Sathyaprakash, Cardiff University, UK*

# Post-Newtonian Dynamics

## Gravity Waves

$$\frac{\tau_{GR}}{\tau_{orb}} = \left( \frac{\tau_{orb}}{t_{light}} \right)^5$$

$\tau_{GR}$ , $\tau_{orb}$ , $t_{light}$

timescale of GR orbit changes
orbital timescale
light crossing time across orbit

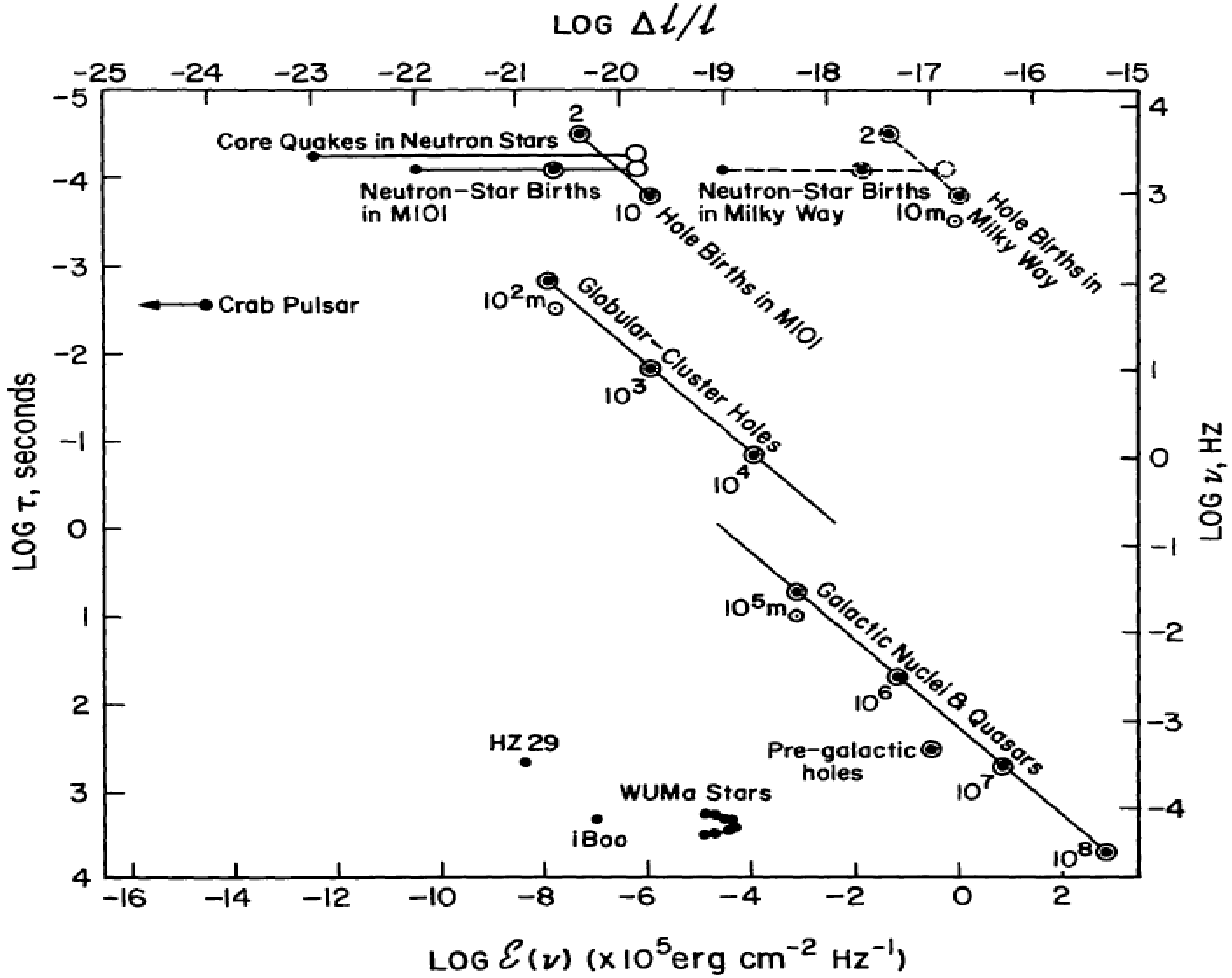
Estimates

$$\log \left( \frac{\tau_{orb}}{t_{light}} \right)^5$$

$\tau_{orb}$

| solar surface | white dwarf | neutron star | stellar mass black hole | supermassive black hole |
|---|---|---|---|---|
| 25<br>300 s | 20<br>0.3s | 15<br>0.001s | 10<br>$10^{-4}$ | 10<br>300s |

Sources In Luminosity

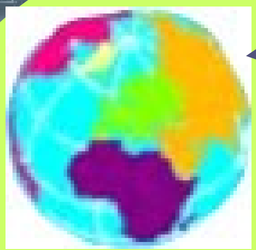



Tyson & Giffard Ann. Rev. Astr. Astroph. 1978 !!

*Figure 2* Numerical estimates of the strongest gravitational radiation signals that might be reaching the earth (from Thorne 1977). The strength of each signal is plotted as a function of its dominant spectral characteristics. Continuous monochromatic signals are denoted by dots corresponding to their dimensionless strain amplitudes. Dots surrounded by circles indicate the strain and spectral energy density of broad-band bursts with frequency ranging from 0.5 $\nu$ to 1.5 $\nu$ and duration $\tau \sim \frac{1}{2}\nu$. A damped ringing wave is denoted by a dot at the appropriate frequency giving the amplitude and an open-circle giving the total energy spectral density. With the exception of those connected by dotted lines, the distances chosen would result in a few events per year of the strength shown.

# LISA: Bin. Black Holes in the Universe
# Terrestrial Detectors: (VIRGO, GEO600, LIGO): Galactic Compact Objects (black holes, neutron stars…) higher frequencies


**Astrophysical Sources**



**Terrestrial Detectors**
**VIRGO, LIGO, TAMA, AIGO**

**Space detectors**
**LISA**

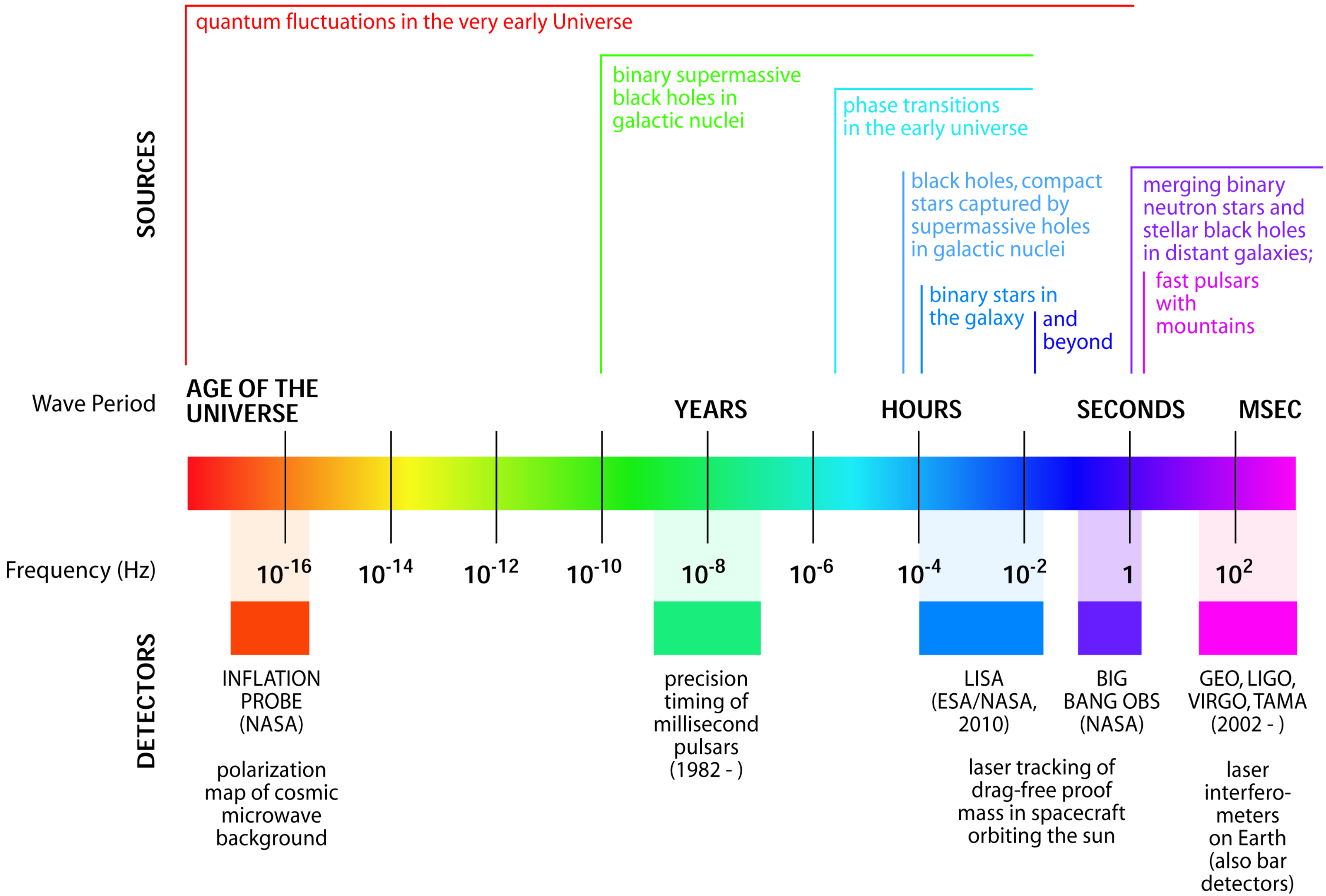

# Example: VIRGO Detector in Cascina near Pisa, Italy

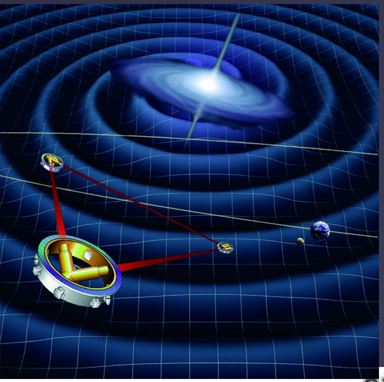
VIRGO – Pisa 3km
LIGO – Livingston, LA
Hanford, WA
4km
GEO600 – Hannover
600m
AIGO – Australien
(planned, 5 km)

http://www.ligo-la.caltech.edu/
http://www.ego-gw.it
http://www.geo600.uni-hannover.de

Outreach to 50 Millionen light years (Neutron Stars)

# THE GRAVITATIONAL WAVE SPECTRUM

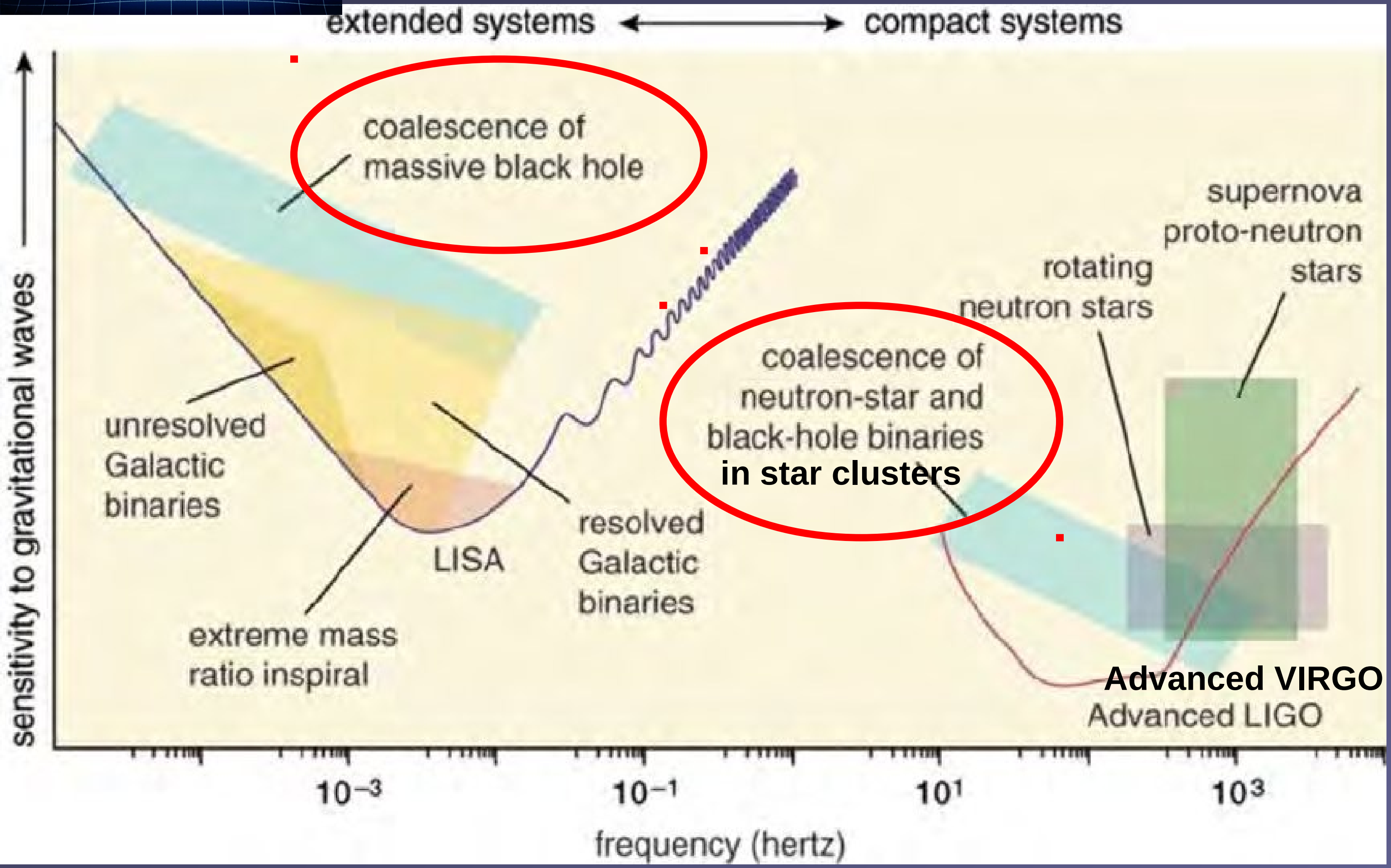


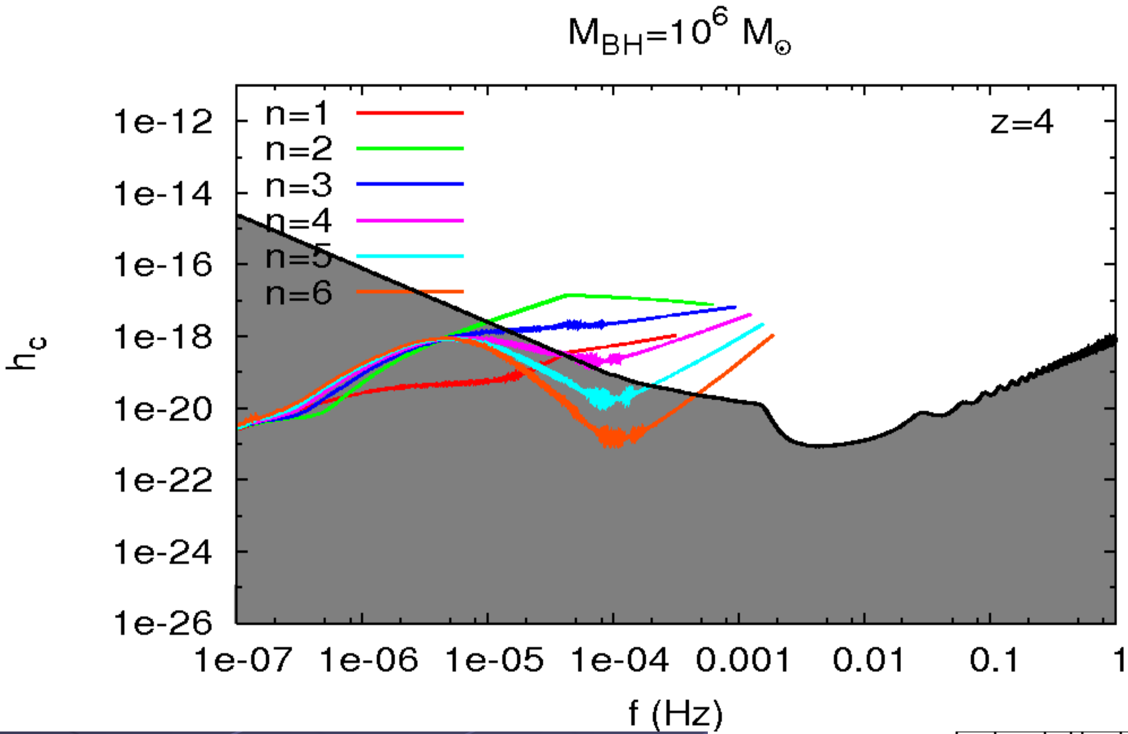


# Gravitational Wave Prediction from Black Holes in Galactic Nuclei and Star Clusters

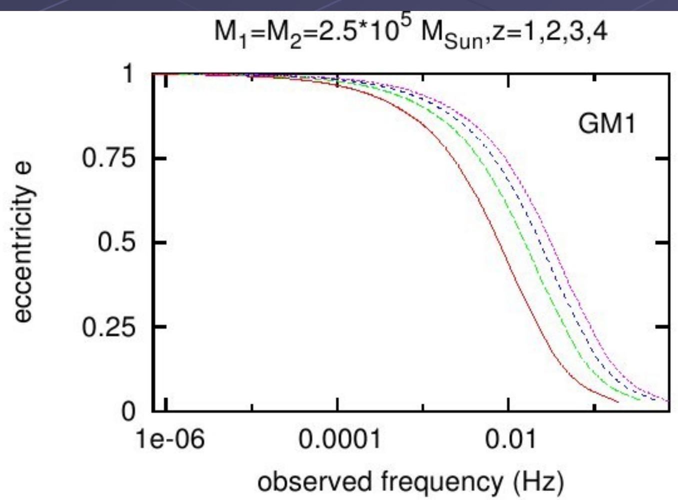
Astrophysical Objects in the realm of LISA (left) and VIRGO or LIGO (right)

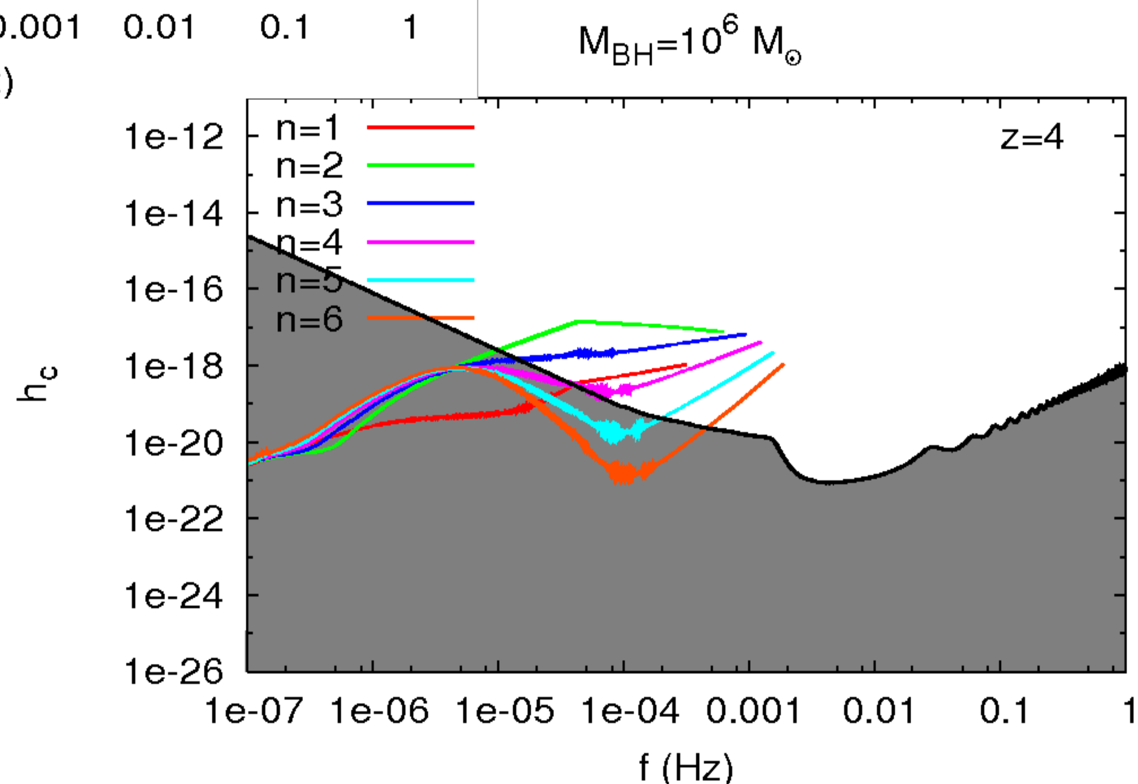
= activities with Nbody Simulations



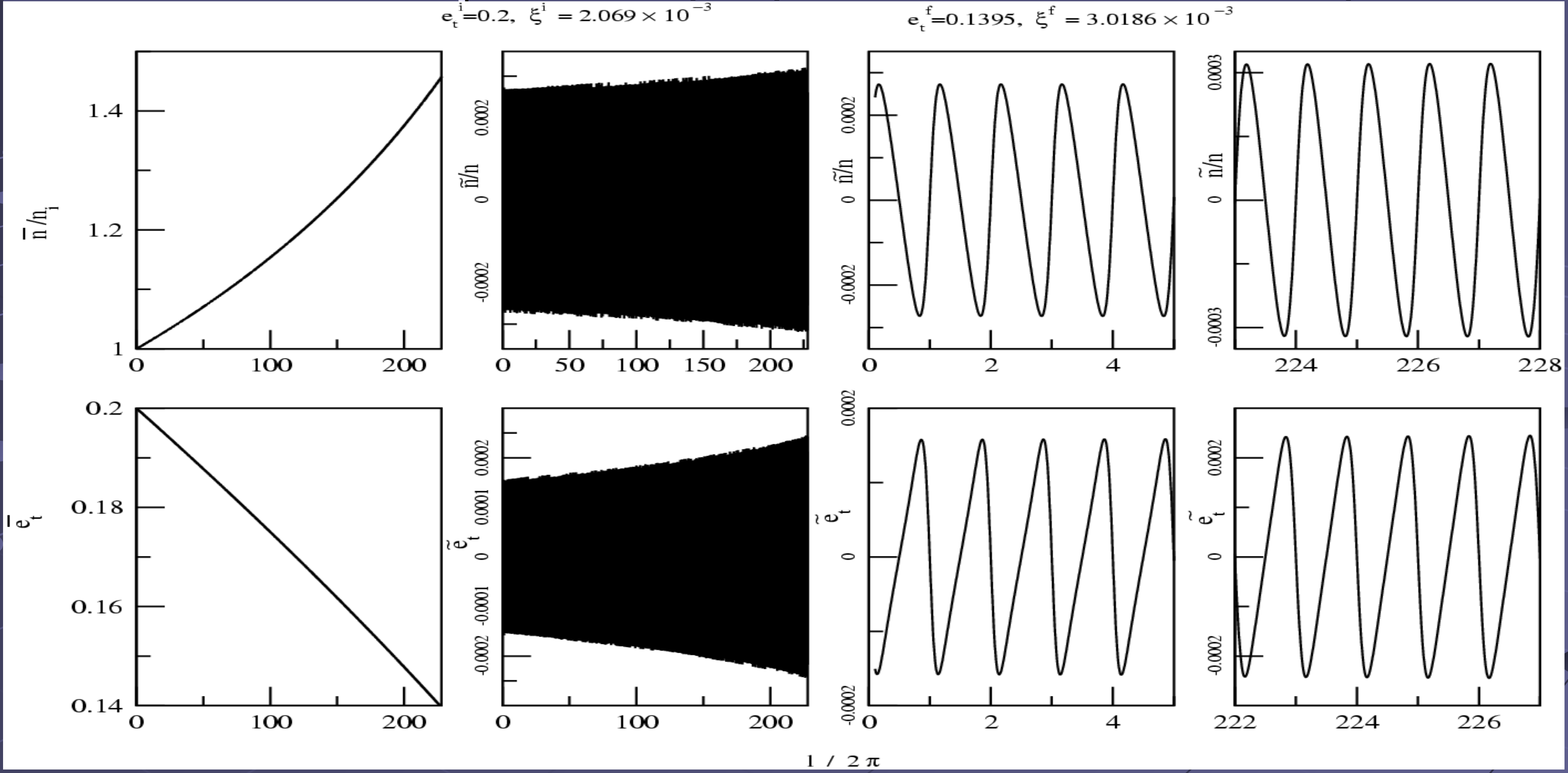



Berentzen, Preto, Berczik, Merritt, Spurzem 2008 (subm. ApJ)



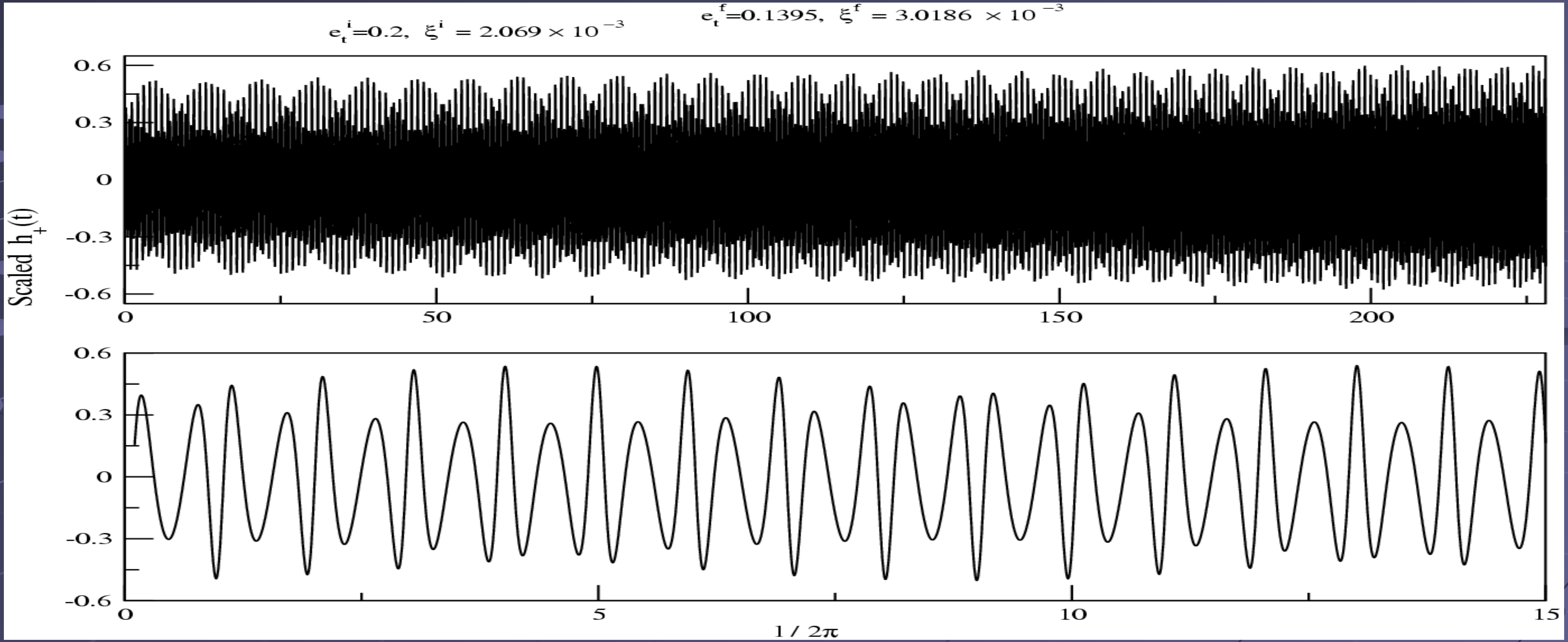



# Our templates (Gopakumar, Schäfer et al.)



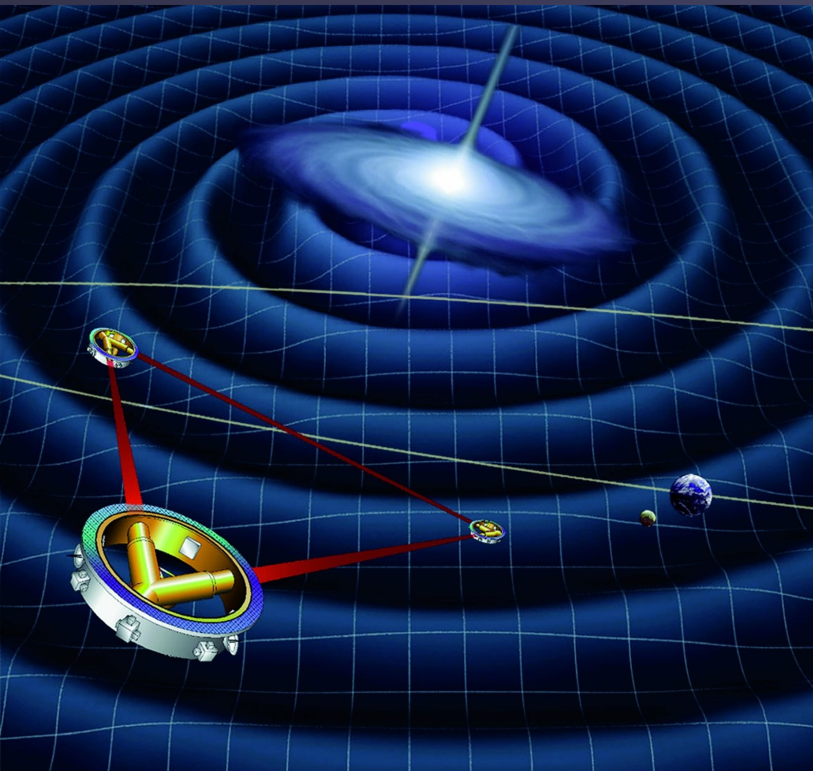

- Quasi-periodic variations in orbital elements
- We can handle arbitrary eccentricities

# Our templates (Gopakumar, Schäfer et al.)




- Plots of $h_+(t)$ showing 3 relevant time scales

Orbital evolution is NOT adiabatic (fully 3.5PN accurate)



1. German LISA cooperation
   funded by DLR (Danzmann et al.)

Heidelberg node: modelling
  astrophysical LISA sources

Jena node: computing waveforms

*Future Work:*

*PN3.5 Spin – Orbit Dynamics*

*PN3.5 Asymmetric GW - Recoil*

1. Virgo/EGO (VESF) cooperation, with LSF
   funded by EGO (Fidecaro et al.)

Heidelberg node: for modelling Virgo sources in Star Clusters (IMPRS)
  another one pending for funding by VESF

Jena node: corresponding waveforms in SFB/TR Gravitational Waves

## *Open Questions for Future Work*

- *How does SMBH (-binary) interact with AGN disk? Circumbinary, individual, exchange of torques between 0.1 and $10^{-3}$ pc?*

  *Stellar Dynamical Shrinking Time Scale*
  *Viscous Timescale of Disk (Thin/Thick)*
  *Migration of Black Holes / Star Formation in Disk*

- *Role of GR/NumRel Spin-Orbit Interaction/ Interaction with gas / Stellar Dynamics*

  *Spin Alignment BH-Disk Bardeen-Petterson Effect*

- *Boundary Conditions (Gas Supply by Outer Astrophysical Conditions, Galaxy, Cosmology)*

# Contents

## Two-Body Relaxation, Core Collapse, Interplay with Most Compact Binaries

So we need (among others):

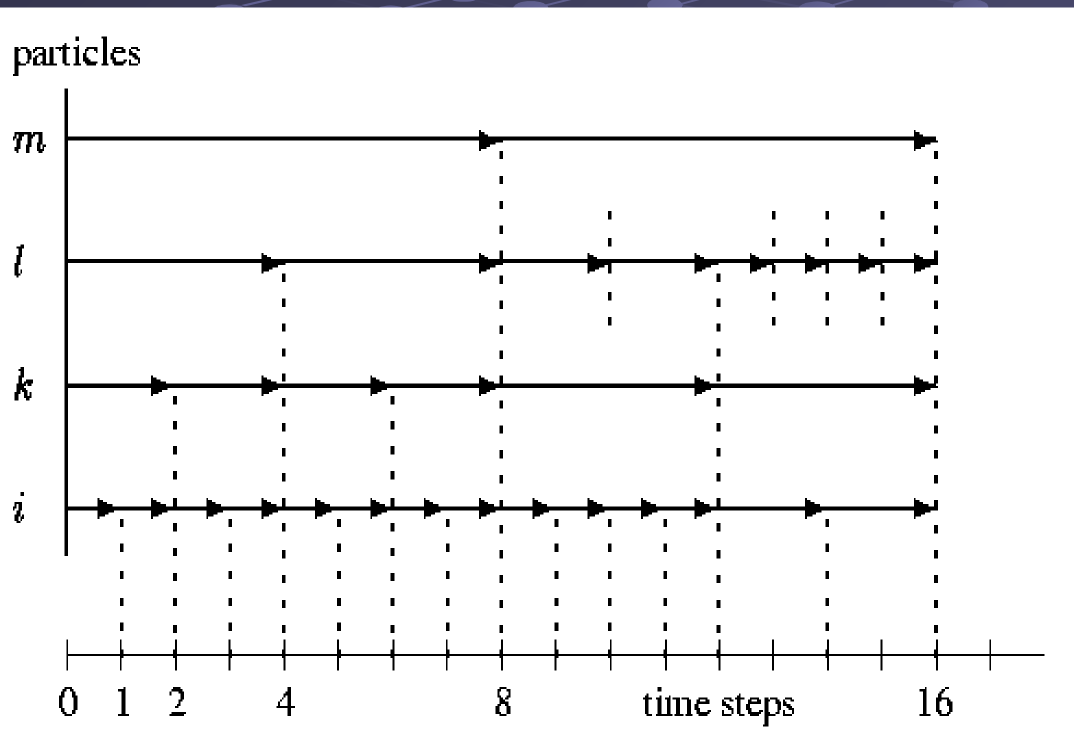
**Supercomputers and Supersoftware …**

- 2-body Regularization (Kustaanheimo & Stiefel 1965)
- 3-body Regularization (Aarseth & Zare 1974)
- Hierarchical Subsystems (Chain, Aarseth & Mikkola)
- Triples, Quadruples, Stability (Mardling & Aarseth 2001)

+ General Relativity....

(if separations get smaller than some critical radius, for galactic supermassive black holes of $10^7$ $M_{\odot}$ it is a=0.1 pc circular, or e.g. a=1.0 pc for e=0.9 )

# Parallelization and Software




Hierarchical Block Time Steps

*S.J.Aarseth, S. Mikkola*
*(ca. 20.000 lines):*

- Hierarchical Block Time Steps
- Ahmad-Cohen Scheme
- Regularisations
- 4th order Hermite scheme

- NBODY6 (Aarseth 1999)
- NBODY6++ (Spurzem 1999) MPI

$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}} \; .$$

# Parallelization and Software

*S.J.Aarseth, S. Mikkola (ca. 20.000 lines):*

- Hierarchical Block Time Steps
- Ahmad-Cohen Neighbour Scheme
- Regularisations

- NBODY6 (Aarseth 1999)
- NBODY6++ (Spurzem 1999) using MPI/shmem
- Parallel Binary Integration in Progress
- Parallel GRAPE Use in Progress
- φGRAPE – NBODY1-like on GRAPE clusters (Harfst et al. 2006)

# Parallelization and Software

- **Copy Algorithm**: parallelize work over block members
  replicate all data on all processors
  (Example: NBODY6++)
- **Ring Algorithm:** domain decomposition
  partial forces shifted
  blocking or non-blocking, systolic or hyper-systolic
  (Gualandris et al. 2005, Dorband et al. 2003)

***Our present workhorse:***

- **Mixed Algorithm**: φGRAPE – domain decomposition on GRAPE memories, copy algorithm for active particles
  (Harfst el al. 2006, astro-ph/0608125)

Note: Special hypersystolic quadratic algorithm (Makino 2002):

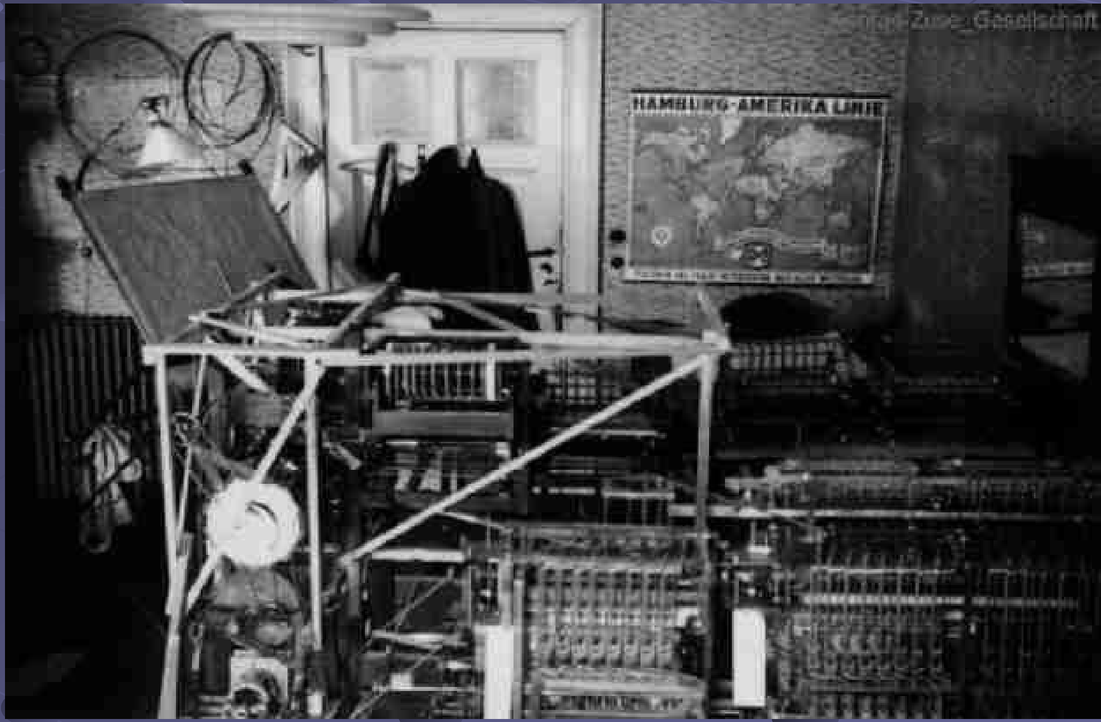
$$O(N/\sqrt{p}) + O(N^2/p)$$

# Contents

# Computational Physics

- Konrad Zuse (1910-1995) Berlin

**Invented freely programmable Computer**




**Z1 in parental flat 1936**

# Hardware

Experience with Custom PC Clusters

with special accelerator hardware:

Heidelberg/Mannheim (2x, GRAPE, FPGA, GPU)

Rochester, Vienna, Munich, Kiev, Japan, ...

Possible Future Plan: KIAA Peking, CAMK Warsaw

Possible Research with Mannheim:

MPRACE/FPGA for near field

Custom Network Fabric with Hypertransport


***Frontier Cluster***

*41x8=328 Cores*

*40 Tesla GPU*

*20 Tflop/s*

*Klessen, Banerjee,*

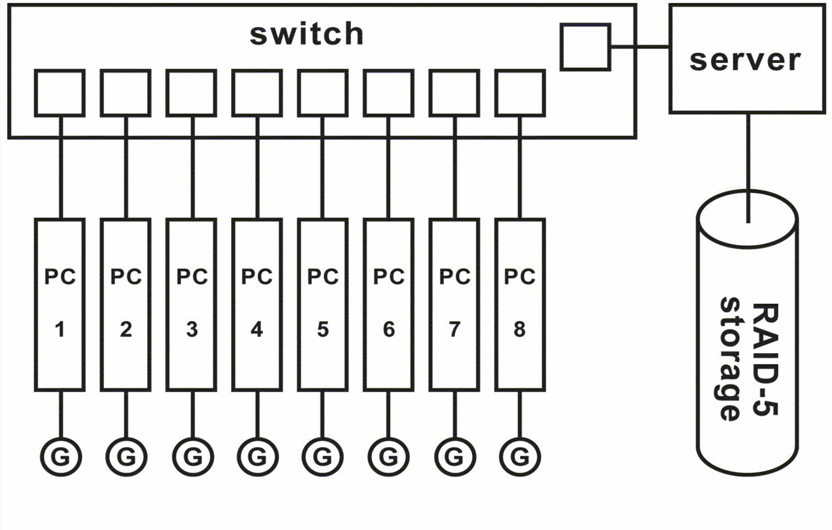
*Spurzem, Männer et al.*

# Hardware

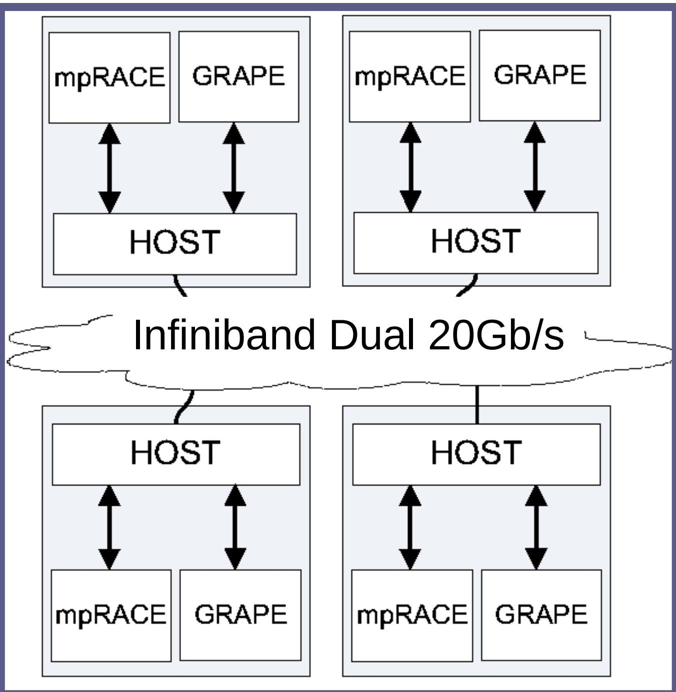
## ARI 32 node cluster

**GRACE = GRAPE + MPRACE**

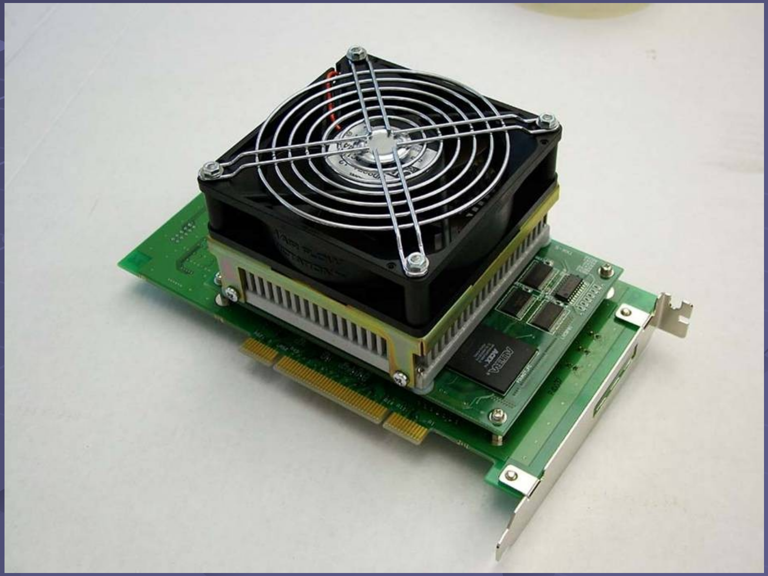
- 32 dual-Xeon 3.0 GHz nodes
- 32 GRAPE6a
- 14 TB RAID
- Infiniband link (10 Gb/s)
- Speed: ~4 Tflops
- N up to 4M
- Cost: ~500K USD
- Funding: NSF/NASA/RIT




- 32 dual-Xeon 3.2 GHz nodes
- 32 GRAPE6a
- 32 FPGA
- 7 TB RAID
- Dual port Infiniband link (20 Gb/s)
- Speed: ~4 Tflops
- N up to 4M
- Cost: ~380K EUR
- Funding: Volkswagen/Baden-Württemberg



# GRAPE Hardware

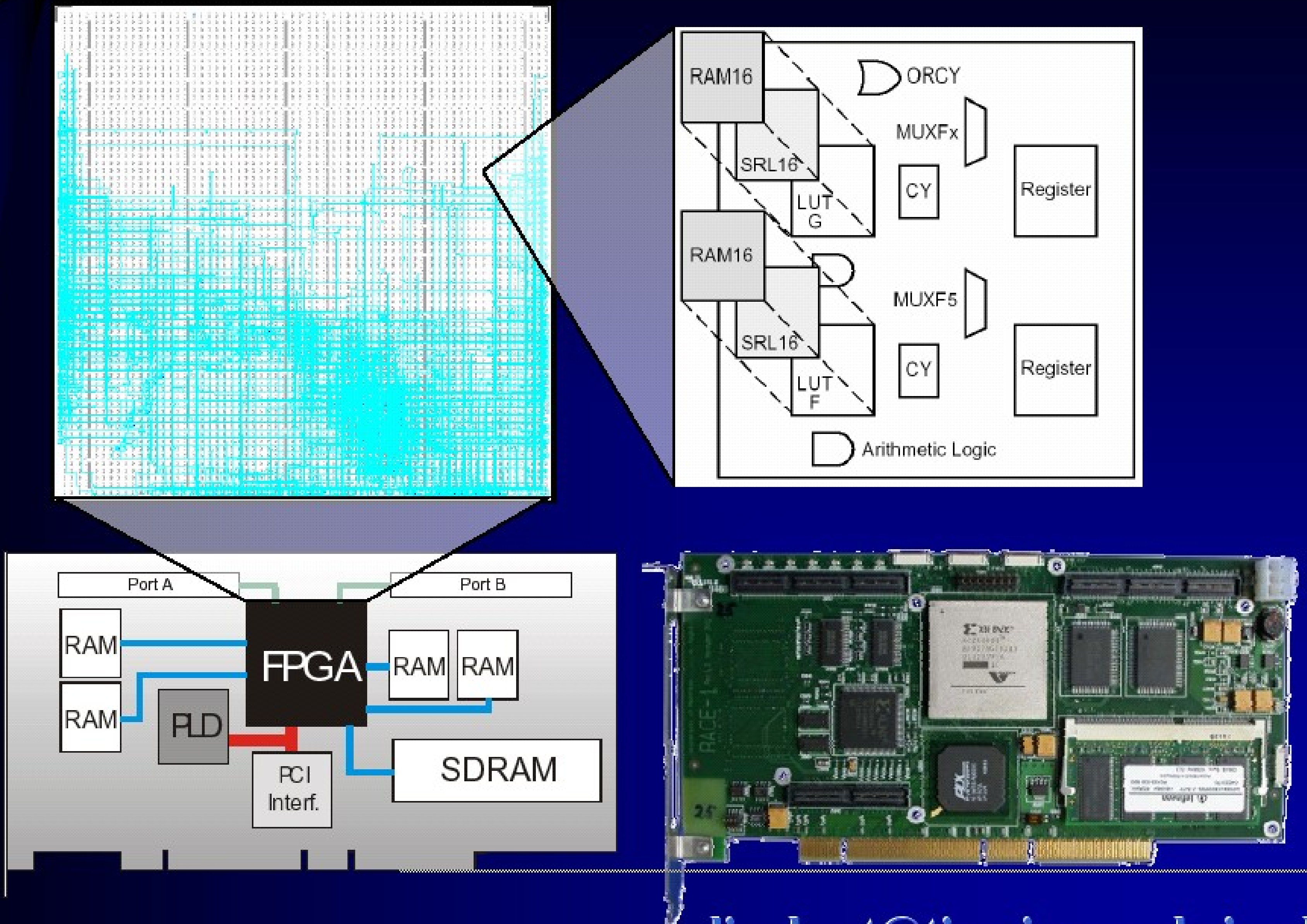


## **GRAPE6a PCI board**

**GRAPE6a - PCI Board for PC-Clusters, development of the University of Tokyo**

**~128 Gflops for a price ~5K USD**
**Memory for N, up to 128K particles**

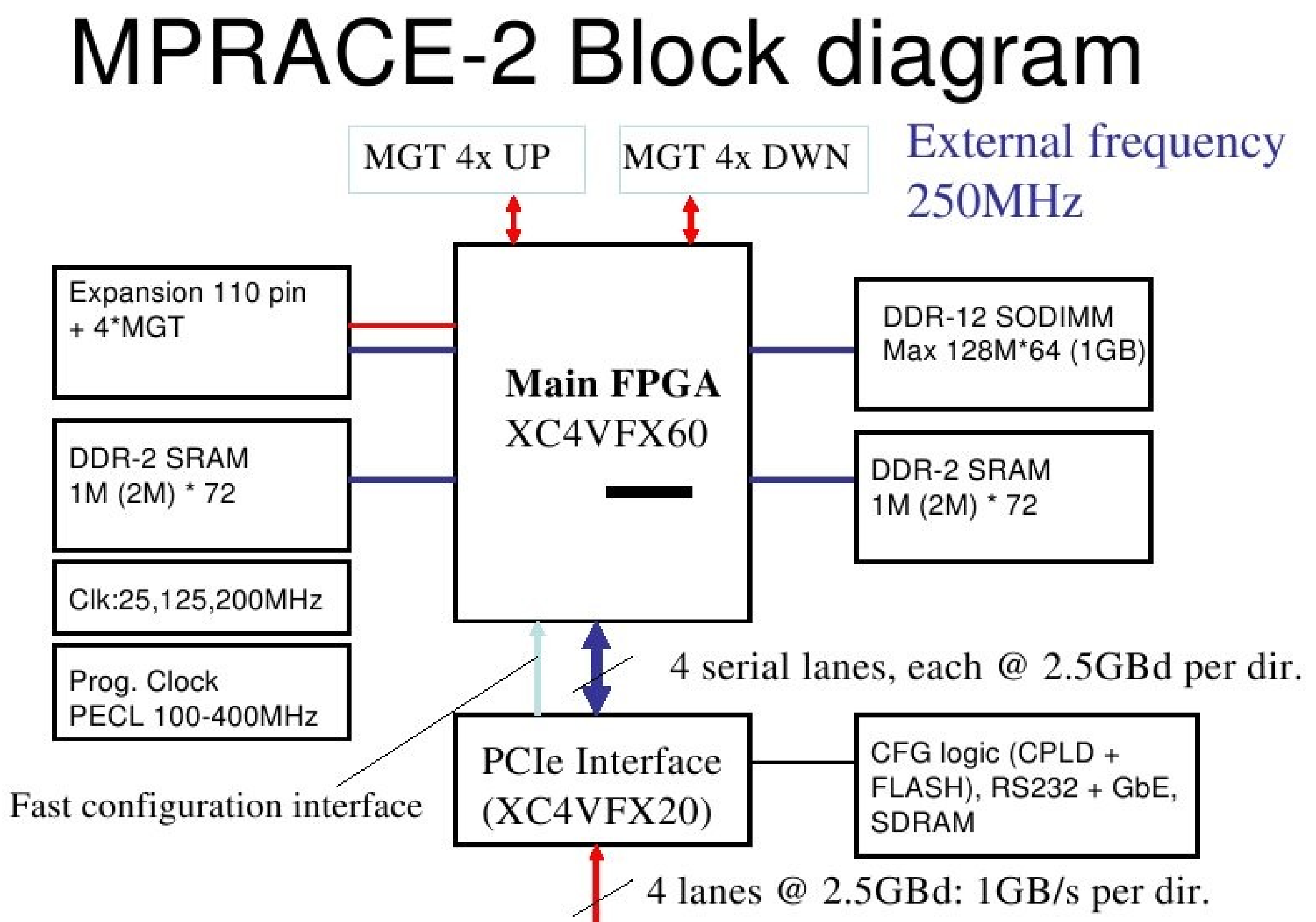
# FPGA-Plattform MPRACE



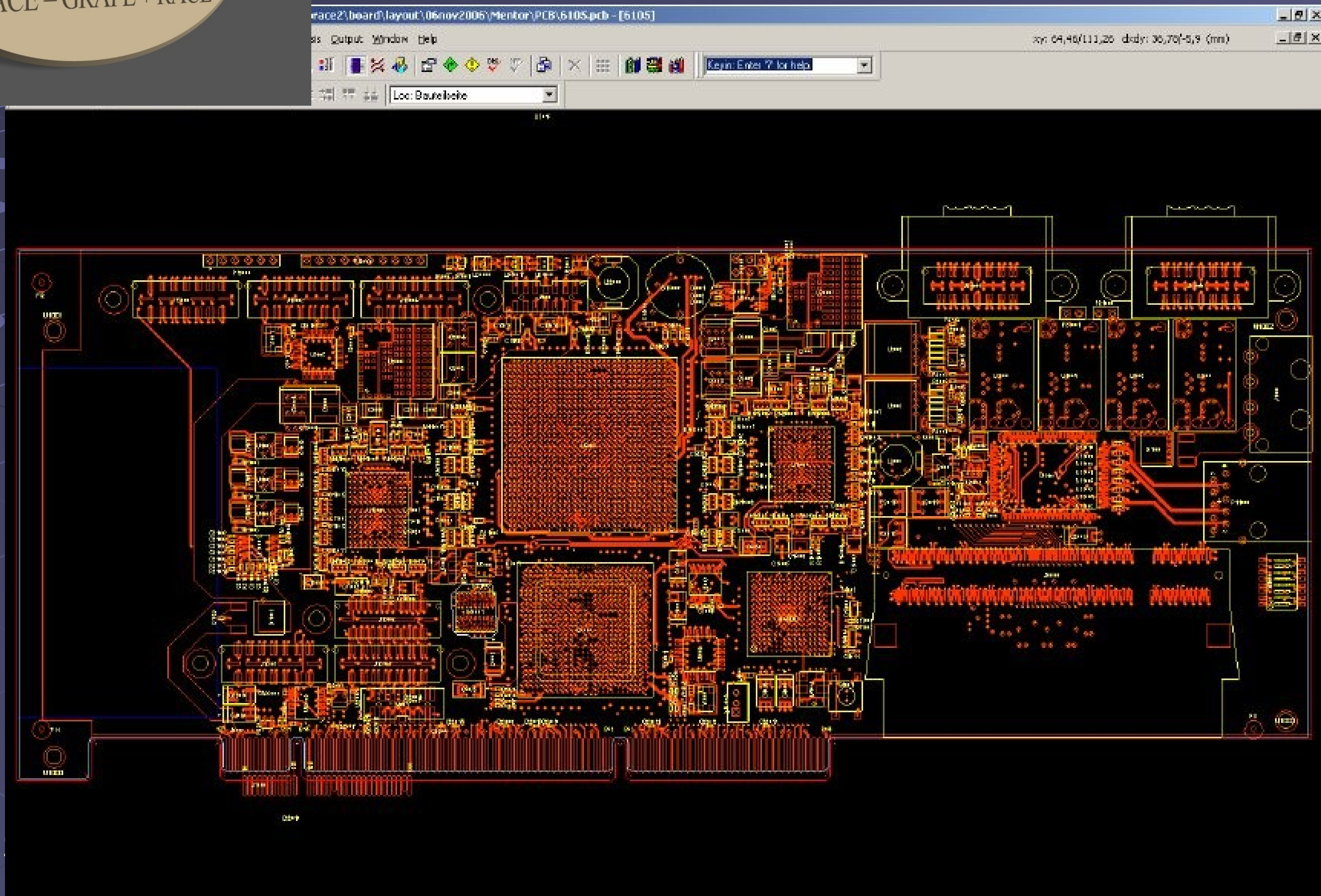

lienhart@ti.uni-mannheim.de

# Computers


GRACE
GRACE = GRAPE + RACE


MPRACE-2 Block diagram
MGT 4x UP
MGT 4x DWN
External frequency 250MHz
Expansion 110 pin + 4*MGT
Main FPGA XC4VFX60
DDR-12 SODIMM Max 128M*64 (1GB)
DDR-2 SRAM 1M (2M) * 72
DDR-2 SRAM 1M (2M) * 72
Clk:25,125,200MHz
Prog. Clock PECL 100-400MHz
4 serial lanes, each @ 2.5GBd per dir.
PCIe Interface (XC4VFX20)
CFG logic (CPLD + FLASH), RS232 + GbE, SDRAM
Fast configuration interface
4 lanes @ 2.5GBd: 1GB/s per dir.

# Computers


race2\board\layout\06nov2006\Mentor\PCB\6105.pcb - [6105]
Output Window Help
Keyin: Enter ? for help
Loc: Bauteilseite

# Hardware

***Frontier-Projekt***

Funded by Excellence Scheme Heidelberg Univ.
and NVIDIA

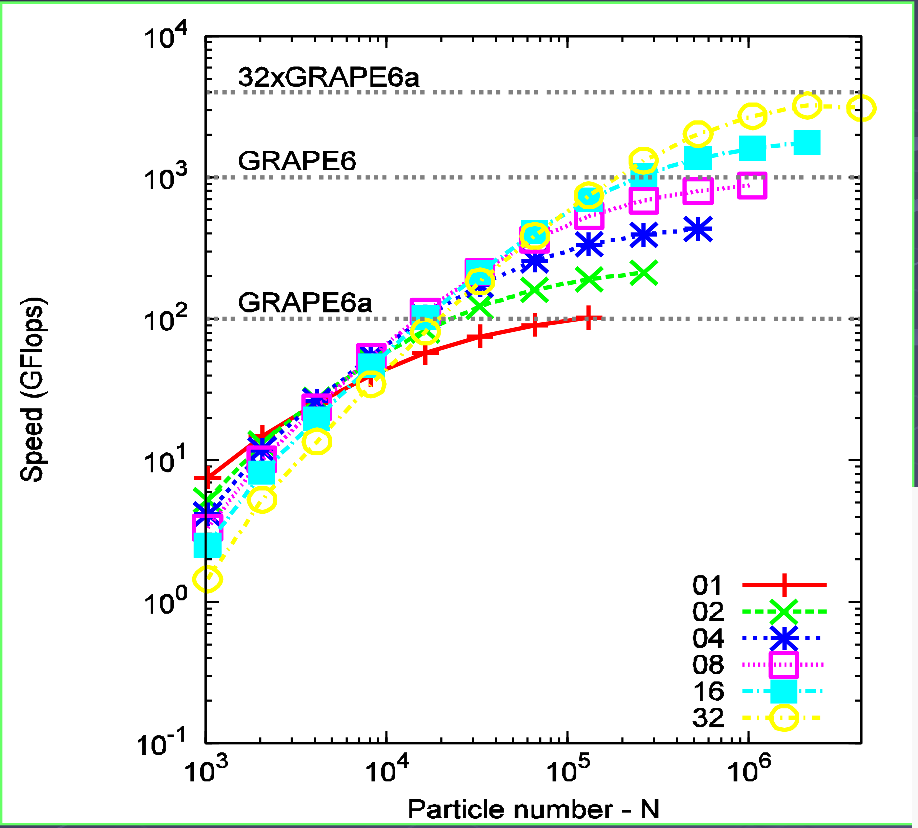

40 nodes with
Tesla GPU cards
Approx. 20 Tflop/s

Future Scale-Up?

**R4-Collaboration:**

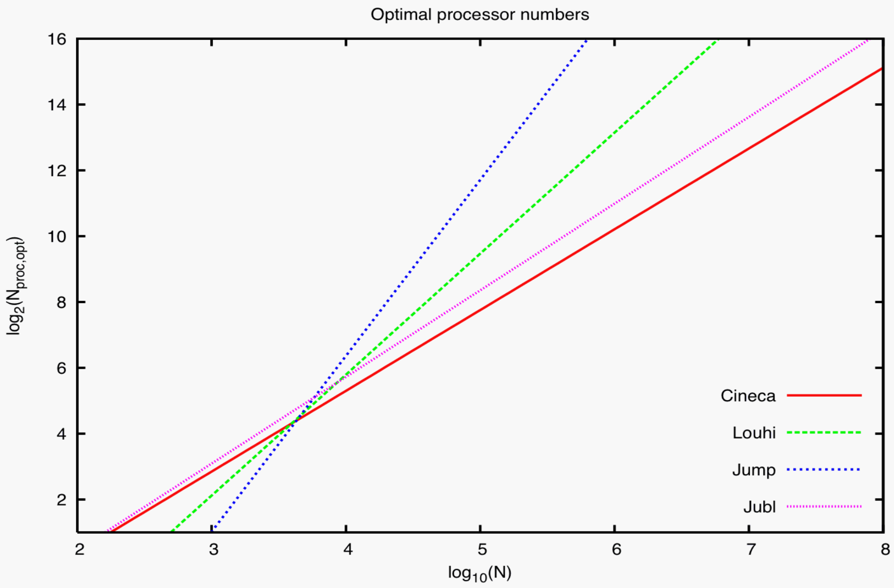
R. Banerjee
R. Klessen
R. Männer
R. Spurzem
I. Berentzen

# Software



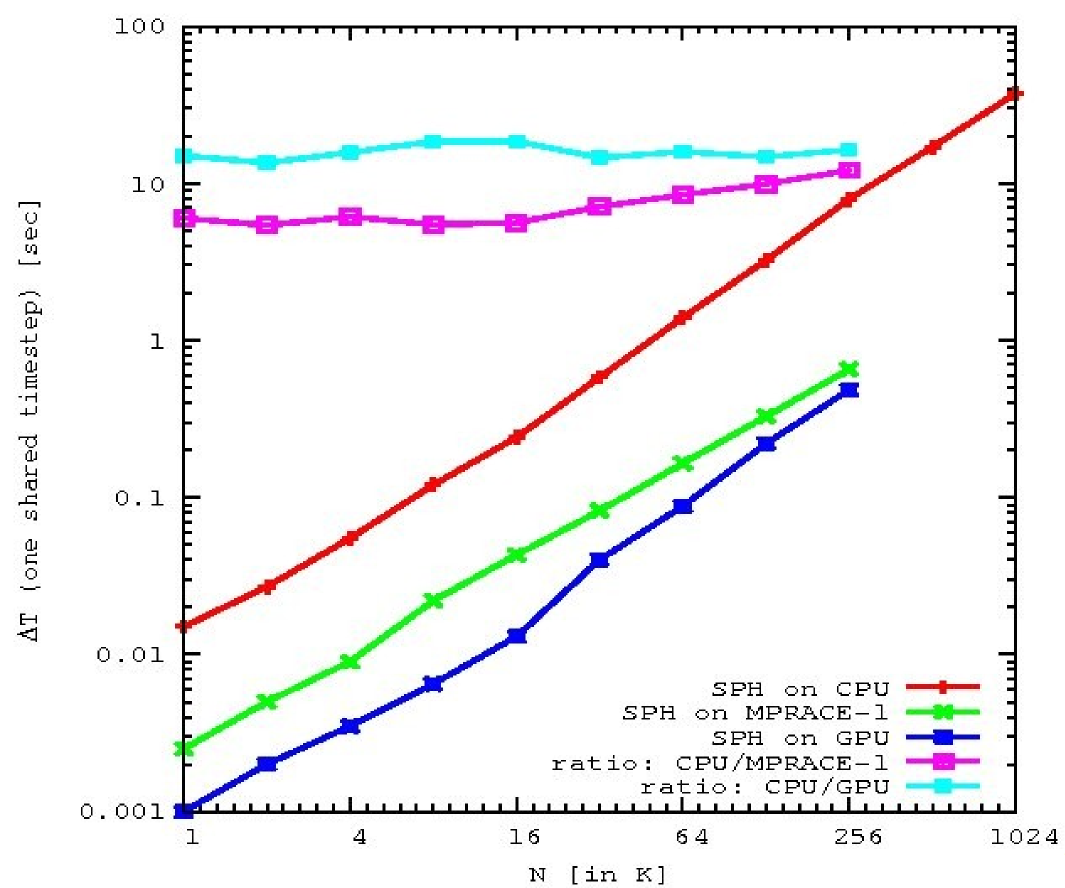

**Left: Using GRACE + φGRAPE:**
Harfst, Gualandris,
Merritt, Spurzem,
Portegies Zwart, Berczik
2007, New Astron.

***Right: DEISA / JUMP / JUBL***
*Benchmarks, Optimal Processor Numbers*
*With Oliver Porth, Andreas Ernst*
*And NIC Support Team*
*Marc-Andre Hermanns, Boris Orth,*
*Wolfgang Frings*



# Hardware

*Recent Development: GPU – Graphics Cards*




*GeForce 8800 GTX (NVIDIA)*
*Using CUDA Library*
*Special Interfaces and API from GRACE project ported.*

***Spurzem et al. 2007,*** *Jl.Phys.Conf.Ser.*
***Berczik et al. 2008, Marcus et al. 2008***
*(SPHERIC)*

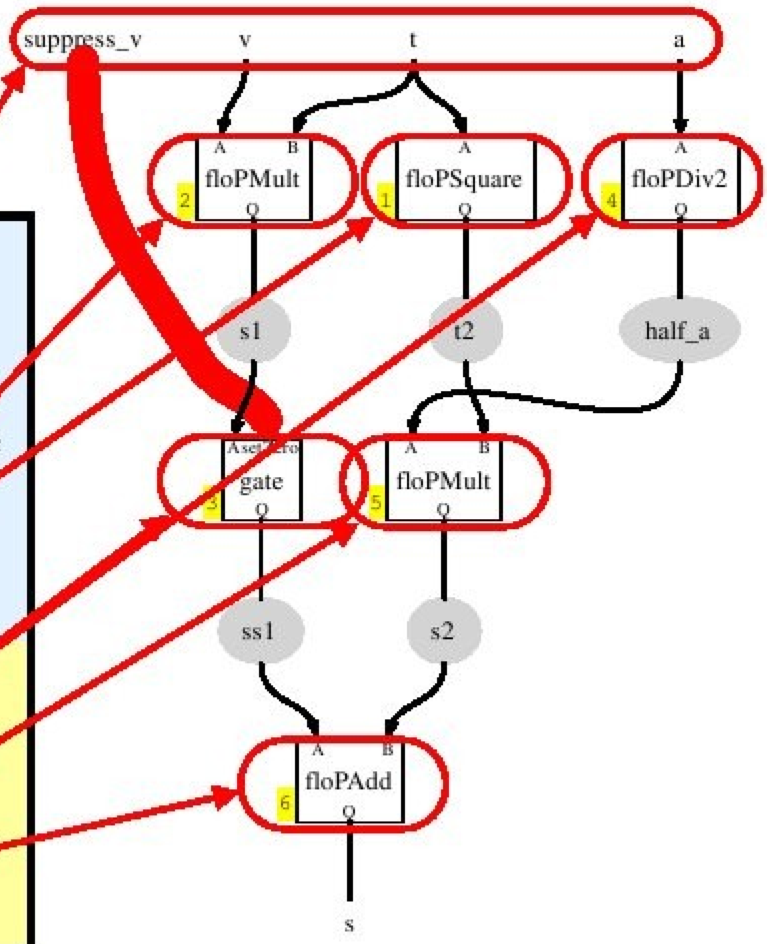
GRACE

GRACE = GRAPE + RACE

# Pipeline Generation on FPGA I (see talk by Gerhard Lienhart)

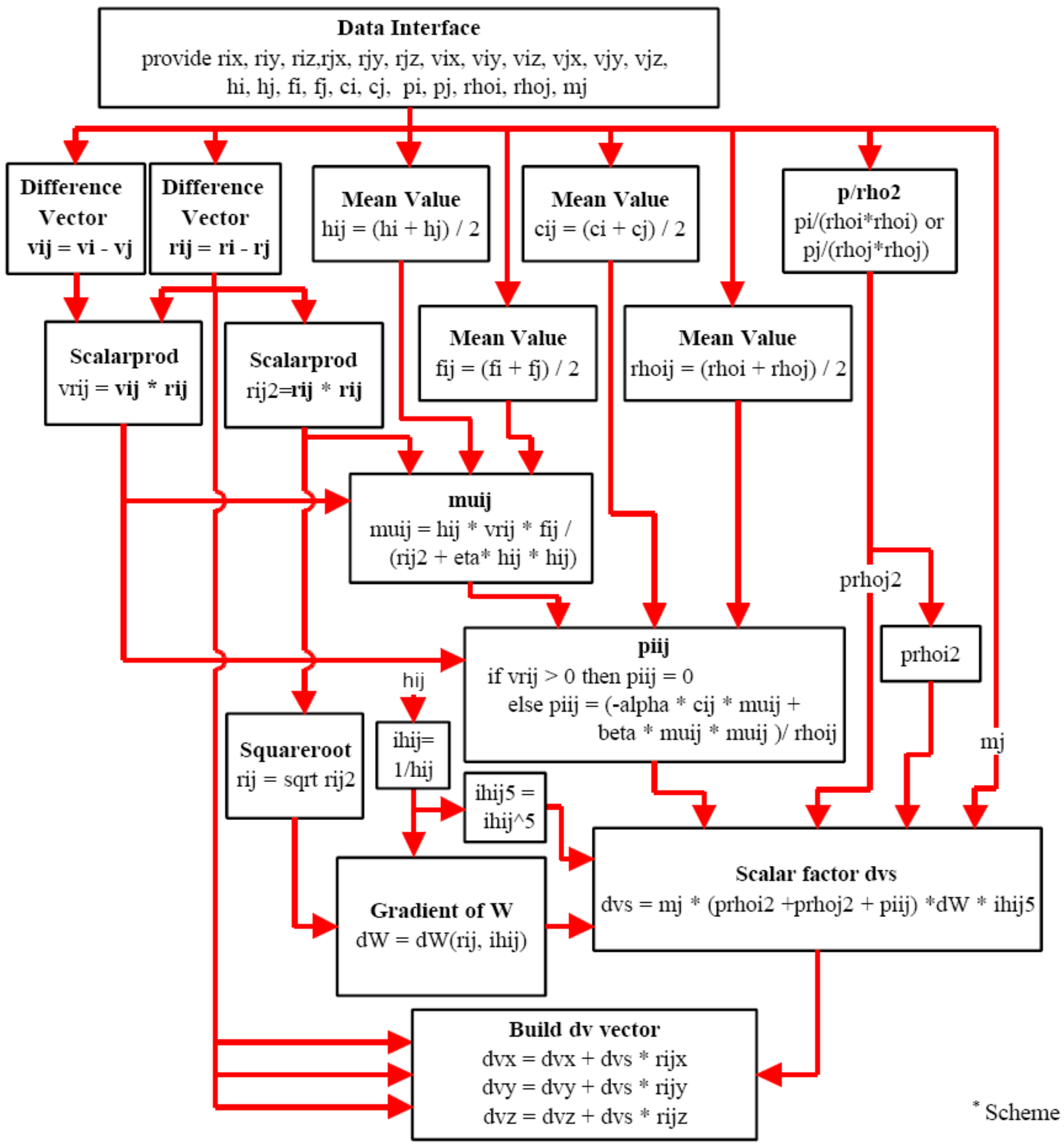
$$s = \begin{cases} \frac{1}{2} a\, t^2 + v\, t & : \quad \text{suppress\_v} = 0 \\ \frac{1}{2} a\, t^2 & : \quad \text{suppress\_v} = 1 \end{cases}$$

```
entity distance;
clock clk;

# parameters
floPValDef fpDef(signifLength=>24,
 expLength=>8,useSign=>1,useIsZero=>0);

# inputs
signal (suppress_v);
floPVal (v,a,t)(fpDef);

# calculate
t2 = <floPSquare> t;
s1 = v <floPMult> t;
ss1 = gated(s1,suppress_v);
half_a = <floPDiv2> a;
s2 = half_a <floPMult> t2;
s = ss1 <floPAdd> s2;
```



# Computers

**Pressure force pipeline:**




* Scheme doesn't show energy term

GRACE

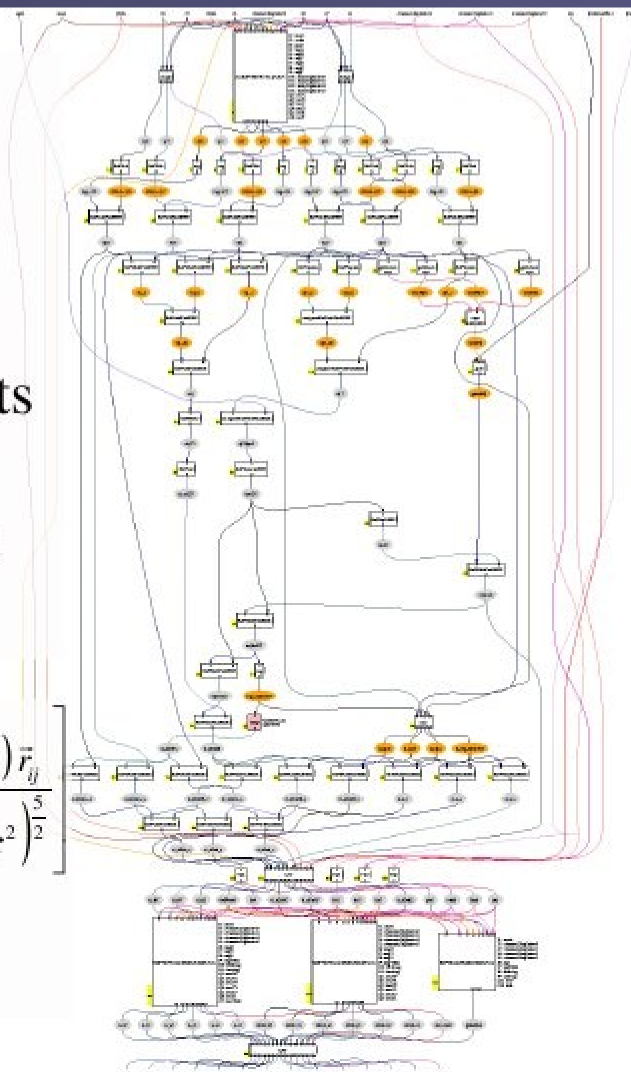
GRACE = GRAPE + RACE

# Pipeline Generation on FPGA II (see talk by Gerhard Lienhart)

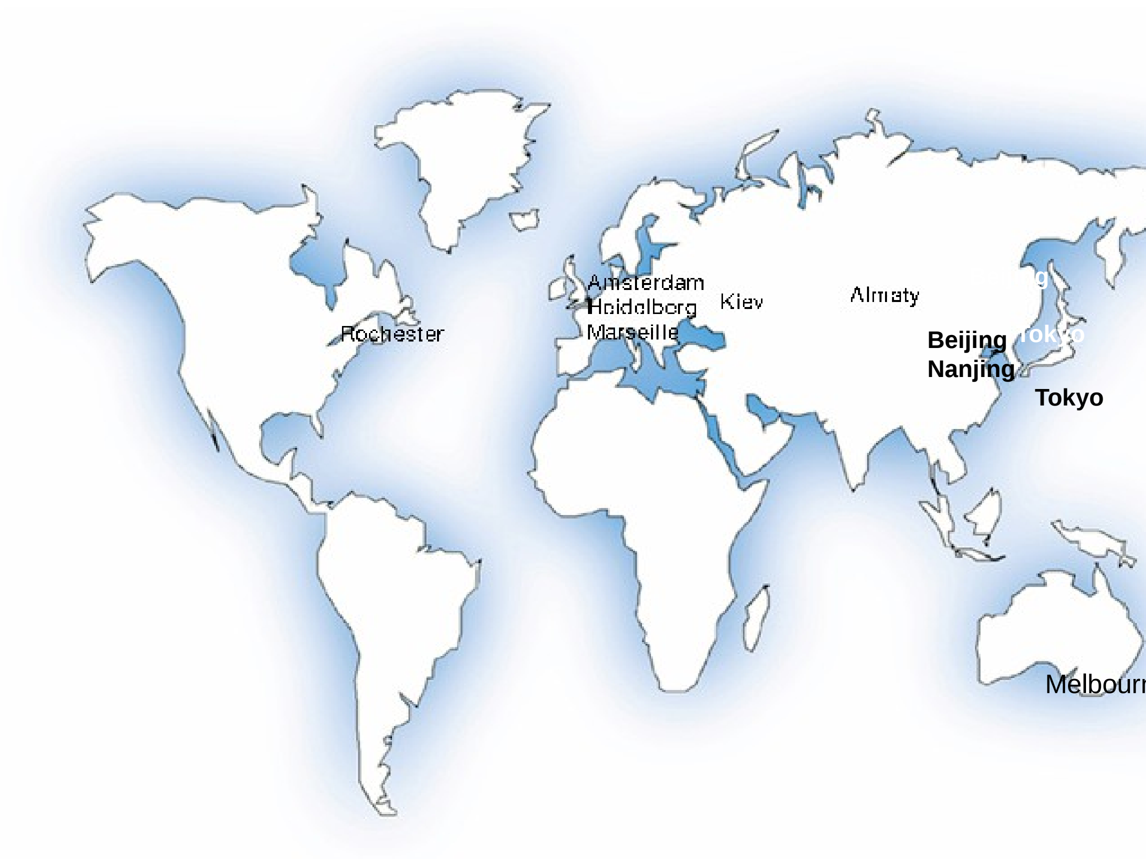
- Designs for Ahmad-Cohen neighbor scheme
- Neigborlist processing of gravity interaction similar to SPH design
- Processing of shared neighbor lists #iParticles >= 2*nrPhysicalPipes
- Pipeline provides a, aDot and pot
- software integration not finished

$$\vec{a}_i = \sum_j G m_j \frac{\vec{r}_{ij}}{\left(\left|\vec{r}_{ij}\right|^2 + \varepsilon^2\right)^{\frac{3}{2}}}, \quad \dot{\vec{a}}_i = \sum_j G m_j \left[ \frac{\vec{v}_{ij}}{\left(\left|\vec{r}_{ij}\right|^2 + \varepsilon^2\right)^{\frac{3}{2}}} - \frac{3\left(\vec{v}_{ij} \cdot \vec{r}_{ij}\right)\vec{r}_{ij}}{\left(\left|\vec{r}_{ij}\right|^2 + \varepsilon^2\right)^{\frac{5}{2}}} \right]$$

$$\phi_i = \sum_j G m_j \frac{1}{\left(\left|\vec{r}_{ij}\right|^2 + \varepsilon^2\right)^{\frac{1}{2}}}$$



# International Cluster-Grid Collaboration
## with GRAPE/GRACE/GPU



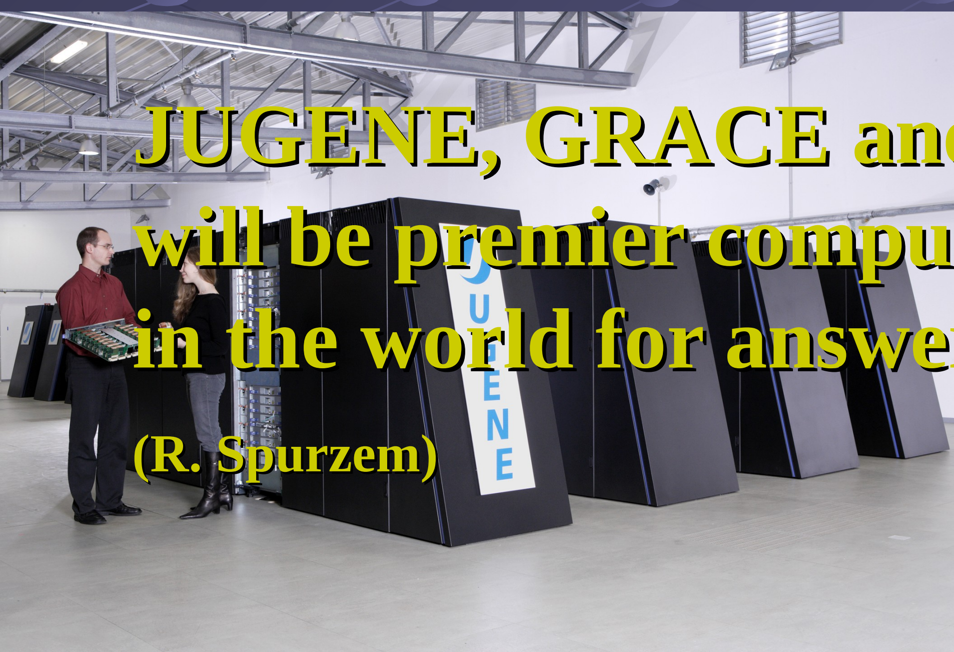

<u>Members of Astrogrid-D:</u>
ARI-ZAH Univ. Heidelberg, D
Main Astron. Obs. Kiev, UA

<u>Candidates for Intl. Grid-Cluster:</u>
Univ. Amsterdam, NL
Obs. de Marseille, F
Fessenkov Obs., Almaty, KZ
Kavli Instititute Astron. Astroph., Beijing, CN
Rochester Inst. Techn., USA
Centre for Supercomp. Japan
Monash Univ. Melbourne, Austr.

T. Prince (Caltech), Project Scientist, LISA:

**Two of the outstanding theoretical issues to be addressed if LISA is to succeed are:**

- “Formation and evolution of nuclear star clusters around supermassive black holes”
- “Understanding the fate of supermassive black holes in galaxy mergers”



**JUGENE, GRACE and future GGAPP will be premier computational instruments in the world for answering these questions.**

**(R. Spurzem)**